CHAPTER I

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"Mathematics should be visualised as the vehicle to train a child to think, reason, analyse and articulate logically. Apart from being a specific subject, it should be treated as a concomitant to any subject involving analysis and reasoning".

National Education Policy (1986)

Mathematics is the backbone of science and technology and no nation can hope to achieve any measure of scientific and technological advancement without proper foundation in school mathematics. According to Abiodun (2005), while science is the bedrock that provides the spring board for the growth of technology, mathematics is the gate and key to the sciences. It is a core subject that is compulsory at primary and secondary schools, and in addition, a credit level pass in mathematics at Senior Secondary Certificate Examination (SSCE) is a requirement for admission into all science related courses such as medicine, pharmacy, Engineering among others. If and when Mathematics is removed, the back-bone of our material civilization would collapse.

Despite important role of mathematics in growth of science and technology, it still remains one of the subjects in which many students at all levels of the school system persistently perform very poorly (Obodo, 2004; Emedo, 2004; Buhari, 2006 and Ifamuyiwa, 2007). Poor achievement of students in mathematics is mostly due to teaching approaches adopted by mathematics teachers in presenting instruction. Most teachers adopt the conventional approach in the teaching of mathematics which is characterized by rote memory of basic mathematical processes and abstract presentation of facts and principles rather than who is being taught and as such, it is a teacher or subject – centered approach. The curriculum is held as absolute in many schools and teachers are reticent to tamper with it even when student do clearly not understand the most important concepts.

The teaching and learning of mathematics is a complex activity and many factors determine the success of this activity. The nature and quality of instructional material, the presentation of content, the pedagogic skills of the teacher, the learning
environment, the motivation of the students are all important and must be kept in view in any effort to ensure quality in teaching-learning of mathematics.

Teaching of mathematics is not only concerned with the computational knowhow of the subject but is also concerned with the selection of the mathematical content and communication leading to its understanding and application. So while teaching mathematics one should use the teaching methods, strategies and pedagogic resources that are much more fruitful in gaining adequate responses from the students then we have ever had in the past.

Teaching mathematics has roots in human-being history and has varied in different times according to the educational goals and teaching situation. Teachers, Philosophers and scientists have mentioned so many methods and techniques for teaching. In the past years child-centered teaching presented by John Dewey was popular in the United States and contemporary in other countries. Dewey and his followers in progressive education programs have defined teaching as guiding students to construct their knowledge. He said: teachers should prepare a real life situation at schools, to learn children living in a democratic society. In the last fifty years, a number of research studies have been conducted on teaching-learning process. New methods and techniques have been developed on the basis of research findings. The traditional methods and techniques are being replaced by new methods and techniques in the last two decades in western countries. The traditional explanation of the term ‘teaching’ which equates teaching with telling is not acceptable to the educators of today, because the recent studies in the field of psychology of teaching and learning have thrown light on some new concepts of teaching. The old concept of teaching as giving off information is being discarded.

According to the changed concept, teaching is to cause the child to learn and acquire the desired knowledge, skills and also desirable ways of living in the society. The main aim of teaching is to help the child to respond to his environment in an effective way. Singh (2005) has given a very precise but comprehensive definition of teaching: “Teaching is the stimulation, guidance, direction and encouragement of learning”. Stimulation means to cause motivation in learner to learn new things. It is to create an urge to learn. Direction means that teaching is not a haphazard activity but it is a goal directed activity which leads to pre-determined behavior. Direction
also means that the activities of the learner in teaching are directed and controlled, keeping into consideration the economy of time and efficiency of learning. Guidance means to guide the learner to develop his capabilities, skills, attitude and knowledge to the maximum for adequate adjustment in the external environment and encouragement of learning means to encourage the learner to acquire maximum learning. Teaching as a transmitting information (Skill approach)-teaching to understand the meaning of mathematics concepts (conceptual approach)-teaching to think and solve mathematical problems (problem solving approach) and finally fostering mathematical power of students to understand mathematics meaningfully and better reasoning, connecting mathematical concepts; communicating to understand and solve problems (NCTM 1989).

Teaching may consist of a description of those acts that teachers demonstrate which reflect their commitment to a particular philosophy of education. Psychologists and educators have explained it from different angles. Some of the explanations are as follows:

- Teaching is communication between two or more persons, who influence each other by their ideas and learn something in the process of interaction.
- Teaching is to fill in the mind of the learner by information and knowledge of facts for future use.
- Teaching is a process in which learner, teacher, curriculum and other variables are organized in a systematic way to attain some pre-determined goal.
- Teaching is to cause motivation to learn.

Gage, as cited by Singh (2005) considers that the process of teaching and learning must be adapted to each other so as to make whatever combination of procedures pay off best. Thus learning is essential for teaching and the learning structures should be considered for effective teaching.

Education is imparted for achieving certain ends and goals. Various subjects of the school curriculum are different means to achieve these goals. So, with each subject some goals are attached which are to be achieved through teaching of that subject. According to Sidhu (1995) the goals of teaching mathematics are as below:

- To develop the mathematical skills like speed, accuracy, neatness, brevity, estimation, etc.
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➢ To develop logical thinking, reasoning power, analytical thinking, and critical thinking.
➢ To develop power of decision-making.
➢ To develop the technique of problem solving.
➢ To recognize the adequacy or inadequacy of given data in relation to any problem.
➢ To develop scientific attitude i.e. to estimate, find and verify results.
➢ To develop ability to analyze, to draw inferences and to generalize from the collected data and evidences.
➢ To develop heuristic attitude and to discover solutions and proofs with the own independent efforts.
➢ To develop mathematical perspective and outlook for observing the realm of nature and society.

The Education Commission (1964-66) points out that “In the teaching of Mathematics emphasis should be more on the understanding of basic principles than on the mechanical teaching of mathematical computations”. Commenting on the prevailing situation in schools, it is observed that in the average school today instruction still confirms to a mechanical routine, continues to be dominated by the old besetting evil of verbalism and therefore remains dull and uninspiring. Innovations in teaching of mathematics can be diversified in terms of Methods, Pedagogic and strategies.

Although methods of teaching have passed through several developments, teachers all over the world followed fixed ways of teaching. It is because the educational programme for teachers prepares them to follow one of a few mixed ways of teaching. Attempts have been made by researchers to master the different approaches, strategies of teaching with the objectives of instruction and pupils’ learning ability. Dunn and Dunn (1979), Fischer and Fischer (1979), Ellis (1979), and Joyce and Weil (1980), also believe that the strength in education rests in the intelligent use of the powerful variety of approaches – matching them with different goals of education.
The quality of teaching and learning mathematics has been one of the major challenges and concerns of educators. Instructional design is an effective way to alleviate problems related to the quality of teaching and learning mathematics. It is important for educators to adopt instructional design techniques to attain higher achievement rates in mathematics (Rasmussen & Marrongelle, 2006). Considering students' needs and comprehension of higher-order mathematical knowledge, instructional design provides a systematic process and a framework for analytically planning, developing, and adapting mathematics instruction (Saritas, 2004).

1.1 MODELS OF TEACHING

In teaching, the use of models is very old. Socrates, the Greek philosopher, used his own model of question-answer (Dialect). Indian ancient teachers developed their own models of teaching to affect the desirable changes in the behavior of the learner. Several models of teaching have been developed in the last few decades in western countries. There is no one particular model that can help the pupils to grow in all respects-social, intellectual, emotional. For this, Joyce and Weil (1985) have identified a variety of strategies developed by different learning theorists and designed a number of models of teaching. These innovations have been found to be very effective. Several models of teaching have been developed which prescribe different approaches to instructional process to bring desired changes in the behavior of the learners.

Models of teaching have been developed to help a teacher improve his capacity to reach more children and create a richer and more diverse environment for them. A model of teaching has been defined in various ways of (Singh, 2005).

**View of Joyce and Weil:** They have given three meanings of teaching models:

(i) “Teaching models are just instructional designs. They describe the process of specifying and producing particular environmental situations which cause the student to interact in such a way that specific change occurs in his behavior.”

(1972, 2)

(ii) Teaching model is a “pattern or plan”, which can be used to shape a curriculum or course, to select instructional materials and to guide a teacher’s actions. (1972, 3)
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(iii) A Model of teaching consists of guidelines for designing educational activities and environments. It specifies ways of teaching and learning that are intended to achieve certain kinds of goals. (1978, 2)

Paul, Daunal and Robert (1979) models are prescriptive teaching strategies, designed to accomplish particular instructional goals.

Jangira (1983, 10) A model of teaching is a set of inter-related components arranged in a sequence which provides guidelines to realize specific goal. It helps in designing instructional activities and environmental facilities, carrying out of these activities and realization of the stipulated objectives.

Siddique & Khan (1991) model of teaching is a plane of pattern that can be used to shape curricula, to design instructional material and to guide instruction in the classroom and other settings. The most important aim of any model of teaching is to improve the instructional effectiveness in an interactive atmosphere and to improve or shape the curriculum.

Buch (1997) model of teaching is merely a tool of thinking about the teaching situation. It is a set of concepts carefully arranged to explain what teacher and students do in classroom, how they interact, how they use instructional material and how these activities are considered in a sequence of the phases, ultimately leading to certain cognitive abilities and attitude among the students.

Sprinthall & Sprinthall (1999) model of teaching basically represents a specific cluster of strategies designed to reach a particular type of learning outcome with pupils in the best way. However, no single model represents the best way to reach all types of learning. Instead, overall teaching effectiveness will depend on, firstly, teacher’s ability to master specific techniques and secondly, to combine them within a particular model and finally on his ability to use a variety of model flexibly.

Shahid (2000) Model of teaching can be defined as instructional design which describes the process of specifying and producing particular environmental situation that cause the students to interact in such a way that a specific change occurs in their behaviour.

Joyce and Weil (2005) model of teaching is a plan or pattern that can be used to shape curricula, to design instructional material and to guide instruction. Models of teaching are really models of learning. The most important long term
outcome of instruction may be students increased capabilities to learn more easily and effectively in the future, both because of the knowledge and skill they have acquired and therefore they have mastered learning processes.

Singh (2008) model of teaching is designed to achieve a particular set of objectives. Student is not a substitute of any teaching skill. Rather, it creates the conducive teaching-learning environment in which teachers teach more effectively by making the teaching act more systematic and efficient.

Joyce, Weil and Calhoun (2009, 6) Models of teaching are really models of learning. When educators utilize a model of teaching with students, they not only assist students in understanding content, but they also teach them the process of thinking and learning. The models of teaching enhance critical thinking and allow students to gain increased control over both content and the learning process.

Thus, model of teaching may be described as some sort of guidelines, plan or pattern, technique or strategies designed to achieve specific educational goals. It is just a blueprint designed in advance for providing necessary structure and direction to the teacher for realization the stipulated objectives. They help a teacher in his task in the same way as a constructed model or blueprint helps an engineer in his project.

1.1.1 Functions of Model of Teaching

Models of teaching have the following functions:

**Guidance:** A model of teaching serves a useful purpose of providing in definite terms what the teacher has to do. He has a comprehensive design of instruction with him through which he can achieve the objectives of the course. Teaching becomes a scientific, controlled and goal directed activity. Thus a model provides guidance to the teacher as well as to the students to reach the goal of instruction.

**Specification of Instructional Material:** A model of teaching specifies in detail the different types of instructional materials which are to be used by the teacher to bring desirable changes in the personality of the learners.

**Improvement in Teaching:** A model helps the teaching-learning process and improves effectiveness of teaching (Singh, 2005).
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Thus, Models of teaching are known to serve three major functions in a given teaching learning situation namely, designing and specifying instructional objectives, developing and selecting instructional material and specifying the teaching-learning activities for the attainment of stipulated instructional objectives. These Functions of Models of Teaching can be explained with the help of the following diagram (Figure1.1).

![Figure1.1: Functions of Models of Teaching (Mangal, 2011)](image)

1.1.2 Components of Model of Teaching

The components of a teaching model, according to Joyce and Weil (1978) are as follows:

- **Focus**: Focus is the central aspect of a teaching model. For what the model stands is the theme of the focus. All of the teaching models are meant for achieving some specific goals or objectives of teaching in relation to the environment of the learner. Therefore, objectives of teaching and aspects of the environment, generally, constitute the focus of the model.

- **Syntax**: the term syntax or phasing of the model refers to description of the model in action. Each model consists of several phases and activities which have to be
arranged in a specified sequence quite unique to a particular model. The syntax helps a teacher to use the model. It tells him how he should begin and proceed further. Comparing the phases of Model reveals the practical difference between Models.

**Principles of reaction:** while using the model how should a teacher regard and respond to the activities of the student is a concern of this element. These responses should be quite appropriate and selective. Ever model through its principles of reaction provide the teacher with particular and unique rules of thumb by which to “tune in” the student and select appropriate responses to what the student does (Joyce and Weil, 1978).

**Social system:** this element of model refers to a description of following:

(a) Interactive roles and relationships between the teacher and student.

(b) The kinds of norms that are encouraged and student behaviour which is rewarded.

Models differ from each other with regard to the description of the above aspects. In some models, the teacher is the centre of activity, or activities are somewhat equally distributed between teacher and students, while in others the students’ occupy the central place. The leadership role of the teacher comprising the location of authority and amount of control over that emerges from the process of interaction also varies from model to model. On the basis of social system, Models can be classified as highly structured, moderately structured, and low structured models.

**Support system:** Support system refers to additional requirements beyond the usual human skills or capacities from the teachers and technical facilities available in an ordinary classroom. Such type of additional support may demand some special skills, knowledge and capacities from teacher or some special aids material facilities like films, self-instructional system, and visit to some place particular organisational climate suiting to the requirements of a particular model (Mangal, 2009).

