CHAPTER - 4

THEORY OF NEUTRON SCATTERING
4.1 Interaction of Neutrons with matter

On account of the uncharged nature of the neutrons, their absorption in matter does not take place in the same manner as in the case of charged particles. Interactions of charged particles with peripheral electrons will result in the ionization of the atoms according to the laws of inverse squares. But, neutrons, experience a force when they come within extremely close range of nuclei. The absorption of neutrons in matter is due to their short range interactions with nuclei. This interaction can be considered as a collision. The collision may be elastic or inelastic.

Elastic collision:

In this process, incident neutrons are merely scattered by the struck nuclei. Only a portion of energy of the incident neutrons will be transferred to the target nucleus. In the case of heavy nuclei, energy loss will be very small. But, the light nuclei reduce the energy of fast neutrons appreciably. The elastic collision is mainly responsible for the energy loss of neutrons in the hydrogenous materials. When neutrons with high energies are released in a medium, the neutrons begin to lose their energy by collisions with
nuclei in the medium. Under suitable conditions the slowing down process continues until the neutrons have velocity in equilibrium with the velocities arising from thermal agitation of the nuclei.

Consider an elastic collision between a neutron of initial energy $E$ and velocity $v$ with a target nucleus of mass $A$ at rest. Application of the laws of conversion of energy and momentum gives the ratio between the final energy of the neutron $E'$ and the initial energy;

$$\frac{E'}{E} = \frac{A^2 + 1 + 2A \cos \theta}{(A+1)^2}$$

(4.01)

Where $\theta$ is the scattering angle in the centre of mass system. In the elastic collision, great attention is given to the loss of energy by the neutrons. There are two limiting cases for the value of scattering angle $\theta$ in the centre of mass system. For no scattering ($\theta=0$), the Eq.(4.01) gives $E'/E = 1$.

The maximum energy loss occurs for a head-on collision ($\theta=180^\circ$). When $\theta=180^\circ$, one can get
It is important to note that, the neutron gives all of its energy to the struck proton \((A=1)\) in a head-on collision.

For neutron energies below 10 MeV. the scattering is mostly \(S\) wave scattering and hence largely independent of scattering angle \(\theta\).

**Inelastic collision:**

Can be subdivided into two categories;

a) Partial  

b) Total

The first kind is called "inelastic scattering". In this process, the neutron is scattered anomalously and the struck nucleus undergoes internal changes and is excited to higher energy level. In the inelastic scattering, the incident neutrons give up a part of their initial energy to the struck nucleus. The so called inelastic scattering, a term used to describe the re emission of a neutron which has been captured by a nucleus. If the kinetic energy of the incident neutron is sufficient to excite the lowest level of the
nucleus, the nucleus will be left in an excited state after the neutron depart and the neutron is emitted with comparatively less energy than that with which is entered the nucleus.

The inelastic scattering of neutrons can be detected by measuring the energy of the scattered neutrons or by the gamma rays that are emitted by the nucleus when it returns to a state of lower energy.

The second kind of inelastic collision is known as the capture process. The capture process plays an important role in the production of artificially radioactive nuclides by the neutron bombardment.

In the investigation of neutron interaction with matter in detail, it is necessary to have a quantitative measure of the probability of a given interaction. This quantity must be one which can be measured experimentally and calculated in such a way that the theoretical and experimental values can be compared readily. The quantity that is often used for this purpose is the cross section of a nucleus for a particular interaction and is denoted by $\sigma$ with an appropriate subscript. This $\sigma_a$ represents the cross section for neutron absorption, defined as the number of neutrons that are absorbed or disappear per cubic centimetre per second for unit
incident flux, $\sigma_{el}$ the cross section for elastic scattering, and $\sigma_{in}$, the cross section for inelastic scattering and so on. The total number of neutrons removed from the beam is obtained by adding the numbers removed by all the processes which can take place. The cross section which corresponds to the effect of all the possible processes is usually known as total cross section and is designated by symbol $\sigma_t$.

It is simple and convenient to introduce the cross section as a target area and to get a rough idea of its magnitude by calculating the geometrical cross section. However, this procedure must not be taken seriously. The experimental meaning of cross section comes from its use as a measure of the number of nuclear events which occur under a given set of experimental conditions.

The values of nuclear cross sections range from small fractions of a barn to thousands of barns, and these values often differ greatly from the geometrical cross section.