**Instructional and Nurturant Effects:** The description of the effects of a Model can validly be categorized as the direct or instructional effects and indirect or nurturant effects. The instructional effects are those directly achieved by leading the learner in certain directions. The diagrammatic representation of the instructional and Nurturant effects of a model is given below:
1.1.3 Families of Teaching Models

In the recent years of the last century; researchers, educators and teachers build-up many teaching models and have recommended to novice teachers and practitioners. Most of teaching models based on learning theories of two major institutes of educational psychology, behaviorist and cognitive domains. These models have advantages and disadvantages. Some models are compatible with other models; some are apt to extol their virtues. It is reasonable to assume that no model is universally appropriate. Each possesses its own strengths and weaknesses” (Shahid, 2000). So teachers have options to apply appropriate model in their teaching. For example most of mathematics teachers will apply problem-solving model in mathematics classes. Joyce and Weil (2003) categorized the models of teaching in four major families as under:

1. Information processing models
2. Social interaction models
3. Personal models
4. Behaviour modification models

**Information Processing Models**

Information Processing Models share an orientation toward the information processing capability of pupils and the ways that they can improve their ability to master the information. These refer to the way people handle stimuli from the environment, organize data, sense problems, generate concepts and solution to problems and employ verbal and non verbal symbols. Some information processing
models are concerned with the ability of the learner to solve problems and thus emphasize productive thinking. A large number of these emphasize on concepts and information derived from the academic disciplines. Models belonging to this family are: (i) Concept Attainment, (ii) Inductive Thinking, (iii) Advance Organizer (iv) Cognitive Model/ Growth Model.

Social Interaction models

These models emphasize on the relationship of the individual to society and the other persons. Models from this orientation give priority to the improvement of the individual ability to relate to others, to engage in democratic processes and work productively in the society. Models belonging to this family are: (i) Group Investigation, (ii) Role Playing, (iii) Jurisprudential Inquiry, (iv) Laboratory Method Model.

Personal Models

These models share orientation towards the individual and the development of selfhood. It is expected that the focus on helping individuals to develop a productive relationship with the environment and to view themselves as capable persons will produce richer interpersonal relations. Models belonging to this family are: (i) Awareness Training Model, (ii) Synectics Model (iii) Conceptual System Model.

Behavioural Modification Models

The models belonging to the behaviour modification family are related to behaviour modification theories. They have evolved from the attempts to develop efficient systems for sequencing learning task and shaping behaviour by manipulating reinforcement. Models belonging to this family are: (i) Contingency Management Model, (ii) Training Model (iii) Mastery Learning Model (iv) Stress Reduction Model.

Each family of teaching model contains subfamily have been represented in figure 1.3 below:
They stress that the different instructional goals would be realized by putting these models of teaching into action. These models present students with a diversity of learning environments that is responsive to varying needs and learning styles. The categorization of these different teaching models does not represent a watertight compartmentalization. These families are by no means antithetical or mutually exclusive. The instructional activities and learning environments emerging from some of the models, even though classified in the different families, are remarkably similar.

Above mentioned models under different families of models of teaching aim at the development of different aspects of human personality i.e. social, personal, informational and behavioural. Since education is meant for all round development of child's personality, no single model can be selected for his or her development. All of them will have to be employed according to the requirements of the situation.

1.1.4 Families of Cognitive Teaching Model

(Model relevant to explain the Mathematics Learning Process)

Teaching models prescribe tested steps and procedures to effectively generate desired outcomes. The number of emerging models and the ones that have emerged is uncountable. Each emerging new model either explores a new approach or attempts a modification of the conventional ones as to cater the uniqueness of individuals. Most
importantly, any teaching model should optimize learning experiences to the needs of
each learner by carefully exploring the learning problems and offering tailored
assistance.

In the recent years of the last century researchers, educators and teacher build-
up many teaching models and have recommend these to novice teachers and
practitioners. Most of teaching models are based on learning theories of two major
institutes of educational psychology, behaviourist and cognitive domain model. There
are many Models of Teaching that are built around the mental process as ranging from
systems for teaching general problem solving ability to procedures for teaching
process.

As any Model of teaching must refer to the organisation of a field of
phenomena and describe its way of working. With regard for the mathematics
learning, Clements and Battista (1992) have discussed Piaget and Inhelder
(developmental models) theory Van Hiele’s theory (cognitive models) and Cognitive
Science Models; Duval (2000) refer two great kind of models: (i) Developmental

Thus, three major theoretical model (theoretical perspective) for cognitive
growth relevant to explain the mathematics learning processes are:

- Cognitive Development model
- Cognitive model
- Cognitive science model

**Cognitive Development Model**

Cognitive development theories hold that growth and development occurs in
progressive stages. The developmental models focus on the increase in knowledge as
individual passes through various stages. Piaget, the father of cognitive approach to
development, see the child passing through various stages, from sensori-motor
through concrete options and then on to formal operation. This model is also called
Piaget’s cognitive development model.

**Cognitive Model**

Cognitive models focus on the cognitive complexity of the working of human
thought. At first sight, they seem for from mathematics learning. And classical models
developed in psychology laboratories cannot be used as they are (Fischbein, 1999). By
the simple reason that the learning of mathematics raises specific and fundamental
questions about reason modes, about the treatment of figures-about the understanding
of mathematical concepts and infinity is very important instance- which are not
envisaged by psychologist. Three cognitive model of geometrical reasoning are (i)
Van Hiele model of thinking in geometry (ii) Duval’s cognitive model of geometrical
reasoning (iii) Fischbein’s theory of figural concepts (Jones, 1998).

Cognitive Science Model

Cognitive science model posits a way of understanding, the way in which
individuals’ possess information (Clement & Battista, 1992). The cognitive science
prospective in students learning of geometry endeavours to integrate research and
practical work in the fields of psychology, philosophy, linguistics and artificial
intelligence. cognitive science models of geometric knowledge includes (i) Greeno’s
model of geometry problem solving (ii) Anderson’s model of cognition and (iii)
Parallel distributed processing(PDP) networks (Battista,1992; vander sandt,2000).

Figure 1.4: Families of Cognitive Model of Teaching

Jones (1998) stated that the teacher can apply the appropriate model of
teaching in his / her teaching situation. He has suggested three theoretical frameworks
for developing reasoning: the Van Hiele model of thinking in geometry, Fischbein’s
theory of figural concepts, and Duval’s cognitive model of geometrical reasoning.
Each of these frameworks provides theoretical resources to support research into the
development of geometrical reasoning in students and related aspects of visualization and construction.

1.1.5 Cognitive Development Model

Piaget’s Cognitive Development Theory in Mathematics Education

One major focus of Piaget’s work examined how children organize and construct ideas about geometry, as well as how they form a representation of space. The research of Piaget and Inhelder (1967) focused on a child’s conception and representations of space. Their theory was comprised of two parts, the first being how a child constructs their own representation of space. Second, Piaget and Inhelder claim that a child’s organization of geometric ideas follows a definite, logical order. They found that preschool children could discriminate objects based on topological features, but could not discriminate between curvilinear and rectilinear objects until later in their development. Preschool age children also had difficulty drawing copies of geometric shapes, lending to the assumption that hand-eye coordination also impacts a child’s conception of space. According to Piaget and Inhelder, children can only truly begin to discriminate among Euclidean shapes around the age of 4.

Jean Piaget identifies four stages of cognitive development that all children will progress through at some point in their lives. These stages are:

- Sensorimotor: Birth to 2 years old
- Pre-operational: 2 to 7 years old
- Concrete operation: 7 years old to adolescence
- Formal operation: Adolescence to adult (as cited in Pusey, 2003).

Sensorimotor Stage

The first stage (generally between birth and two years old) of development that Piaget identifies is the Sensorimotor Stage. At this stage, Children are egocentric and can only see the world from their own perspective. They learn and understand concepts using their five senses and need concrete experiences to understand concepts and ideas.

In mathematics, children at Sensorimotor Stage begin to have an understanding of object permanence in which they are able to find objects that have been taken out of their view. Piaget demonstrates this through an experiments in
which he displaces an object under a pillow, and children are able to correctly find the object, illustrating they know the object still exists even though they cannot see it. Children are also beginning to see how numbers link to objects and therefore approach being capable of counting on their fingers. They understand that one finger matches with the number one and can count concrete objects using this concept. At this point in development, children don’t understand point of view but can distinguish themselves from the rest of the world (Ojose, 2008). Children begin to match one object to one person or one toy to one person. Individuals struggling in this stage of understanding this concept will be able to recite the first ten numbers but might not be able to match the number to objects.

**Preoperational Stage**

The second stage of cognitive development identified by Jean Piaget is the Preoperations Stage, during two to seven years old. During this period, children are able to do one step logic problems, develop language, continue to be egocentric, and complete operations (Blake, Barbara, and Pope, 2008).

In mathematics, children at this stage are able to solve one step logical problems like addition and subtraction but still are primarily limited by working with concrete materials. The children need to incorporate any materials that are available such as blocks, and counters. Because they are still egocentric as in the previous stage, children are “…restricted to one aspect or dimension of an object…”(Ojose, 2008). Since, children are restricted to one dimensional thinking; they are primarily influenced by the visual representation of things. For example, if a child was presented with the sequence 1, 2, 3, 4, 5 and asked to compare it to 2, 4, 6, 8, 10, the child would recognize there is a difference but might not articulate how the first sequence transforms into the second sequence.

**Concrete Operations Stage**

The next stage of development that Piaget discusses is the Concrete Operations Stage which generally recognizes a child between the ages of seven to eleven years old. A child would be able to think logically and start classifying based on several features and characteristics rather than solely focusing on the visual representation (Ojose, 2008).
Mathematically speaking, this stage represents a remarkable new development for a child. Since children can now classify based on several features, they are able to consider 2 or 3 dimensions. While children were previously limited to their own point of view, they can now take into account others perspectives. They can also begin to understand the ideas of seriation and classification more thoroughly while also developing how to present solutions in multiple ways (Blake, Barbara, and Pope, 2008). In order to develop the ability in a child of presenting multiple solutions, discussions in a classroom can be very helpful.

**Formal Operations Stage**

The last stage of development that Piaget identifies is the Formal Operations Stage, which children enter roughly between the ages of eleven to sixteen years old and continues throughout adulthood. This marks the distinct change of a child’s thinking to a more logical, abstract thinking process” (Blake, Barbara, and Pope, 2008).

At this point in cognitive development, children do not need the concrete experiences they required to understand mathematics in the previous stages. They form their own hypotheses and determine possible consequences which stems from seeing situations from differing perspectives. The child can also begin to understand abstract concepts which lead to much more complicated mathematical thinking (Ojose, 2008). They also begin to think about the concept of infinity and understand how to estimate what an infinite series converges to.

### 1.1.6 Cognitive Models of Teaching

**Van Hiele Model**

According to the theory of Pierre & Dina Van Hiele, student progress through levels of thought in learning geometry from a Gestalt-like visual level through increasingly sophisticated levels of description, analysis, deduction, and abstraction. The theory has three aspects: the existence of levels, properties of the levels & the movement from one level to the next. The Van Hiele believe that cognitive development in geometry (progression from one level to another) can be accelerated through instruction; therefore, the Van Hiele gives detailed explanation of how teachers should operate to lead students from one level to the next.
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Van Hiele (1986, p36-47) distinguished five levels of geometric thought which can be summarized as follows:

**Level one:** Visual

Figures or objects are seen as a whole; individual properties of such are not distinguished.

**Level two:** Descriptive/analytical

Figures or objects can be identified by their properties; each property is seen in isolation, i.e. there is no comparison to other figures or objects.

**Level three:** Abstract/relational

Figures or objects are still determined by their properties, but the relationships between properties and figures evolve.

**Level four:** Formal deduction

Theories and deductive proof can be constructed.

**Level five:** Regor/mathematical

Proofs counter to intuition can be accepted as long as the deductive argument is valid; learners can manipulate geometric statements such as axioms, definitions, and theorems.

Instruction should be geared toward finding out the level at which a child operates and then building up from there, otherwise the child can be the teacher may be at different wavelengths and instruction is bound to fail. A child at level N will answer most questions at that level but will not answer questions at level N+1 (Mayberry, 1983; 58).

There are five phases/stages of learning that facilitate geometric thinking (Teppo, 1991; 212, & Van Hiele 1986, 54). Students progress from one level to the next as a result of passing through the five instructional phases. Van Hiele (1986, 50) states that if a child comes to conclude that ‘every square is a rhombus’ that is not as a result of maturity but is a result of the learning process. These instructional phases can be summarized as follows:

**Phase one:** Information

Learners get reacquainted with material for instruction.

**Phase two:** Bound orientation

Learner explores the field of inquiry through carefully guided activities.
Phase three: Explication

The learner and the teacher discuss the object of study. Language appropriate to the particular level is stressed.

Phase four: Free orientation

Students learn by general tasks to find different types of solutions.

Phase fifth: Integration

Students built an overview of all they have learned of the subject. At this stage, rules may be composed and memorized.

Teppo (1991) states that Van Hiele currently characterized his model in terms of three rather than the five foregoing levels and these can be summarized as follow:

<table>
<thead>
<tr>
<th>Theoretical Level</th>
<th>Use deductive reasoning to prove geometric relationship</th>
</tr>
</thead>
<tbody>
<tr>
<td>The process of learning</td>
<td>The phase of learning</td>
</tr>
<tr>
<td></td>
<td>Integration</td>
</tr>
<tr>
<td></td>
<td>Free orientation</td>
</tr>
<tr>
<td></td>
<td>Explication</td>
</tr>
<tr>
<td></td>
<td>Directed orientation</td>
</tr>
<tr>
<td></td>
<td>information</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Descriptive Level</th>
<th>Recognise objects by their geometric properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>The process of learning</td>
<td>The phase of learning</td>
</tr>
<tr>
<td></td>
<td>Integration</td>
</tr>
<tr>
<td></td>
<td>Free orientation</td>
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<td>Explication</td>
</tr>
<tr>
<td></td>
<td>Directed orientation</td>
</tr>
<tr>
<td></td>
<td>information</td>
</tr>
</tbody>
</table>

| Visual level | Recognise geometric objects globally |

Table1.1: Van Hiele Model of Teaching Geometry (Teppo, 1991)

Thus, Van Hiele theory offers a theoretical framework for teaching and learning of geometry. The theory point out to the levels of geometric thinking a child goes through and that it is through instruction that a learner will proceed from a lower level to a higher level.
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Duval's Cognitive Model

Raymond Duval (1998) argues that geometry involves three kinds of different cognitive processes in relation to discursive processes:

- **Visualisation processes**: Visualisation processes with regard to space representation for illustration of a statement, for the heuristic exploration of a complex situation, for a synoptic glance over it or for a subjective verification;

- **Construction processes**: Construction processes by tools, Construction of configurations can work like a model in that the actions on the representative and the observed results are related to the mathematical objects which are represented;

- **Reasoning processes**: Reasoning in relationship to discursive processes for extension of knowledge, for proof, for explanation.

The synergy of which is necessary for proficiency in geometry. Approaching geometry from a cognitive point of view, he has distinguished four cognitive apprehensions connected to the way a person looks at the drawing of a geometrical figure: perceptual, sequential, discursive and operative. To function as a geometrical figure, a drawing must evoke perceptual apprehension and at least one of the other three. Each has its specific laws of organization and processing of the visual stimulus array. Particularly, perceptual apprehension refers to the recognition of a shape in a plane or in depth. In fact, one’s perception about what the figure shows is determined by figural organization laws and pictorial cues. Perceptual apprehension indicates the ability to name figures and the ability to recognize in the perceived figure several sub-figures. Sequential apprehension is required whenever one must construct a figure or describe its construction. The organization of the elementary figural units does not depend on perceptual laws and cues, but on technical constraints and on mathematical properties. Discursive apprehension is related with the fact that mathematical properties represented in a drawing cannot be determined through perceptual apprehension. In any geometrical representation the perceptual recognition of geometrical properties must remain under the control of statements (e.g. denomination, definition, primitive commands in a menu). However, it is through operative apprehension that we can get an insight to a problem solution when looking at a figure. Operative apprehension depends on the various ways of modifying a given
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figure: the mereologic, the optic and the place way. The mereologic way refer to the division of the whole given figure into parts of various shapes and the combination of them in another figure or sub-figures (reconfiguration), the optic way is when one made the figure larger or narrower, or slant, while the place way refer to its position or orientation variation. Each of these different modifications can be performed mentally or physically, through various operations. These operations constitute a specific figural processing which provides figures with a heuristic function. In a problem of geometry, one or more of these operations can highlight a figural modification that gives an insight to the solution of a problem.

1.1.7 Cognitive Science Models

Anderson's Model of Cognition

ACT-R is a general theory of cognition developed by John Anderson it is an elaboration of the original ACT theory (Anderson, 1976). Anderson (1983) provides a complete description of ACT-R.

Anderson’s Adaptive Control of Thought (ACT) model distinguishes between two kinds of knowledge, namely declarative and procedural. Declarative knowledge is “knowing that” or “knowing why”, for example, postulates and theorems would be stored in schemas along with knowledge about their function, form, and preconditions. Procedural knowledge is “knowing how” is stored in the form of production systems, or sets of condition-action pairs. The diagrammatic representation of Anderson’s Adaptive Control of Thought (ACT) model is given below in figure1.6:

Figure 1.5: Anderson’s Adaptive Control of Thought (ACT) Model
According to the ACT model, all knowledge initially comes in declarative form and must be interpreted by general procedures (for example, one uses general, recipe-following procedures to cook a new dish using the declarative knowledge read in a cookbook). Thus, procedural learning occurs only in executing a skill; one learns by doing. When declarative information is in the form of direct instruction, step by step interpretation is straightforward. However, information is not that direct. In the case of high school geometry, students may use declarative information to provide data required by general problem-solving operation, such as, general search, sequential decomposition of problems, means-ends analysis, inferential reasoning, or making analogies between worked examples and new problems. Importantly, geometry textbooks assume student facility with such operations and virtually never directly specify which procedure should be applied. This assumption is something mistaken. For example, several students studied by Anderson all had misunderstandings about how one determines whether a statement is implied by a rule. Thus, learning involves the acquisition of declarative knowledge which is then interpreted by general procedure (procedural knowledge). These two types of knowledge are used in geometry learning.

**Greeno’s Model of Problem Solving**

According to Greeno’s theory three domain of geometry required for students to solve the problems they are:

1. **Propositions**: Proposition are used in making inferences (familiar statements about geometric relations, such as “corresponding angles formed by parallel lines and a transversal are congruent”) that constitute the main steps in geometry problem solving.

2. **Perceptual Concepts**: Perceptual concepts are used to recognise patterns mentioned in the antecedents of many propositions (for example, corresponding angles).

3. **Strategic Principle**: Strategic Principles are used in the setting goals and planning (for example, when solution requires showing that two angles are congruent, one approach is to use relations such as corresponding angles; another is to prove that triangles containing the angles are congruent).

Of these three domains, the first two are included explicitly in instructional materials; however, strategic knowledge is not. References to that knowledge in the...
materials are indirect at best, and most teachers do not explicitly identify principles of strategy in their teaching. Students must acquire this knowledge through induction from sequences of steps observed in the form of tacit procedural knowledge, involving processes the student can perform but cannot describe or analyze. These strategic principles are quite specific to the domain of problem. Greeno suggests that unguided discovery is more effective than a more explicit form of instruction, if direct teaching is interpreted as the teacher imposition of prescribed steps on students, it contrasts with Van Hiele’s characterization of students finding their own way in the network of relations; if it is interpreted as teacher facilitation of students construction and development of explicit awareness of strategies, the two positions are not disparate.

1.1.8 DUVAL’S COGNITIVE MODEL (Detailed version)

Theoretical Consideration

As a mathematical domain, geometry is to a large extent concerned with specific mental entities, the geometrical figures. During the past twenty five years several mathematics educators have investigated students’ geometrical reasoning, based on different theoretical frames, such as Van Hiele’s model referring to levels of geometric thinking (Van Hiele, 1986), Fischbein’s theory of figural concepts (Fischbein, 1993), Duval’s cognitive analysis of geometrical thinking (Duval, 1998).

As Duval (1998) has noted, geometry can be used to discover and develop different ways of thinking. The content of geometry can be used to develop lower mathematical reasoning, such as recognizing figures, and higher mathematical reasoning, such as discovering properties of figures, inventing geometrical patterns, or solving problems (NCTM, 1989). Additionally, geometric ideas are useful in representing and solving problems in other areas of mathematics as well as in real-world situations (NCTM, 2000). Considering the importance of geometrical reasoning in mathematics education, there is a need for a framework of abilities that could be used in fostering student’s performance. Although there are many studies on different aspects of geometrical reasoning in the literature there are no structural models describing the way students’ experiences arising from geometry teaching at school are built into structure.

Most researchers agree that there is a certain hierarchic development of cognition as far as geometry is concerned and the Van Hiele levels provide a valuable
framework for studying geometry thinking. According to Van Hiele model, the development of students’ thinking in geometry is not dependent upon age or biological maturation, but on the form of instruction received (Clements & Battista, 1992; Fuy, Geddes & Tischler, 1988; Hoffer, 1983; Van Hiele, 1984). Senk (1989) claimed that van hiele did not acknowledge the existence of a “non-level”; instead he asserted that all students entered geometry at ground level which is level 0 (level 1 on a scale of 1 to 5), with the ability to identify common geometric feature by sight.

Del Grande (1990) claims that “Geometry has been difficult for pupils due to an emphasis on the deductive aspects of the subject and a neglect of the underlying spatial abilities” (p. 19). Recognising the complex nature of visualization and imagery especially its role in development of geometrical reasoning French psychologist Raymond Duval (1998, 38-39) has suggested that “differentiating between different visualization processes—in development of geometrical thinking. According to Duval (1998), any model of mathematics learning in which different ways of reasoning are organised according to a strict hierarchy is inappropriate. Rather than being representative of higher (or lower) level of thinking, he argues that different kinds of cognitive activity have their own specific and independent development. He explains that geometry involves three cognitive processes which fulfil specific epistemological functions: 1) visualisation processes; 2) construction processes; 3) reasoning processes. Duval argue that “these three kinds of cognitive processes are closely connected and their synergy is cognitively necessary for proficiency in geometry”. To incorporated the above mentioned aspects in theoretical structure, the matter has been organised under the following heads

- Duval’s cognitive model
- Description of cognitive interaction processes

1.1.8 (a) Duval’s Cognitive Model

Geometry may be exciting for mathematician and for anyone who like mathematics. According to Duval (1998) geometry involves three kinds of cognitive processes in relation to discursive processes which fulfil specific epistemological functions is:
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- Visualisation processes
- Construction processes
- Reasoning processes

Duval pointed out that these different processes can be performed separately. For example, visualisation does not necessarily depend on construction. Even if construction a leads to visualisation, construction processes actually depends only on the connection between relevant mathematical properties and the constraints of the tools being used. Similarly, even if visualisation can be an aid to reasoning (for instance, by aiding the finding of a proof) but visualization can also be misleading. However, Duval argues, “these three kinds of cognitive processes are closely connected and their synergy is cognitively necessary for proficiency in geometry” (ibid, p38). Duval illustrates the connections between these three kinds of cognitive processes in the way represented in figure 1.7.

Figure 1.6 : The underlying cognitive interactions involved in geometrical activity (Duval, 1998)

In figure 1.7, each arrow represents the way one kind of cognitive process can support another kind in any geometrical activity. Duval makes arrow 2 dotted because,
as argued above, visualisation does not always help reasoning. Arrows 5 (A) and 5(B) illustrate that reasoning can develop in a way independent of construction or visualization processes. Duval’s argument that the synergy of these three cognitive processes is cognitively necessary for proficiency in geometry.

1.1.8 (b) Description of Cognitive Interaction Processes

- Visualization processes

  Visualization is an important aspect of mathematical understanding, insight and reasoning. Presmeg (1997) considers visualization to be “the process involved in constructing and transforming visual mental images…” (p. 304). Perceiving and processing visual information by sensory and mental processes are referred to as visual perception processes. Humans display a robust tendency to rely more on visual information than other forms of sensory information (visual dominance effect: e.g., Sinnett, Spence & Soto-Faraco, 2007).

  Duval (1998) refers to visualization as one of three independent cognitive processes that fill specific epistemological functions in geometry: visualization, construction and reasoning. He believes that these three kinds of processes must be developed separately; various ways of seeing figures and reasoning should be included in curriculum. Seeing, constructing and describing a geometrical figure and its properties with mathematical sense is not an easy task for many students.

  Visualisation processes with regard to space representation for illustration of a statement, for the heuristic exploration of a complex situation, for a synoptic glance over it or for a subjective verification. Furthermore, different figures for the same mathematical situation may induce variations in performances. Duval (1995) uses four cognitive apprehensions: perceptual, sequential, discursive and operative as an analytical framework to explain why students’ performance varies with different geometrical figures. He believes that there are several ways of looking at a drawing or a visual stimulus array and points out that “mathematical perception is not simple and it overlaps the different apprehensions” (Duval, 1995, p.155). The following is a summary of his four apprehensions:

(i) Perceptual Apprehension

  It is about physical recognition (shape, representation, size, brightness, etc.) of a perceived figure. We should also discriminate and recognize sub-figures within the
perceived figures since a relevant discrimination or recognition of these sub-figure units may help and give cues for problem solving in geometrical situations.

(ii) Sequential Apprehension

It is about construction of a figure or description of its construction. Such construction depends on technical constraints and also mathematical properties since construction of a figure may merge different figural units. It is believed that construction can help recognition of relationships between mathematical properties and technical constraints.

(iii) Discursive Apprehension

Mathematical properties represented in a drawing can only be clearly defined with speech determination. Making denomination and hypothesis is very important for one to know and derive the mathematical properties within the representation; otherwise, it is only an ambiguous representation.

(iv) Operative Apprehension

It is about making modification of a given figure in various ways:

- The mereological way: dividing the whole given figure into parts of various shapes and combine these parts in another figure or sub-figures;
- The optic way: varying the size of the figures;
- The place way: varying the position or its orientation.

These modifications can be performed mentally or physically. A relevant way of figural changes can give insights to the solution of a problem. However, operative apprehension seems to be the most difficult one among the four since there are various possible figural modifications, finding an appropriate and relevant one to bring solution to a given question becomes a difficult task for many students.