### 4.2 Resonance Scattering

The theoretical estimation of total cross section for the intermediate and heavy nuclei with low energy neutrons can be obtained by Breit-wigner formula. The Breit-wigner formula explains the neutron scattering through the formation of resonance
levels. To obtain a quantitative understanding of the formation of resonance levels, the nuclear potential seen by the captured neutrons is represented by a square well potential. The oscillatory wave-functions inside and outside the wall must be matched smoothly. Depending on the phase of the wave function inside the nucleus, the smooth matching can be result in substantial variations between the relative amplitudes of the wave functions inside and outside the nucleus. The resonances occur at more or less well defined energies of neutrons scattered from nucleus. The resonances can be viewed as non stationary states of nucleus. When only one partial wave with angular momentum ‘l’ is important for the resonant state, there will be a scattering resonance where \( \eta_l = \exp (2i\delta_l) = -1 \), corresponding to a phase shift \( \delta_l = \pi/2 \). The Breit-wigner formula for the shape of single, isolated resonance is given by:

\[
\sigma_{sc} = \frac{\pi \Gamma^2}{(2l+1)k^2} \frac{1}{(E-E_0)^2 + \Gamma^2/4} \quad (4.03)
\]

Where \( \Gamma \) is the width of the resonance state i.e., the range of energies over which the resonance has a large effect on scattering, \( E_0 \) determines the location of the resonance, \( E \) is the energy of the incident neutron and \( K \) is the wave vector of the incident neutron.
The cross section in the neighborhood of an isolated resonance is given by

\[ \sigma_{sc} = \frac{\pi (2l + 1)}{K^2} \left( \exp(-2i\delta_{p}) - 1 + \frac{i\Gamma}{(E - E_0)^2 + i \Gamma/2} \right)^2 \]  

(4.04)

The \( \delta_{p} \) is the phase shift of the partial wave of angular momentum \( l \), due to potential scattering. The term \( \exp(-2i\delta_{p}) - 1 \) represents the potential scattering.

When \( E - E_0 \gg \Gamma/2 \), the potential scattering term dominated and then scattering cross section approaches the value:

\[ \sigma_{sc} \approx \sigma_{pot} = \frac{4\pi (2l + 1)}{K^2} \sin^2 \delta_{p} \]  

(4.05)

At resonance \( E = E_0 \), the resonant term dominates and then scattering cross section approaches the value:

\[ \sigma_{sc} \approx \sigma_{res} = \frac{4\pi (2l + 1)}{K^2} \]  

(4.06)

When there is interference between potential scattering and resonance scattering, the shape of the cross section becomes more complicated. This leads to destructive interference. The phase shift \( \delta_{l} \) which always enters the expression for the elastic scattering cross section, increases by \( \pi \) radians as the energy traverses each
resonance. This can be seen by the expression for the resonant contribution to the phase shift

\[ \delta_r = \cot^1 \left[ \frac{(E - E_0)}{\Gamma/2} \right] \quad (4.07) \]

The smaller the width \( \Gamma \), the more rapidly occurs the traversal of the phase shift by \( \pi \) radians.

The phase shift for potential scattering is different for each case, it increases by \( \pi \) radians as the energy traverses the resonant energy \( E_0 \). The position of \( E_0 \) is defined to be the energy at which the resonant portion of the phase shift has increased by \( \pi/2 \) radians.

The radioactive capture reactions show a resonant structure. The most important reactions between neutrons of low energy and nuclei of intermediate and heavy atomic weight are elastic scattering and radioactive capture \( (n, \gamma) \) reaction. The radioactive capture cross section \( \sigma(n, \gamma) \) is given by;

\[ \sigma(n, \gamma) = \frac{\pi}{K^2} \frac{\Gamma_n, \Gamma_\gamma}{(E-E_0)^2 + (\Gamma/2)^2} = \frac{\lambda^2}{4\pi} \frac{\Gamma_n, \Gamma_\gamma}{(E-E_0)^2 + (\Gamma/2)^2} \quad (4.08) \]

Where \( \Gamma_\gamma \) is the partial level width for the emission of \( \gamma \)-ray; \( \Gamma_n \) is the partial level width for the entrance of neutron, and \( \Gamma \) is the
total level width. The Breit-wigner formula is an example of the application of wave mechanics to nuclear physics, and it is typical of such applications that the wavelength $\lambda$ of the incident neutron appears in the formula.