Duval (1995) believes that these apprehensions are important for students to become proficient in geometry and they are closely related to each other as he has mentioned that “operative apprehension does not work independently of the others, particularly of discursive apprehension. …Special and separate learning of operative as well as discursive and sequential apprehension are required, and a mathematical way of looking at figures only results from coordination between separate processes of apprehension over a long time” (Duval, 1995,p.155).
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Construction Processes

Duval (1998) refers to construction as one of three independent cognitive processes that fill specific epistemological functions in geometry: visualization, construction and reasoning. Much time should be spent with construction at earlier stages of secondary school and need for this kind of activity does not diminish in the senior secondary phase. By construction or by observing construction, learners should be led to consider the separate components of each diagram. According to Duval “construction” processes by tools such as compass, rulers, available primitives in a geometrical software etc. Construction of configurations can work like a model in that the actions on the representative and the observed results are related to the mathematical objects which are represented.

Reasoning Processes

Duval (1998) pointed out that “geometry more than other areas in mathematics, can be used to discover and develop different ways of thinking”. According to Duval, “the word ‘reasoning’ is used in a very broad range of meanings. Any move, any trial and error, any procedure to solve a difficulty is often considered as a form of reasoning.” (ibid).

For developing students’ reasoning in geometry, Duval introduced three cognitive processes in geometry involved in proof, namely:

- **A purely configural process:** operative apprehension,
- **A natural discursive process:** in ordinary speech through description, explanation and argumentation,
- **A theoretical discursive process:** deduction.

Thus, reasoning in relationship to discursive processes for extension of knowledge, for proof, for explanation.

1.1.9 Description of the Duval's Cognitive Model

The model of teaching developed here is based on Duval’s theory/ ideas about subject matter, cognitive structure, active reception learning.

Focus

The main focus of teaching-learning through Duval’s cognitive model stands for (i) understand/testing the nature of cognitive interaction processes of student while
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solving mathematical problems (ii) develop visualisation ability (iii) develop sense of construction/drawing and (iii) develop concepts of logic, reasoning ability through visualisation and construction processes.

Syntax

The Duval’s cognitive model has three phases of activity.

Phase I: Visualization processes

In this starting phase, the students are provided with space representation of illustration, exploration or verification of different geometric situation through following activities:

(Activities) (i) Perceptual apprehension
(ii) Discursive apprehension
(iii) Operative apprehension

Phase II: Construction processes

In this phase, after visualizing mathematical/ geometric concepts via visual imagery, mental imagery students are persuaded to construct a configuration of concepts in their mind or with the help of restricted tools and geometrical requirements. This phase include following activities:

(Activities) (i) Construction of concept via mind (mental imagery)
(ii) Construction using tools: ruler and compass and available primitives in a geometrical software etc.

Phase III: Reasoning processes

In this phase, attempts are made to see whether the students are able to develop their ability to think and reason by providing problematic situations or ask for explanation and discursive processes for proof. This phase includes activities as under:

(Activities) (i) Purely configural process
(ii) Natural discursive process
(iii) Theoretical discursive process
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A Summary of the Syntax appears in Table 1.2.

<table>
<thead>
<tr>
<th>Phases</th>
<th>Activities</th>
</tr>
</thead>
</table>
| Phase one: Visualization processes | • Perceptual apprehension  
                             • Discursive apprehension  
                             • Operative apprehension  |
| Phase two: Construction processes | • Construction of concept via mind (mental imagery)  
                             • Construction using tools: ruler and compass and available primitives in a geometrical software etc.  |
| Phase three: Reasoning processes | • Purely configural process  
                             • Natural discursive process  
                             • Theoretical discursive process  |

Table 1.2: Syntax of Duval's cognitive model

Principles of Reaction

This element is concerned with the reaction of the teacher to the response of the students. In Duval’s cognitive model, the teacher’s task is to nurture the reasoning ability, proof performance in the students by emphasizing the cognitive interaction processes. The teacher needs to appreciate student’s ideas, argument, concepts about geometrical figures/ shapes and their properties in terms of mathematical language and symbols. During phase one and phase two, teacher is to remain supportive for turning the student’s attention towards analysis of visual imagery, visual perception and construction of geometrical figures. During phase three, teacher task is to encourage, promote and help student to develop different ways thinking/ reasoning.

Social system

In Duval’s cognitive model, social system is comprised of mixed interaction between teacher and students. In phase one and phase two of syntax i.e. visualisation and construction processes both teacher and students is equally active but in the phase three i.e. reasoning processes students are more active than teacher. Here, the question of passive learner is ruled out. The students can express themselves freely and can make conclusions. Ample freedom is given to the students for carrying out
their own thinking. They may formulate their own ideas, concepts for making argument and test these in the light of the available data. It is however true that teacher formulates instructional objectives, decides about the instructional process to be carried out in the limelight of this model. In the whole, teacher acts as a facilitator with democratic attitude in order to attain equal sharing between her and the students.

Support system

Apart from ordinary nicely equipped classroom with blackboard, chalk, pointer etc. in Duval’s cognitive model, teacher requires resource material like moving model, charts, geometry instruments, paper cut-outs, daily life used material cube, cuboids and cylindrical shaped box and puzzle toy, geometry software (according to mood of instruction) etc. to provide maximum support to the learners to encounter to the problem and to develop thinking /reasoning ability through visual imagery, mental imagery and drawing geometrical figures/ shapes.

Instructional and Nurturant effects

Each model is developed around some focus/goal. The success of the model is measured by the extent it has attained the goal. But every model affects some other aspects of the student’s behaviour also. The primary purpose of Duval’s cognitive model is to teach learners how to reflect on the reasoning and inquiry, how to define their problems, how to work with others in exploring facts, different ways of looking at geometrical figures and how to conclude on the basis of available. Besides achievement, instructions are used to develop reasoning ability, visualisation and construction skills. Here teacher plans some activities and create a climate that nurture understanding of facts and ideas, visual imagery, logical reasoning, openness as well as cognitive interaction processes. It also nurtures a spirit of cooperation and as ability to work with other in a reasoning process.

1.1.10 Comparison of Duval’s Cognitive Model with Piaget Developmental Model and Van Hiele Model

Each of the three major theoretical models often insights into how students learn geometry. By comparing Duval’s Cognitive theory with that of Piaget’s theory and Van Hiele’s theory the researcher intends to make an attempt to signify the importance and role of Duval cognitive model of geometrical reasoning more clearly and analytically in the development of geometric thinking of children. It is assumed
that comparing and contrasting with Piaget’s features of mental growth and Van Hiele model of geometric thinking would help to provide clearer picture of the Raymond Duval perspectives. In the following paragraph, Duval cognitive theory has been compared with (i) Piaget’s theory (ii) Van Hiele theory.

**Duval’s Position As Compared To Piaget’s Theory in the Development of Geometric Reasoning/Thinking**

- Piaget theory describes how the learning of geometric concepts follows a definite order which is more dependent on the age of the learner than any other criteria. Duval’s theory, on the other hand, emphasizes that stage of learning geometric concepts are not connected with any particular age, but are strongly connected to teaching/learning process.

- In geometry Piaget (1928) has stressed mainly on the methodical nature of deductive thinking and on the difficulties students have when they try to think in that way in the context of geometry (Van Hiele 1959). Whereas Duval’s (1998) cognitive model of geometrical thinking suggests that geometrical thinking involves visualization, construction and reasoning process, pointed out the role of visual representation of geometrical statement.

- Piaget and Duval possess contrasting position on how geometric reasoning develops in the pupils. For Duval, the ability to reason logically in geometry is curriculum dependent, where as for Piaget, certain logical option are product of maturation and are independent of content specific knowledge.

- Piaget’s cognitive development theory aim at explaining the mechanisms and processes by which the infant and then the child develops into an individual who can reason and think using hypotheses. Piaget described three basic processes, which affect cognitive development: assimilation, accommodation, and equilibration (Sprinthall & Sprinthall, 1977). Whereas approaching geometry from a cognitive point of view Duval has distinguished four cognitive apprehensions connected to the way a person looks at the drawing of a geometrical figure: perceptual, sequential, discursive and operative apprehension (Duval, 1995) for solving geometrical problems.

- Piaget has stated that the first cognitive development is achieved through differentiation and coordination of schemes (Zimmerman, 1991) at further
levels the development implies a differentiation of the first semiotic registers, the native language and the iconic representation of shapes, and their coordination. Duval (1998) think that there is double gap between naive behaviour and mathematical behaviour in geometry. The one is about the visualisation and the other is about reasoning.

Both Piaget and Duval’s theory add an insight to the solution of a geometry problem and geometric thinking.

**Comparison between Duval’s theory and Van Hiele theory**

- In Duval theory there is no developmental hierarchy between the different kinds of cognitive activities: visualisation, natural discursive reasoning, theoretical deductive reasoning, and formal axiomatic proof, analytic or synthetic processes. In fact, since the representative level (about the age of 2-3 years) until the most mature levels, we have visualisation, speech, reasoning, analytic and synthetic processes. He argues that different kinds of cognitive activity have their own specific and independent development. In Van Hiele theory, students move sequentially along the levels of thinking without skipping a certain level, i.e. if a student is at the third level, he/she must first have attained the first two level. However, not all students pass through the five levels in same way. According to Senk (1985); the Van Hiele model states that two persons reasoning at different level may not understanding each other.

- According to Duval (1998) there is no understanding without visualisation correct geometrical reasoning through visualisation process results from the combination and interaction of the verbal propositions and the geometrical figure. Duval (1995, 1999) distinguishing four apprehension for a geometric figure” perceptual, sequential, discursive and operative one or more of these operation can give an insight to the solution of a problem. While in van Hiele the reasoning of students at the visual and descriptive / analytic level is quite different when they identify a figure. For the student at the visual level, the judgement is based on an observation” (Van Hiele, 1986, 110). There is no why, one just see it, for the student at the descriptive / analytic level, the judgement results “from a network of relation”. The thinking of students at the descriptive / analytic level may involve observation; it may be that they see
an image whenever they consider a given figure; the image is not the basis for judgement.

- Duval (1998) believes that there are three types of cognitive processes involved in geometry problem solving and in proof i.e. (i) purely configural process (ii) natural discursive process (iii) theoretical discursive process. He also thinks that visualisation and reasoning is a “double gap” between naive behaviour and mathematical behaviour. However Van Hiele says that students’ development in their thinking about reasoning and proof is a growth i.e. depending on increasing understanding of geometric knowledge and their relationships.

- In Duval theory development of thinking is multi-model and not uni-model. These rules out any model of development in which different kinds of cognitive activities would be organised into a strict hierarchy from the concrete to the most abstract from visualisation to the axiomatic rigour. Whereas in Van Hiele theory development of thinking follow uni-model in which different kinds of cognitive activities would be organised into a strict hierarchy from the concrete to the most abstract from visualisation to the axiomatic rigour.

1.2 REASONING

Reasoning is a thinking activity that is of crucial importance throughout our lives. Consequentially, the ability to reason is of central importance in all major theories of intelligence structure. Whenever we think about the causes of events and actions, when we pursue discourse, when we evaluate assumptions and expectations based on our prior knowledge, and when we develop ideas and plans, the ability to reason is pivotal.

The word ‘reasoning’ is used in a very broad range of meanings. Any move, any trial and error, any procedure to solve a difficulty is often considered as a form of reasoning. More specifically any process which enables us to draw new information from given information is considered as ‘reasoning’. In this way, induction, abduction, inferences are various kinds of reasoning.

NCTM (1980, 1989) specifies the following goals to develop reasoning in students. First, “students should recognize that reasoning is based on specific assumptions and rules.” Second, students should be encouraged to make and
investigate conjectures. NCTM assigns conjecture a great role in reasoning. NCTM envisions a progression of reasoning skills, beginning with trial-and-error strategies (which are then examined and analyzed) to conjecture strategies.

**Perkins and Salomon (1989)** General reasoning abilities, that is, the ability to question or provide a counterargument to test causal knowledge is hypothesized to be a developmental cognitive construct. Students’ reasoning abilities are also referred to ‘strategies as thinking tools’.

**Johnson-Laird & Byrne (1993)** reasoning, in general, involves inferences that are drawn from principles and from evidence, whereby the individual either infers new conclusions or evaluates proposed conclusions from what is already known.

**Duval (1998)** “the word ‘reasoning’ is used in a very broad range of meanings. Any move, any trial and error, any procedure to solve a difficulty is often considered as a form of reasoning”.

In the views of **National Council of Teachers of Mathematics (NCTM; 2000 p. 122)** reasoning is the ability to reason systematically and carefully develops when students are encouraged to make conjectures, are given time to search for evidence to prove or disprove them, and are expected to explain and justify their ideas.

Systematic reasoning is a defining feature of mathematics. It is found in all content areas and, with different requirements of rigor, at all grade levels. For example, first graders can note that even and odd numbers alternate; third graders can conjecture and justify—informally, perhaps, by paper folding—that the diagonals of a square are perpendicular. Middle-grades students can determine the likelihood of an even or odd product when two number cubes are rolled and the numbers that come up are multiplied. And high school students could be asked to consider what happens to a correlation coefficient under linear transformation of the variables (NCTM 2000, p. 56).

**Yackel and Hanna (2003)** defined, “reasoning” is often used in making the assumption that everybody knows what it means, without any elaboration.

**Random House Webster's College Dictionary (2011)** reasoning is the process of forming conclusions, judgments, or inferences from facts or premises.

**Webster Dictionary (2011)** reasoning is that which is offered in argument; proofs or reasons when arranged and developed; course of argument.
Thus, Reasoning means a mental cognitive process to judge concepts logically to acquiesce or corroborate. It is consisted of all the connections, between experiences and knowledge that a person uses to explain what they see, think and conclude. Each school of thought has defined it in its own domain of hypothesis either through discrete or empirical.

1.2.1 Types of Reasoning in Mathematics

In mathematics, reasoning is used to solve problems, and also to decide whether an assertion (e.g., an answer to a problem) is correct. Students tend to engage in mathematical reasoning when they recognize that a logical inference (or series of inferences) is called for, recognize the type and degree of justification needed, and harness language(s), including mathematical terms and symbols, to create an explanation. Reasoning is complex in part because it is situational – sometimes a sentence or two will suffice, at other times a “proof” is required.

Traditionally, reasoning has been divided into two categories – deductive and inductive. Carroll (1993) has proposed three major reasoning abilities: sequential (deductive), inductive, and quantitative. Edwards (1997) has proposed five types of reasoning activities that are commonly noticed before the territory of proof. These reasoning activities are based hierarchically and include:

- Noticing and constructing patterns
- Describing the pattern
- Conjecturing
- Inductive Reasoning
- Deductive reasoning

Apart from this other types of reasoning are:

Logical reasoning: Logical reasoning means a methodical mental cognitive process to acquiesce and corroborate concepts by discrete empirical evidence or deductive/inductive abstractions to arrive at some conclusion. Three kinds of logical reasoning can be distinguished: deduction, induction and abduction.

Abstract Reasoning: The ability to analyze information and solve problems on a complex, thought-based level is sometimes referred to as abstract reasoning.
Abstract problems are often visual and typically do not involve social ideas. Abstract reasoning is usually assessed as part of intelligence testing. Abstract reasoning ability is important because it enables students to apply what they learn in complex ways.

**Quantitative Reasoning:** Quantitative reasoning is defined as the ability to analyze quantitative information, including the determination of which skills and procedures can be applied to a particular problem to arrive at a solution. It is not restricted to skills acquired in mathematics courses, but includes close up reasoning abilities developed over time through practice in almost all high school or college courses, as well as in everyday activities such as budgeting and shopping.

**Proportional reasoning:** Proportional reasoning is at the heart of middle grades mathematics. It is a form of mathematical reasoning that involves a sense of co-variation and of multiple comparisons, and the ability to mentally store and process several pieces of information. Proportional reasoning is very much concerned with inference and prediction and involves both qualitative and quantitative methods of thought. Essential characteristics of proportional reasoning involve reasoning about the holistic relationship between two rational expressions such as rates, ratios, quotients, and fractions. This invariably involves the mental assimilation and synthesis of the various complements of these expressions and an ability to infer the equality or inequality of pairs or series of such expressions based on this analysis and synthesis.

**Adaptive Reasoning:** Adaptive Reasoning is loosely defined as the capacity for logical thinking and the ability to reason and justify why solutions are appropriate within the context of problems that are large in scope, while Strategic Competence refers to the ability to formulate suitable mathematical models and select efficient methods for solving problems (National Research Council, 2001). In mathematics, “adaptive reasoning is the glue that holds everything together,” used by students “to navigate through the many facts, procedures, concepts, and solution methods and to see that they all fits together in some way” (p. 129).

**Statistical Reasoning:** Statistical reasoning may be defined as the way people reason with statistical ideas and make sense of statistical information (Garfield and Gal, 1999). This involves making interpretations based on sets of data, graphical
representations, and statistical summaries. Much of statistical reasoning combines ideas about data and chance, which leads to making inferences and interpreting statistical results. Underlying this reasoning is a conceptual understanding of important ideas, such as distribution, center, spread, association, uncertainty, randomness, and sampling.

**Algebraic reasoning:** Algebraic reasoning is the ability to think logically about unknown quantities and the relationships between them. It allows students to reason about expressions, functions, graphs, equations and so forth, and makes it possible for them to draw conclusions and make conjectures about the relationships between them.

**Geometric Reasoning:** Geometric Reasoning is the ability to think logically about geometrical problems and properties of geometrical figures. It helps in developing deductive reasoning with the use of logic, which helps one to expand both mentally and mathematically.

### 1.2.2 Geometrical Reasoning

One of aspect of basic competencies mathematics is reasoning which include step of higher mathematics thinking; include capacity for thinking logically and systematically. Reasoning is a process to reach logical conclusion based on facts and relevant resources. Geometry promote a way of thinking and reasoning about shapes—that is, to observe and describe the relationships within and among geometric shapes, analyze what changes and what stays the same when shapes are transformed, and make generalizations through multi-modal mathematical communication to reinforce the learning of geometric concepts and relevant academic language as the basis for building mathematical convincing arguments. By studying geometry, students develop logical thinking abilities, spatial intuition about the real world, knowledge needed to study more mathematics, and skills in the reading and interpretation of mathematical arguments (Suydam, 1985).

**National Council of Teachers of Mathematics (1989)** mathematical reasoning requires the attainment of abilities to construct mathematical conjectures, develop and evaluate mathematical arguments, and select and use various types of representations.
Mason, Becker & Georges (1991) defined reasoning ability or mathematical thinking as “A dynamics process which, by enabling us to increase the complexity of ideas we can handle, expands our understanding”.

Schoenfeld (1992) has defined reasoning ability as development of a mathematical point of view- valuing the process of mathematization and abstraction and having the predilection to apply them; and the development of competence with tools of the trade, and using those tools in the service of the goal of understanding structure.

Department of education (1997) Reasoning is fundamental to mathematical activity. Active learners question, examine, conjecture and experiment. Mathematics programmes should provide opportunities for learners to develop and employ their reasoning skills. Learners need varied experiences to construct arguments in problem settings and to evaluate the arguments of others.

National Council of Teachers of Mathematics (2000) Reasoning mathematically is a habit of mind, and like all habits, it must be developed through consistent use in many contexts.

Mansi (2003) defined mathematical reasoning as the ability to think coherently and logically and draw inferences or conclusions from mathematical facts known or assumed.

Informally, geometric reasoning is the process of defining and deducing the properties of a geometric entity using the intrinsic properties of that entity, its relationships with other geometric entities, and the rules of inference that bind such properties together in a geometric (Euclidean) space.

Key elements of reasoning in geometry

Through the study of geometry, students are expected to learn about geometric shapes and structures and how to analyze their characteristics and relationships (NCTM, 2000), building understanding from informal to more formal thinking and passing from recognizing different geometric shapes to geometry reasoning and geometry problem solving. Key elements of reasoning in geometry are:

• **Inductive Reasoning** - process of reasoning in which the assumption of an argument supports the conclusion, but does not ensure it
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- **Deductive Reasoning** - process of reasoning in which the argument supports the conclusion based upon a rule
- **Conjecture** - a mathematical statement which has been proposed as a true statement, but which no one has yet been able to prove or disprove
- **Theorem** - a proposition that has been or is to be proved on the basis of explicit assumptions
- **Hypothesis** - a proposed explanation which can be a proposition ("A causes B")
- **Postulate** - a mathematics statement which is used but cannot be proven
- **Axiom** - a formal logical expression used in a deduction to yield further results

NCTM (2010) four key elements of reasoning in geometry are:

- **Conjecturing about geometric objects**: Analyzing configurations and reasoning inductively about relationships to formulate conjectures.
- **Constructing and evaluating geometric arguments**: Developing and evaluating deductive arguments (both formal and informal) about figures and their properties that help make sense of geometric situations.
- **Taking multiple geometric approaches**: Analyzing mathematical situations by using transformations, synthetic approaches, and coordinate systems.
- **Making geometric connections and modelling**: Using geometric ideas, including spatial visualization, in other areas of mathematics, other disciplines, and in real-world situations.

Thus, Mathematical reasoning and proof offer powerful ways of developing and expressing insights about a wide range of phenomena. Those who reason and think analytically tend to note patterns, structure, or regularities in both real-world situations and symbolic objects; they ask whether those patterns are accidental or whether they occur for a reason; and they conjecture and prove. Ultimately, a mathematical proof is a formal way of expressing particular kinds of reasoning and justification. Through the study of geometry, students are expected to learn about geometric shapes and structures and how to analyze their characteristics and relationships (NCTM, 2000), building understanding from informal to more formal
thinking and passing from recognizing different geometric shapes to geometry reasoning and geometry problem solving. The national council of teachers of mathematics advocates the inclusion of geometry throughout the k-12 grades and emphasizes the importance of learning mathematical reasoning skills through the study of geometry (N T C M, 2002). The NTCM further emphasizes the need for students to develop spatial reasoning and visualization skill in the learning of geometry.

1.2.3 Aim and Importance of Reasoning in Geometry

Reasoning and proof are essential in mathematics. If proving is the main activity in geometry, deductive reasoning is its main source. The concept of proof in mathematics is often first introduced in high school Geometry and not seen in a broader view as the more formal aspect to reasoning and justification. When students are asked to comment on proof, some common responses are “I hated proofs,” “Why did I need to prove something that seemed so obvious?” and “I would rather trust the brilliant mathematician who came up with the theorem” (Sowder & Harel, 1998). These words could probably be heard in any typical Geometry classroom. The NCTM (2000) makes its view of the importance of mathematical reasoning clear, emphasizing that being able to reason is essential to understanding mathematics. Aim of teaching geometry, according to Royal society\textsuperscript{\textregistered} J M C (2001), as cited in king (2002) is:

- to provide a breadth of geometrical experiences in two and three dimensions;
- to develop knowledge and understanding of and the ability to use geometrical properties and theorems;
- to develop the skills of applying geometry through modelling and problem solving in real world contexts;
- to encourage the development and use of conjecture, deductive reasoning and proof;
- to develop useful ICT (Information communication technology) in specially geometrical context;
- to engender a positive attitude to mathematics;
- to develop spatial awareness, geometric intuition and the ability to visualize;
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- to develop an awareness of the historical and cultural heritage of geometry in the society, and of the contemporary application of geometry.

In support of this description of geometry and its aims, Jones (2002) suggests that geometry helps the students to develop the skills of visualization, critical thinking, intuition perspective, problem solving, conjecturing deductive reasoning logical argument and proof.

1.2.4 Cognitive Analysis of Geometrical Reasoning Process

Raymond Duval had done a series of extensive studies regarding the cognitive factor in geometry reasoning. Duval (1998) refers to reasoning as one of three independent cognitive processes that fill specific epistemological functions in geometry: visualization, construction and reasoning. According to Duval, “the word ‘reasoning’ is used in a very broad range of meanings. Any move, any trial and error, any procedure to solve a difficulty is often considered as a form of reasoning”.

In geometry the given information, or the available information, is given under visual organisation of nD/2D gestalts and under some semantically networks from which not only gestalts and objects can be named, but also from which questions, hypotheses, conjectures about gestalts, objects and their relations can be generated. And this given information must be processed at a representational and symbolic level, even if some models can be physically constructed. Duval (1998) introduced three cognitive processes in geometry involved in proof, namely

- **Purely configural process**: which means operative apprehension;
- **Natural discursive process**: which is performed in speech through description, explanation, argumentation;
- **Theoretical discursive process**: which is performed through deduction.

He argued that purely configural process can be embedded into a natural discourse but cannot be embedded into a theoretical discourse. The gap between the purely configural process and the theoretical discursive process in proof contributes to most of the difficulties encountered by students. Duval further argued that there is always a gap between the natural discursive process, which is closer to ‘everyday’ language and some things can be designed as “figural reasoning”, and the theoretical
discursive process, which is performed in a purely symbolical register or in the natural language register.

Duval (2007) suggested three approaches in analyzing the cognitive complexity of reasoning which echo the various factors involved:

• Distinction of operative status of proposition in deduction (hypotheses, theorem, etc.)
• Distinction between truth value and epistemic value of a proposition;
• Complexity of proposition meaning within the different possible organizations of propositions underlying various type of reasoning.

Analysis of students’ cognitive factors on geometry reasoning should also take into account students’ developments in geometry learning. Duval’s theory offers a broad framework in which to view cognitive growth in geometry.

1.2.5 Cognitive Growth Processes in Geometrical Reasoning

Reasoning skills develop at a rapid pace in early childhood. Even before they enter school, children have many experiences with refining their thinking as their reasoning skills advance. For example, when young children first learn the word “ball,” anything in their environment that is round is called a ball: The moon is a ball. A watermelon is a ball. But as children begin to identify additional characteristics of a ball (round, spherical, a toy), they refine their thinking about the items that do not fit this more sophisticated understanding. New words become part of their vocabulary to describe the items that were once known as “ball.”

Reasoning with mathematical concepts follows a similar pattern. Children begin to reason about big mathematical ideas, and, as they mature, they refine or expand their ideas with the use of logical thinking. By making conjectures -- guesses made on the basis of experience or information -- and testing those conjectures, students make sense of the mathematics they are learning.

Visualisation in Developing Reasoning

In geometry, visualisation covers together perceptive, discursive and operative apprehension of a representation of space. And because it does not require mathematical knowledge, visualisation plays a basic heuristic role and, through the
operative apprehension, can give something like convincing evidence Duval (1998). He investigated the relationships among different apprehensions, especially perceptual with discursive and perceptual with operative. Relationships with the different kinds of reasoning can start from the two following typical behaviours:

### NAIVE BEHAVIOUR

**Perceptive apprehension** → **operative apprehension** → **a natural discursive process**

1) Dimensional change
2) Anchorage change?
3) A discursive apprehension

Among possible other ones

### MATHEMATICAL BEHAVIOUR

**Perceptive apprehension** → **operative apprehension** ---→ **a theoretical discursive process**

**Of a gestalt configuration**

**Dimensional change**

**Anchorage change?**

**A discursive apprehension**

Among possible other ones

**Figure 1.7: Two typical behaviours of reasoning (Duval, 1998)**

In figure 1.8, starting from the same geometrical situation (on left), two wholly different behaviours are possible. One reacts at what is spontaneously visible (0) and reasoning works like a description of the steps of the configural change leading to a solution (2). In order, reasoning starts only from the discursive apprehension and is independent from visualisation (3). The purely configural change does not give the steps and the organisation of deductive reasoning for the proof, but it shows some key points or an idea which allows to select the main theorems to be used (dotted arrow 2).