If the neutron width $\Gamma_n$ is proportional to the velocity $v$ of the neutron and energy of the incident neutron $E$ is very very less than the resonant energy $E_0$, then the formula for $(n, \gamma)$ reaction reduces to

$$\sigma(n, \gamma) = \frac{\lambda^2 \Gamma_n \Gamma_\gamma}{4\pi \left( E_0^2 + \Gamma^2/4 \right)} \frac{\text{Constant}}{v} \quad (4.09)$$

Thus the $\sigma(n, \gamma)$ is inversely proportional to the velocity of the incident neutron.

The cross section $\sigma(n, \gamma)$ for intermediate and heavy nuclei decreases as velocity of the incident neutron increases. The $(n, \gamma)$ reaction is important for scattering of low energy neutrons by intermediate and heavy nuclei. When highly excited compound nucleus is formed by capture of an incident neutron by a target nucleus, the strong interactions among the constituent particles produce the effect that the excitation energy is immediately distributed among a large number of them the transferred frequently
from one to another. In the case of heavy nuclei, the excitation levels are sharp, i.e., the distance between neighbouring levels will be of the order of few eV. In the case of light nuclei, the distance between the neighbouring levels will be much larger. The large number of closely packed excitation levels in the compound nucleus explains the frequent occurrence of resonance of slow neutrons. The resonance levels will be different from nucleus to nucleus, and hence each nucleus have its own groups of neutrons which can be easily captured. The $1/v$ law holds if the neutron energy is comparatively smaller than the width of the resonance level. For heavy nuclei, the resonance levels are sharp and the $1/v$ law holds only for a very narrow energy region. But, for light nuclei, it holds up to rather high energy region corresponding to the greater width of the resonance levels.

4.3 Method of Partial Wave Analysis

The theoretical estimation for total cross section for intermediate and heavy nuclei with high energy neutrons in the absence of absorptive effects can be obtained by the method of partial wave expansion. According to this method, if the neutrons of momentum $P = hK$ interact with impact parameter $b$ (perpendicular
distance from the centre of target nucleus to the line of flight of incident neutron) than the relative angular momentum is given by

\[ \frac{l}{\hbar} = p \cdot b \quad (4.10) \]

\[ b = \frac{l}{h} = \frac{l}{\lambda} \quad (4.11) \]

Where \( \lambda = \frac{\lambda}{2\pi} \) is the reduced deBroglie wave length of the incident neutron.

In quantum mechanics angular momentum \( l \) can be expressed in integer units. The neutrons with angular momenta \( 0h \) and \( l \hbar \) will interact through impact parameters between 0 and \( \lambda \), and thus over a cross section of \( \pi \lambda^2 \). With \( h \leq l \leq 2h \), the cross section is a ring of inner radius \( \lambda \) and outer radius \( 2\lambda \) and the cross section gets a value \( 3\pi \lambda^2 \). Thus, interaction area can be divided into a number of zones, each corresponding to a specific angular momentum \( l \) and each having area \( \pi [(l+1)\lambda^2] - \pi \lambda^2 = (2l+1)\pi \lambda^2 \). The maximum impact parameter for scattering can be estimated about \( R \) (the sum of radii of the incident neutron and target nucleus).

The maximum angular momentum \( l \) is estimated about \( R/\lambda \).

The elastic scattering cross section is given by
\[
\frac{R}{\lambda_c} = \sum_{l=0}^{\infty} (2l + 1) \pi \lambda_c^2 = \pi (R + \lambda)^2 \tag{4.12}
\]

If there are other processes in addition to elastic scattering, the total microscopic cross section for removal of a neutron from the incident beam can be written as

\[
\sigma_t = \sigma_s + \sigma_r \tag{4.13}
\]

The first term is the microscopic cross-section for elastic scattering and the second term is the reaction cross section. The reaction cross section is the cross section for all events in which the internal state of the product nucleus differs from that of the target nucleus. It must be noted that inelastic scattering is included in \(\sigma_r\). Here, "reaction" means all processes except elastic scattering.

The maximum value attributed to \(\sigma_s\) is \(\pi (R + \lambda)^2\) and the maximum value attributed to \(\sigma_r\) is \(\pi (R + \lambda)^2\). Therefore the total microscopic cross section is given by

\[
\sigma_t \approx 2\pi (R + \lambda)^2 \tag{4.15}
\]
4.4 Optical Model

Optical model is used to account elastic scattering in the presence of absorptive effects. The optical model is also called the 'cloudy crystal ball model'. In the optical model, the nucleus is not 'black' to the wave representing the incident neutron, but it acts like a gray sphere which partly absorbs and partly reflecting the incoming wave. The single particle and compound nucleus pictures are combined successfully in the 'optical model'. In this model, the scattering can be expressed with simple potential

$$U(r) = \begin{cases} -V_0 - iW_0 & \text{for } r < R \\ 0 & \text{for } r > R \end{cases}$$

(4.16)

Where $r$ is the radial distance from the centre of the potential well and $R$, the value of $r$ such that the potential vanishes for $r > R$.