For some geometrical situations naive behaviour is efficient, but under very narrow conditions. The gap between the perceptive apprehension and the discursive one, due to the dimensional change and the anchorage change, must be small (arrow 1
in figure 1.6). He claims that there are several triggering or inhibiting factors which may influence the visibility of relevant re-configuration of a given figure. Duval (1998) thinks that visualisation and reasoning is a “double gap” between naive behaviour and mathematical behaviour. The one is about the visualisation and the other is about reasoning. Thus some specific skills must be developed from the common way of looking at figures and from the natural discursive reasoning. The main problem of teaching and learning geometry is how to get pupils to step over this double gap. More recent researches show that “…it is the task that determines the relation with figures. The way of seeing a figure depends on the activity in which it is involved” (Duval, 2006).

Duval (2006) analyses and classifies the different ways of seeing a figure depending on the geometrical activities presented to pupils. He distinguishes four ways of visualising a figure: by a botanist, a surveyor, a builder and an inventor. Botanists and surveyors have ‘iconic visualisation’, and perceive the resemblance between a drawing and the shape of an object. Builders and inventors on the other hand have ‘non-iconic visualisation’, and their perception is based on the deconstruction of shapes. Duval analyses the introduction of supplementary outlines, which he thinks fundamental in ‘non-iconic visualisation’, in particular he discusses re-organising outlines which allow to reorganise a figure and thus to reveal in it parts and shapes that are not immediately recognizable. He also discusses the méreologique decomposition of shapes, a division of the whole into parts which can be juxtaposed or superimposed, with the aim of reconstructing another figure, often very different to the starting figure. This allows the detection of geometrical properties needed to solve a problem, using an exploration purely visual of the figure initial. He distinguishes three kinds of mereological decomposition: material (with cutting and rebuilding as in a jigsaw puzzle), graphic (using reorganising outlines) and by looking (with the eyes, not “mentally”).

1.3 ACHIEVEMENT

In the preceding sections of this chapter theoretical structure considerations have been made on Duval’s cognitive model of geometrical reasoning and its comparison with other perspective. It is utmost important to note that Raymond Duval developed cognitive model of geometrical reasoning is an attempt to encounter
difficulties in high school geometry. The importance of the model lies on explaining not only why students has trouble in learning geometry but also what could be done to remove these stumbling block to impede learning (Usiskin, 1982). This is why the model is under active investigation for the improvement of performance in geometry. Alternatively speaking, the importance of the model of teaching lies in improvement of its performance. This is why performance in geometry (dependent variable of this present study) has been another consideration of this chapter.

The prominent goal of secondary school geometry is to develop the ability to reasons/write proof, so in the course of considering performance in geometry. Criterion Performance has been the main focus. Though more emphases has been on reasoning, proof performance in geometry; performance in school geometry, on whole has also been considered.

**Concept of Achievement**

Achievement is one of the most important goals of education. Academic achievement is a measure of understanding or skills in a specified subject or group of subjects combined. The problem, why students achieve or fail to achieve in the school has always interested psychologist and educators. Different authors have defined the concept of achievement/academic achievement differently and they cover different dimensions of achievement.

Wolman (1973) academic achievement is the degree or level of proficiency attained in scholastic or academic work.

Hawes & Hawes (1982) defined achievement of a successful accomplishment or performance in particular subject, area or course, usually by reasons of skills, hard work and interest, typically summarized in various types of grades, marks, scores or descriptive commentary. The terms achievement is defined in terms of performance of particular course or subject, and performance in terms of skills and interest.

The meaning of academic achievement as mentioned in the dictionary of education (Good, 1989) is the knowledge attained or skill developed in the school subject, usually designated by test score or marks assigned by the teacher.

A test designed to measure the knowledge or proficiency of an individual is something that has been learned or taught (Webster, 1998).
Ladson-Billings (1999) academic achievement represents intellectual growth and the ability to participate in the production of knowledge. At its worst, it represents inculcation and mindless indoctrination of the young into the cannons and orthodoxy of the old. Ladson-Billing version seem to be wider and concerns with processes of obtaining knowledge and intellectual growth at best.

Megargee (2000) achievement tests how well students have mastered the subject in the course of instruction.

Vibha (2001) Achievement in the school may be takes to mean any desirable learning that is observed in the students since the word desirable implies a value judgement it is obvious that a particular piece of learning may be referred to as achievement or otherwise depending on whether it is considered desired or not.

Nash & Stevenson (2004) Achievements are accomplishments such that goals we have set ourselves are achieved. As we socially construct our selves, they usually compare favorably against similar goals of other people.

Tuan, Chin & Shieh (2005) Achievement goals refer to a desire to accomplish learning tasks to increase one’s own competence.

Best & Kahn (2006) Achievement tests attempt to measure what an individual has learned his or her present level of performance. They are used in diagnosing strengths and weaknesses and as a basis for awarding prizes, scholarships, or degrees. Many of the achievement tests used in schools are nonstandard zed, teacher-designed tests.

Random house dictionary (2011) achievement connotes final accomplishment of something noteworthy, after much effort and often in spite of obstacles and discouragements.

The above definitions indicate that academic achievement represents the core of the terms educational growth or intellectual growth. In a broader sense, it has been taken as process of obtaining knowledge and intellectual growth, not necessary limited to knowledge, skills and understanding in the school and college subjects. It seems customary for schools and colleges to be concerned to a greater extent with the development of knowledge, understanding and acquisition of skills in different subjects or disciplines. Achievement is concerned with quality and quantity of
learning in a subject or group of subjects, and it is typically summarized in various grades, marks, score or descriptive commentary (Hawes & Hawes, 1982).

In short, achievement as a test designed to measure knowledge, understanding and skill in a specified subject or group of subjects. Achievement has been regarded as a very important indicator to judge the quality of education. Hence academic achievement is concerned with the quantity of learning attained in a subject of study, or group of subjects after a period of instruction.

1.3.1 Assessment of Academic Achievement

Student assessment is clearly central to standards. If the work of students is not assessed by valid and reliable methods, standards cannot be rigorous. (Higher Education Quality Council, 1997, pp. 8, cited in Webster et. al, 2000)

Perhaps no one would deny the importance of academic achievement in child life. The success or failure of a student is measured in terms academic achievement. It is the common observation that success in the academic field serves as an emotional tonic and any damage done to a child in the home or neighbourhood may be partially repaired by the success in the school. High achievement in school built self-esteem, self-confidence and strengthens self-efficacy that leads to better adjustment with the groups. Good academic record to certain extent predicts future of the child. Today, at the time of admission for entrance in job, for scholarship, for future studies, good academic record is the only yardstick. Whatever one’s interest, attitude or aptitude may be one can’t underestimate the importance of academic record. It also helps the teacher to know whether teaching methods/models are effective or not and helps them in bringing improvement accordingly. Thus, assessment of academic achievement helps both the students and teacher to know where they stand.

The assessment of academic achievement has long been a routine part of all educational processes. It has two purposes:

1. Specifying and verifying problems; and
2. Making decision about students

It aims to assist professionals in making decisions about referral, screening, classification, instructional planning and student progress.
Psychological tests are among the most useful tools of testing achievement for they provide the data for most experimental and descriptive studies. Achievement as performance in school or college is done through tests, usually through teachers made tests. Teacher made test are used to identify specific objectives that have previously been taught and to evaluate the degree to which students have mastered these objective. The continuous process of assessing achievement, school have relied on larger scale evaluation of student’s achievement.

Several methods are used to measure child academic performance, including standardized achievement test scores, teacher ratings of academic performance, and report card grades. Standardized achievement tests are objective instruments that assess skills and abilities children learn through direct instruction in a variety of subject areas including reading, mathematics, and writing (Sattler, 2001). Teacher rating scales allow teachers to rate the accuracy of the child’s academic work compared to other children in the class, and allow for ratings on a wider range of academic tasks than examined on standardised achievement tests (DuPaul, Rapport & Perriello, 1991). Report card grades allow teachers to report on classroom academic performance. Both standard tests and teacher made tests have their advantages and limitations towards effective evaluation of achievement. While standardized tests are available for much different purpose, the content of such tests may not conform closely enough to the local program of instruction. Teacher made tests have the advantage of being suit entirely in the context of the local teaching situation, but it is not so well defined as in the case of standardized tests.

Methods of assessing or measuring achievement can also be categorized into the following types

- **Norm Reference tests**
- **Criterion Reference tests**
- **Performance tests**
- **Curriculum-based Assessment**

**Norm Reference tests:** A norm reference test compares students’ achievement relative to other student’s achievement (Cohen, Manion & Marrison, 2007). Such tests are used to compare a student’s performance to a norm or average of performances of
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similar grade/age peers. Thus a major feature of the norm reference tests is its ability to discriminate between students and their achievement.

Criterion Reference tests: This type of test provide information about a student’s level of proficiency in or mastery of some skill or set of skill and this is accompanied by comparing a students’ performance to a standard mastery called a criterion (Kubiszyn & Borich, 1987). A criterion reference test does not compare students with students, rather, requires a student to fulfil a given set of criteria, a pre-defined and absolute standard or outcome (Cohen, Manion & Morrison, 2007). A criterion referenced test provides the researcher with the information about exactly what students have learned, what they can do, whereas a norm referenced test can provide the researcher with information on how well one student has achieved in comparison to another to provide variability of greater range of scores.

The question in use of data from criterion referenced examination results arises when such data are used in a norm referenced way to compare student with student, school with school and region with region. More recently an outgrowth of criterion reference testing has been the rise of domain referenced tests, where considerable signification is accorded to the careful and detailed specification of the content or domain (Cohen, Manion & Morrison, 2007) test items are selected from the full content or domain with careful attention to sampling procedure so as to represent wider field, and then inferences are made from the limited number of items to the student’s achievement in the whole domain.

Performance based tests: Performance tests, usually administered individually, require the subjects to manipulate objects, or mechanical apparatus to provide indication that the subject learned skills as demonstrated through materials that is produced while their actions are observed and recorded by the examiner (Best & Kahn, 1999)

Curriculum- based Assessment: Curriculum based assessment represents attempts to assess a student’s performance using expected curriculum objectives as the data for evaluation. There are multiple model of curriculum based assessment, but all models are focused on evaluating student progress in an ongoing manner directly form a curriculum.
More than mentioned above, there are many ways of assessing achievement one most commonly used is achievement test. Under ideal conditions, achievement or aptitude tests measure the test performance of which individual are capable (Best & Kahn, 1999). Achievement tests measures the current status of individuals with respect to proficiency in the given areas of knowledge for curriculum areas such as reading and, mathematics, and also in the form of comprehensive batteries in several different areas. Standardized achievement tests are carefully developed and are available for curriculum such as reading and mathematics, and in terms of comprehensive batteries in several different areas. A diagnostic test is also a type of achievement test yielding multiple scores for each area of achievement. In the school survey for the past several decades, achievement tests have been used extensively in the appraisal of instruction (Best & Kahn, 1999).

1.3.2 Factors Affecting Achievement

Academic achievement is considered to be the unique responsibility of educational institutions. Knowledge of level of correction between different factors and academic achievement is, therefore, necessary for a teacher in ascertaining what contributes high and low achievement of students. This is also of great concern to the parents, institutions and society. Truly speaking the future of any institution depends on the academic achievement of its students.

There are innumerable factors which affects academic achievement viz. intelligence, personality, motivation, school environment, heredity, home environment, learning experiences of school and class in particular etc. The factors like interest, aptitudes, family background and socio-economic status of the parent also influence of academic achievement.

There are several factors that are responsible for high and low achievement of the students and these factors can be grouped into two broad classes: subjective factors and objective factors.

Subjective and Psychology factors

These are related to individual himself while influencing one achievement e.g. intelligence, learning ability, motivation, self-efficacy, learning style, study habits etc.

Objective or/ & Environmental factors
Introduction

These factors conforming to the environment of the individual include socio-economic status, educational system, family environment, evaluation system, value system, teacher’s efficiency, school situation and environments etc.

Factors affecting achievement have been classified into following categories of their sources:

**Cognitive factors:** like intelligence, creativity, ability, learning rate, reasoning ability etc.

**Affecting factors:** like values, interests, self-efficacy, perseverance, stress etc.

**Home related factors:** socio-economic status, family size, birth order, gender bias, parental involvement, and parental expectation working status of parents.

1.3.3 Achievement in Geometry

Geometry is the mathematics of space (Bishop, 1983) and the study of geometry helps students represent and make sense of both the world in which we live and the world of mathematics. Geometry offers a means of describing analyzing, and understanding the world and seeing beauty in its structure (NCTM, 2003). It connects the real world to true mathematical constructs. The most obvious connection is architectural design and constructs. An architect can design a beautiful building that incorporates many geometric shapes and figures. When the design becomes a blueprint, a contractor must use tools such as the distance formula or trigonometry to determine the length of materials or the angle in which corners need to meet so that the building can properly stand. Geometry helps us bring our design and ideas of life in the real world.

Cohen (1995) defined Geometry as “Geometry is vehicle that provides much of the basic core of knowledge that the student of mathematics should possess-basic geometric facts, structures of the system, applications, a study of two-and-three dimensional space……. introduction of forms of argument, introduction to forms of proof, the deductive skills that reach for away places (an author writing in a lucid style or a lawyer preparing a strong court case),building the foundation for the coursework of trigonometry, analytic geometry, or calculus”.

The ability to write proofs, reason is considered one of the prominent goals of high school geometry, while the main goal of geometrical instruction in junior high school is to help the students learn to carefully, organize and extent their
understanding of geometrical concepts and relationships. In other words, the geometry of junior high school which focuses mainly on concepts and relations gained through experiment, intuition and conjecture, geometry course of high school level focuses mainly on giving student deeper insight into how those geometric facts and relations may be structured by logical processes. So the students’ performance in geometry (achievement in geometry) includes:

- Naming and recognizing common geometric forms;
- Illustrating and defining geometrical concepts;
- Relating geometrical relations for elementary and junior high school grades, and
- Ability to establish the relations logically (deductively) in geometric context for high school grades.

Traditionally, it has been a common practice to attach the label informal or intuitive geometry in elementary and junior high schools and formal geometry in senior high schools (Usiskin, 1987).

Achievement processes usually are explained by characteristics of students and their learning environments. The achievement levels of children studying in English medium schools were analyzed separately. The mean score was 47.8% in mathematics. About 38 percent of children failed to score more than 40% in mathematics. Therefore the general impression that all is well with English medium schools is not correct (Aggarwal, 2001).

Mathematics remains the most preferred subject, with a third of students in classes six to eight rating it as number one, and over 21% still feeling the same way in classes 11 and 12. (Shukla, 2005)

Learning geometry may not be an easy and large numbers of students fail to develop an adequate understanding of geometry concept, geometry reasoning and geometry problem solving skills (Clements & Battista, 1992; Mitchellmore, 2002). The lack of understanding in geometry offer, causes discouragement among students which invariably will leads to poor performance in geometry. A number of factors have been put forward to explain why learning geometry is difficult-- visualization abilities and ineffective instruction apart from these other factors are:
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1.3.4 Factor Affecting Geometry Performance

Despite geometry being an important branch of mathematics, there are many challenges in teaching and learning it. According to Freudenthal (1971) geometry failed because it was taught in such a manner that its deductivity could not be reinvented by the learner but imposed. Other factors are:

Use of physical objects (Models & Manipulative)

Manipulatives can come in a variety of forms and they are often defined as “physical objects that are used as teaching tools to engage students in the hands-on learning of mathematics”. Manipulative materials are concrete models that involve mathematical concepts appealing to several senses, including the socio-cultural needs that learners can touch and move. These are physical objects, such as tangrams, cubes, base-ten blocks, and geometric solids that can make abstract ideas and symbols more meaningful and understandable to students (Heddens, 2005).

“Manipulative help student learn by allowing them to from concrete experience to abstract reasoning”

“I hear and I forget,
I see and I remember,
I do and I understand”. Confucius (551-479 BC)

Manipulative can be used in teaching a wide variety of topics in mathematics, including the objectives from the five N C T M standards: Problem solving, communicating, reasoning, connections, and estimation. Manipulatives help students learn by allowing them to move from concrete experiences to abstract reasoning. When students manipulate objects, they are taking the first steps toward understanding math processes and procedures. The effective use of manipulatives can help students connect ideas and integrate their knowledge so that they gain a deep understanding of mathematical concepts.

Manipulatives help relieve boredom in learners as they offer a change from the textbook (abstract) method of learning, thereby allowing learners to explore and use their imagination. Manipulatives provide a picture of a mathematics concept that appeals to visual/spatial learners and they provide stimulation for those who are not. Visualisation is the natural way one begins to think, since before actually verbalising
(using words), images first need to emerge before one can write or speak. Manipulatives can also be placed within cooperative groups, which is appealing to the interpersonal learners.

Manipulatives can be extremely helpful for young children, but they must be used correctly. Smith (2009) stated that there are probably as many wrong ways to teach with manipulatives as there are to teach without them. The math manipulatives should be appropriate for the students and chosen to meet the specific goals and objectives of the mathematical program.

Manipulatives not only allow students to construct their own cognitive models for abstract mathematical ideas and processes, it also provides a common language with which to communicate these models to the teacher and other students.

Resnick (1998) presented the following advantages of manipulative use:

• Manipulatives are extraordinary tools to help reach weaker students, but that is not their only purpose: they are a useful way to improve education in any math class.

• Manipulatives provide an environment to teach math as well as pedagogy to teachers. Often, teachers are ineffective because of their own limited understanding of the material.

• Manipulatives do not make math "easy", and teachers may need to learn something in order to use them. The increased understanding will serve them whether or not they use manipulatives in their class in the immediate future.

• There is no sense in using manipulatives in a "do as I say" algorithmic model which only perpetuates antiquated pedagogy. It is far more effective to use them as a setting for problem solving, discussion, communication, and reflection.

• Manipulatives should be a complement to, not a substitute for other representations. In particular, Cartesian graphing and other pictorial representations are extremely important.

• Deliberate attention must be paid to help students transfer what they know in the context of the manipulatives to other representations, including symbolic, numerical, and graphical.
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Transfer of learning does not happen spontaneously one can deduce from this that Manipulatives have an important role to play in the conceptual understanding of mathematics. According to Duval model(1998) learning geometry includes three types of cognitive processes, each of them having specific function: visualisation (space representation of 2D or 3D configurations), construction (of models representing geometrical structures), and reasoning (to organise descriptive discourses). To do processes of visualisation and construction, students have necessarily to manipulate instruments to make drawings, or to interacts with computers. Also to do some types of reasoning processes, namely those which are not totally abstract, students take advantage of instruments as help for and base of their reasoning.

Clements (1999) recommends that manipulatives should not be used as an end, without careful thought, rather than as a means to an end. He states further that a manipulatives physical nature does not carry the meaning of a mathematical idea but it should be used in the context of educational tasks to actively engage children's thinking with teacher guidance.

However, the researcher believes that there are no obstacles that cannot be overcome to make the use of manipulatives enjoyable, fun, and beneficial in the learning of mathematics. Furthermore, the researcher strongly believes that if students are having fun, then they are more likely to learn. Therefore, any manipulative that could provide fun and learning opportunities simultaneously is likely to be more effective than the traditional "chalk and talk" or "kill and drill" method.

Use of Semiotic Representations

Mathematics is a powerful tool for solving practical problems and a highly creative field of study. One of the reasons is that ideas can be expressed with symbols, charts, graphs, and diagrams (Van de Walle, 2004). Symbols, graphs, and charts, as well as physical representations such as counters, fraction bars, and Cuisenaire rods are also powerful learning tools.

Representation refers to “the act of capturing a mathematical concept or relationship in some form and to form itself” (NCTM, 2000). Representations, in the broadest sense, according to Kaput (1985), are something that indicates something else, and so must essentially involve some kind of relationship between symbol and referent, although each may itself be a complex entity. Moving from one
representation to another is an important way to add understanding to an idea. In addition, different representations of mathematics ideas could be transformed with each other (Van de Walle, 2004).

In addition to number systems there are geometric figures, algebraic and formal notations, graphic representations and natural language, even if it is used in a different way than in everyday language. With regard to the property of semiotic representations that is basic for mathematical activity (Duval, 1995, p. 21) keeping also its modern meanings call them “representation registers” (word already used by Descartes, in La Geom’etrie (Descartes, 1954, p. 8 (p. 300)), stated following four very different types of registers.

<table>
<thead>
<tr>
<th>MULTI-FUNCTIONAL REGISTERS:</th>
<th>NON-DISCURSIVE REPRESENTATION (Shape configurations 1D/2D, 2D/2D, 3D/2D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Processes CANNOT BE made into algorithms</td>
<td></td>
</tr>
<tr>
<td>SYMBOLIC SYSTEMS</td>
<td></td>
</tr>
<tr>
<td>Only written: impossible to tell orally otherwise than by spelling</td>
<td></td>
</tr>
<tr>
<td>Computation, proof</td>
<td></td>
</tr>
<tr>
<td>D2 COMBINATION OF D1 AND D0 SHAPES, oriented (arrows) or not.</td>
<td></td>
</tr>
<tr>
<td>TRANSITIONAL AUXILIARY Representations</td>
<td></td>
</tr>
<tr>
<td>No rules of combination (free support)</td>
<td></td>
</tr>
<tr>
<td>IN NATURAL LANGUAGE: two non equivalent modalities for expressing</td>
<td></td>
</tr>
<tr>
<td>ORALLY explanations,</td>
<td></td>
</tr>
<tr>
<td>WRITTEN (visual): theorem, proofs . . .</td>
<td></td>
</tr>
<tr>
<td>ICONIC: drawing, sketch, pattern</td>
<td></td>
</tr>
<tr>
<td>NON-ICONIC: geometrical figures which can be constructed with tools</td>
<td></td>
</tr>
</tbody>
</table>

**Figure 1.8: Classification of the different representation registers which can be mobilized in mathematical processes**

Depending on the domain or the phase of problem-solving one register may explicitly dominate, but there must always be the possibility of passing from one register to another. The symbolic representations pose problems for the students.
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Duval (2002) has claimed that there is no direct access to mathematical objects other than through their semiotic representations.

It helps people to understand mathematical ideas, and to use these ideas. When students develop flexibility with a variety of representations for mathematical ideas, not only do they add to their own understanding, but also obtain skill in applying mathematical ideas to new areas and communicating ideas to others. (Van de Walle, 2004). More generally, the aim of representations is to develop abstract ideas.

Use of Genuine Artefacts

'Genuine artefacts' included brochures, menus, bus timetables and photographs. Such artefacts were relevant to the children because they allowed them to make connections to real-world experiences, offered significant reference to concrete situations, allowed them to keep their reasoning processes meaningful (Kaput 1994; van Oers 1996), and enhanced their capacity to think metacognitively (Lowrie and Clancy 2003). Bennett, Harper and Hedberg (2002) commented that the quality and nature of authenticity of an artefact will depend on (a) the level of sensory fidelity in task representation so that practical skills may be developed; (b) the extent to which critical thinking or problem solving can be enhanced; and (c) the potential for social interaction and engagement. Genuine artefacts provide opportunities for students to develop skills in knowing when and how to use mathematical knowledge for representing and solving problems in both practical and realistic situations (Lowrie 2004).

As Boaler (1993) noted: The reasons offered for learning in context seem to fall into two broad categories, one concerning motivation and interest of students through an enriched and vivid curriculum, the other concerning the enhanced transfer of learning through a demonstration of links between school mathematics and real world problems.

The challenge for teachers is to establish a learning environment that encourages students to personalise learning in ways that allow individuals to extend, adapt, revise and adopt mathematical ideas to a context that they can place themselves within. Bonotto (2002) proposed that classroom-based activities that aim to create connections between reality and mathematics should be founded on the use of cultural (genuine) artefacts.
An artefact may be considered generally as any human creation, such as physical tools, production schemes, language or skills. Artefacts used for supporting learning, such as concrete materials designed for educational use, are ‘secondary’ as compared to ‘primary’ artefacts used directly in the production (Wartofsky, 1979). To make an artefact an ‘instrument’, for example for learning, it is necessary for the user to develop ‘utilisation schemes’, i.e. ways to use the artefact (Strässer, 2004).

According to Houdement (2007) when a person encounters a problem, he/she handles the problem with his/her personal Geometrical Working Space. It is the place organised to ensure the geometrical work. It puts in a network the objects whose nature depends on the geometrical paradigm, the artefacts like drawings tools, computers but also rules of deduction used by the geometer and the theoretical system of reference possibly organised in a theoretical model depending on the geometrical paradigm. An expert solving a problem of geometry creates a suitable geometrical working space. When he/she has decided what geometrical paradigm is convenient for the problem, he/she can organize the use of artefacts and the type of reasoning.

Use of Language

Mathematics is a universal language. Therefore the mathematical knowledge begins with the acquisition of linguistic knowledge. Geometry has today become the most widely used heuristic language, and this is why it is felt that students should have advantage of learning it (Gleaser, 1986). Language plays a critical role in regulating cognitive activity-the vocabulary an individual has at his disposal affects what he think about (Ellis & Siegler 1994). Language helps students to formulate conjectures and convincing argument or proofs, especially in geometry. Schiro (1997) explains that language helps students to make connections. It assists them to see how mathematical ideas can be expressed in different ways, to link informal and intuitive mathematical meanings to more formal, abstract symbolism; to see connections among various forms of mathematical representation (for example, oral, written, concrete, pictorial, numerical, graphical, algebraic and geometrical).

In mathematics, reasoning is used to solve problems, and also to decide whether an assertion (e.g., an answer to a problem) is correct. In problem solving, a learner’s Language helps students learn mathematics by constructing new meanings.
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acquiring new understandings and information, and developing new skills (Schiro, 1997). For example, a student can examine examples and non-examples by using the material presented to him or her. Here he or she will use his or her own everyday language to describe and think about the problem (Van Hiele, 1986). By so doing, they will be able to put their mathematical thoughts into words or symbols. This will result in discovering, objectifying, and confronting their meanings. As a result of the manipulation of materials and the completion of simple tasks set by the teacher, the need to talk about the subject matter becomes important (Pegg, 1995). This encourages students to better reflect on, organise, clarify, evaluate, comprehend, revise, and express their ideas.

The language further help students to access, understand, monitor, and orchestrate their own mathematical constructs that will facilitate their enhancement, development and reconstructions (Schiro, 1997). Furthermore, through language, students learn to share the specialised language, knowledge, traditions, and affective stance of mathematics in general and geometry in particular. When students are given a variety of activities and are expected to find their own way to a solution, language will help them relate new experiences to previous experiences in ways that facilitate assimilation and accommodation (Schiro, 1997). This personalisation will help them to accept the relevancy, meaningfulness, and usefulness of mathematics in their everyday lives (Genz, 2006).