The real part $V_0$ is responsible for the elastic scattering and the imaginary part $W_0$ is responsible for the absorption.

The outgoing scattered wave can be taken in the form of $e^{ikr}$ with $R$

$$K = \frac{\sqrt{2m(E + V_0 + iW_0)}}{\sqrt{\hbar^2}}$$

which follows from solving Schrödinger equation

in the usual manner.
The theory accounts very well for the behaviour of neutron cross section with values:

\[ V_0 = 42 \text{ MeV} \]

\[ W_0 = \xi V_0, \]

\[ \xi = 0.03 \text{ to } 0.05 \]

\[ R = 1.45 \text{ A}^{1/3} \text{ fm} \]

For the chosen potential, the Schrödinger equation can be solved and equating boundary conditions at \( r = R \), gives the complex scattering amplitudes \( \eta = \exp(2i\delta) \). Which can be used to compare calculated cross sections with experimental values.

### 4.5 The Scattering Cross Section for Hydrogen

The scattering cross section for hydrogen with high energy neutrons is obtained by using the formula:

\[
\sigma = \frac{4\pi h^2}{M(E/2 + |\epsilon|)} \tag{4.17}
\]

Where \( \epsilon \) is the binding energy of the deuteron, \( E \) is the energy of incident neutron and \( M \) is the reduced mass of proton – neutron system.
After calculating the total microscopic cross section $\sigma_i$ by an appropriate theory, the attenuation expected is given by $\exp(-N\sigma_i x)$ where $N$ is the number of nuclei per cubic centimetre of the target material and $x$, the thickness of the target material.

### 4.6 Macroscopic Cross Section & Relaxation Length

In the transmission measurements the total microscopic cross section $\sigma_i$ is used to describe the removal of neutrons from a collimated beam of neutrons in crossing the thickness $dx$ of material, the neutrons will encounter $Ndx$ nuclei per unit surface area of the neutron beam or material. If $\sigma_i$ is the total cross section, then the loss intensity $I$ is

$$dI = -I \sigma_i Ndx \quad (4.18)$$

and the intensity decreases with absorber thickness according to an exponential relationship

$$I = I_0 \exp(-N\sigma_i x) \quad (4.19)$$

Where $I_0$ is the intensity of the incident neutron beam and $I$, the intensity of the neutron beam after traversing a thickness $x$ of the material.
The symbol $\sigma_i$ just signifies the nuclear cross section. The total macroscopic cross section $\mu_i$ is given by $N\sigma_i$.

The total macroscopic cross section $\mu_i$ is expressed in cm$^{-1}$ and is therefore an absorption coefficient. In terms of $\mu_i$, the Eq. (4.19) becomes

$$I = I_0 \exp (-\mu_i x) \quad (4.20)$$

The average distance neutrons will travel before capture is given by $1/\mu_i = \lambda$

Where $\lambda$ is the mean free path for capture in a target for which the macroscopic cross section is $\mu_i$ in terms of $\lambda$ the Eq. (4.20) becomes

$$I = I_0 \exp (-x/\lambda) \quad (4.21)$$

Hence for a beam of neutrons, $\lambda$ is obviously the distance the neutrons will travel before they are reduced to $1/e$ times their initial intensity. The $\lambda$ is frequently called the relaxation length.
4.7 Macroscopic Removal Cross Section

Attenuation of fast neutrons can be accomplished by use of a hydrogenous shielding material. The attenuation through hydrogenous shielding material or through backed by hydrogenous material can be determined by use mass removal cross section\(^{[4.02]}\). To determine the total macroscopic removal cross section for a compound or mixture, the following formula is used;

\[
\mu_{\text{Rem}} = \sum_{i=1}^{N} \rho_i \mu_{\text{Rem}} \quad (4.22)
\]

Where \(\mu_{\text{Rem}}\) is the macroscopic removal cross section for the compound or mixture, \(\rho_i\) is the density of \(i^{\text{th}}\) element and \(\mu_{\text{Rem}}\) is the macroscopic removal cross section of the \(i^{\text{th}}\) element.