Clarkson (2003) explains that language is vital in the learning process, because students need to discuss and share experiences and ideas and to describe, explain and record mathematics in their own language. It follows that reflecting on learning and recording mathematical ideas in written language can clarify, demonstrate understanding and prompt new thoughts. Geometry is taken as cultural subject matter and as a tool for developing reasoning.

Role and importance of language in the development of geometry thinking has been one of the main concerns of the Duval’s – according to Duval (1995a) in elementary geometry, there is no algorithm for using figures in a heuristic way and the way a mathematical proof runs in natural language cannot be formalized but by using symbolic systems. Proofs using natural language cannot be understood by most students (Duval, 1991).
Use of Figures/diagram

There is a long tradition of using diagrams as visual aids to learning geometry. There is good evidence that diagrams were integral to mathematics in much of ancient geometry in India (Swetz, 1995; van der Waerden, 1983), China (Swetz, 1995; Swetz & Kao, 1977), and the Near East (Netz, 1998; van der Waerden, 1983). Modern mathematicians, however, have been suspicious of diagrams, suggesting that the universal mathematical essence that is to be abstracted from a proof cannot be captured by the particularity of a diagram. Geometrical figures are the basis elements which play a significant role in the history of geometry and mathematics generally from the first scientific foundations of geometry (Euclidean elements) up to now. The teaching also of in education uses the geometrical figure, as necessary tools for the foundation of the geometrical concepts.

A figure constitutes the external and iconical representation of a concept or a situation in geometry. It belongs to a specific semiotic system, which is linked to the perceptual visual system, following internal organization laws. As a representation, it becomes more economically perceptible compared to the corresponding verbal one, because in a figure various relations of an object with other objects are depicted (Mesquita, 1996).

Geometrical figures are simultaneously concepts and spatial representations. In this symbiosis, the figural facet is the source of invention, while the conceptual side guarantees the logical consistency of the operations (Fischbein & Nachlieli, 1998).

In most cases in geometry class, diagrams are intended as models and are meant to represent a class of objects. Nevertheless, every diagram has characteristics that are individual and not representative of the class.

An early attempt to distinguish between diagrams and figures is present in the work of Parzysz (1988) who distinguished between figure and drawing. He says, “the figure is the geometrical object which is described by the text defining it” (p. 80). Drawings, on the other hand, are two-dimensional representations of figures. Further addressing the complex distinction between diagrams (drawings) and figures, Raymond Duval (1995) identified four different processes (grasps or apprehensions of the figure) that individuals could deploy when working with diagrams: (1) perceptual, (2) sequential, (3) discursive, and (4) operative apprehension. Perceptual grasp
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involves paying attention to characteristics of parts of a diagram or the diagram as a whole. By applying perceptual grasp an individual looks at characteristics of the diagram (such as the size, the shape, and the orientation of the diagram), divides the diagram into smaller sub-figures, and recognizes a figure by its name. Sequential grasp involves organizing the construction of a diagram into a series of steps, especially when using construction tools. The choice of construction tools limits the kind of procedure involved for making a diagram. For example, if one were to construct the angle bisector of a given angle with a compass and a straightedge, the procedure would involve a sequence of steps in a particular order. In contrast, if one were to draw freehand the angle bisector of an angle, the procedure would not require that many steps and a sketch of the angle bisector would suffice. Therefore, sequential grasp yields different “figures” depending on what kind of tools are being used to produce them. Discursive grasp is the process of producing or using information about the diagram. This information could be encoded in the diagram by means of imposing symbols, but also could be expressed in descriptive statements about the diagram. Finally, operative grasp is the process of modifying a diagram, for example, by subdividing the diagram into its components or by adding other elements to the diagram such as auxiliary lines.

Duval has argued that the work with geometric figures involves perceptual grasp plus another one of the other three processes. According to Duval (1995), mathematical work with diagrams uses the four kinds of grasps. These grasp processes are not isolated, but connected, especially when considering the kinds of resources that students have available to solve problems. For example, if students were to have available dynamic geometric software to solve a problem, it is likely that they would have to use sequential grasp to consider the order of steps to make a diagram. Knowledge of the geometric object—the figure—involves coordinating the different apprehensions of a diagram, work that may rely notably on proving, for example when showing that the diagram produced by executing as series of steps (sequential grasp) indeed has the properties described in informational text about the diagram (discursive grasp).

As mentioned by Mitchelmore (1980), use of more manipulative have been found effective in teaching geometry at elementary level and more diagrams at higher levels. Figures have been increasingly used in the subject of geometry, Analyzing from a curricular
perspective the various ways of conceptualizing geometry. Geometry is taken as the study of visualization, drawing and construction of figures (Usiskin, 1987). Geometry has been a vehicle for representing mathematical or other concepts whose origin is not visual or physical and the figures or diagrams have been one of the main agents for such purposes.

Therefore, one could expect that mathematical reasoning would involve the active use of diagrams not just for communicating ideas, but as a means towards working on a problem. With regards to diagrams, Pólya (1945) said that these are useful not just for solving geometric problems, but to solve other problems as well. He provided different suggestions to work with diagrams.

Clements & Battista (1992) mentioned that diagram accompanying the discussion of a geometric statement is not always helpful in reasoning, and students often attribute characteristics of a drawing to the geometric object it represents and fail to understand that drawing do not necessarily represent all known information about the object represented.

A research by Deliyianni, Elia, Gagatsis, Monoyiou and Panaoura (2009) confirmed the role of perceptual, operative and discursive apprehension in geometrical figure understanding using Confirmatory Factor Analysis (CFA). Moving a step forward, Elia, Gagatsis, Deliyianni, Monoyiou and Michael (2009) investigated the role the mereologic, the optic and the place way modifications exert on operative figure understanding and they verified a model.

Use of Computer

Technology is promoted and effective tool to teach and learn geometry. When technology is used appropriately, it can provide a rich environment in which students' geometric understanding and intuition can be developed. Calculators and computers with appropriate software transform the mathematics classroom into a laboratory much like the environment in many science classes, where students use technology to investigate, conjecture, and verify their findings (NTCM, 1989). Computer allows its users to explore, investigate and pose problems, and to offer flexible representations of situations on symbolic and formal level. This ability is the main feature of the computer. Computers provide an ideal medium for studying geometry. Geometry permits interesting recent developments based on the new access to direct
manipulation of geometrical drawings. Thanks to manipulation of geometrical drawings for providing to view conceptualization in geometry as the study of the stationary properties of drawings while dragging their components around the screen. The statement of a geometrical property now becomes the description of a geometrical phenomenon accessible to observation in the new fields of experimentation. It has been revealed that computer based learning have some benefits for teaching geometry, and it was slightly better for teaching verbal concepts related to geometry. However, the traditional approach was better for teaching nonverbal ideas (Kantowsky, 1981). In the computer environment, students' geometrical performance was affected by continuous variation of geometric figures (Kakihana & Shimizu, 1994). Computer Software like Logo, The Geometric Supposer, Geometer’s Sketchpad and Cabrigéométre allows mathematics to be taught visually to the class as a whole, to small groups, or to individuals by creating dynamic and productive three way interaction between teacher, student, and computer. It enables students and teachers to investigate and construct unlimited geometric shapes. The shapes are first created and then they are explored, manipulated and transformed to ideal concept. Students cannot be creative enough in a traditional class (Schoenfeld, 1989).

There are numerous studies about the effects of computer based learning and dynamic geometry software to develop students’ understanding in geometry (Hativa, 1984; Işıkşal & Aşkar, 2005; Jones, 2000; Jones, 2001; McCoy, 1991; Marrades & Gutiérrez, 2000; Scher, 2002; Straesser, 2001; Velo, 2001).

1.4 SELF-EFFICACY

The construct of self-efficacy beliefs constitutes a key component in Bandura’s (1977a) social cognitive theory; it signifies a person’s perceived ability or capability to successfully perform a given task or behavior. According to Bandura every individual possess a belief system that exerts control over his/her thoughts, emotions and actions. Among the various mechanisms of human agency, none is more central or pervasive than self-efficacy beliefs (Bandura & Locke, 2003; Pajares, 2000).

Bandura (1977a) defined Self-efficacy expectations as beliefs about one’s ability to perform a given behaviour successfully and thereby bring about desired consequences.
Bandura (1986) and Tanner & Jones (2003) defined self-efficacy construct as people’s judgement of their capabilities to organize and execute courses of action required to attain designated types of performances; it is concerned not with the skills one has but with the judgements of what one can do with whatever skill one possesses.

Schunk (1989) defined self-efficacy as student’s belief about their academic capabilities for learning which includes the evaluation of what the learning context requires and how capable one is in utilizing knowledge and skill to bring about new learning.

Woolfolk (1993) defined self-efficacy as "the ability of a person to organize his / her skills and his / he beliefs that s/he can develop efficiency in a newly acquainted situation.

Bandura (1994) self-efficacy is a person’s belief in his or her ability to succeed in a particular situation. Bandura described these beliefs as determinants of how people think, behave, and feel.

Bandura (1995) self-efficacy is “the belief in one’s capabilities to organize and execute the courses of action required to manage prospective situations.”

Henk & Melnick (1995) discussed Bandura’s (1977b, 1982) theory of perceived Self-efficacy as a person’s judgement of her or his abilities to perform an activity and the effect this perception has on the ongoing and future conduct of the activity.

Madeline (1996) defined Self-efficacy as the degree to which the student thinks he or she has the capacity to cope with the learning challenge.

Bandura (1997) defines self-efficacy as one’s perceived ability to plan and execute tasks to achieve specific goals. He characterized self-efficacy as being both a product of students’ interactions with the world and an influence on the nature and quality of those interactions.

Baron & Byren (1997) defines self-efficacy as self evaluation of an individual’s competency or ability to perform.

Schwarzer (1999) self-efficacy can make a difference to people’s ways of thinking feeling and acting. With respect to feelings, a low sense of self-efficacy is associated with depression, anxiety and helplessness. People with low self-efficacy also harbour pessimistic thoughts about their performance and personal development.
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In contrast, a strong sense of belief in oneself facilitates cognitive and executive processes in multiple contexts, influencing, for example, decision making and academic achievement.

**Eysenck (2000)** defines Self-efficacy as an individual’s assessment of his or her ability to cope with given situations.

**Zeldin & Pajares (2000)** Self-efficacy is defined as one’s beliefs about his or her ability to successfully perform specific tasks in specific situations.

**Bandura (2000, 2001)** Efficacy beliefs are the foundation of human agency. Unless people believe they can produce desired results and forestall detrimental ones by their actions, they have little incentive to act or to persevere in the face of difficulties. Whatever other factors may operate as guides and motivators, they are rooted in the four core belief that one has the power to produce effect by one’s actions. His definition of human agency is characterized by four (PAEI) core features which form a concern structure pattern:

![Concern Structure Pattern](image)

- **P** – **Intentionality**: A proactive commitment to bring about a represented future state of events via specific familiar actions (with some improvisation as needed).
- **A** – **Forethought**: Outcome expectations based on observed conditional relationships that help one set long term goals and anticipate problems, rewards and punishment/costs.
- **E** – **Self-Reflectiveness**: Metacognitive processing of one’s own thoughts, feelings, actions and motivations, underlying the capacity to change one’s agentive stance.
- **I** – **Self-Reactiveness**: Self-regulation of motivation, affect and action, guiding performance by personal standards and taking self-directed corrective action.
Pintrich & Schunk (2002) Self-efficacy is defined as learner’s beliefs about their capability of succeeding on specific tasks. It involves students’ perceptions related to the difficulty of a task and possibility of succeeding it (Parsons, Hinson and Brown, 2001; Fetsco & McClure, 2005).

Nowak and Krcmar (2003) Self-efficacy refers to one’s self-belief in his/her capability to attain a specific goal.

Margolis and McCabe (2006) Self efficacy is commonly defined as the belief in one's capabilities to achieve a goal or an outcome. Students with a strong sense of efficacy are more likely to challenge themselves with difficult tasks and be intrinsically motivated. These students will put forth a high degree of effort in order to meet their commitments, and attribute failure to things which are in their control, rather than blaming external factors. Self-efficacious students also recover quickly from setbacks, and ultimately are likely to achieve their personal goals. Students with low self-efficacy, on the other hand, believe they cannot be successful and thus are less likely to make a concerted, extended effort and may consider challenging tasks as threats that are to be avoided. Thus, students with poor self-efficacy have low aspirations which may result in disappointing academic performances becoming part of a self-fulfilling feedback cycle.

In short, social learning theorist’s defined self-efficacy as one’s belief that he/she is able to organize and apply plans in order to achieve a certain task or a sense of confidence regarding the performance of specific tasks.

1.4.1 Self-Efficacy and Related Beliefs

Self-efficacy beliefs differ conceptually and psychometrically from closely related constructs, such as self-concept, outcome expectations, efficacy, self-esteem and self-confidence.

Self-Efficacy versus Self-Concept

Pajares (2008) Self-efficacy beliefs refer to matters related to one’s capability and revolve around questions of “can”, whereas self-concept beliefs refer to matters related to being and reflect questions of “feel”. Academic self-concept is referred as self-perceptions of ability, which affects students’ effort, persistence, anxiety (Pajares, 1996b), and indirectly their performance.
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Self-efficacy is a context specific assessment of competence to perform a specific task or a range of tasks in a given domain an individual’s judgment of his or her capabilities to perform given actions (Schunk, 1991). Whereas Self-concept can be viewed as an “Umbrella” term which encompasses three components: the self image, the ideal-self and the self esteem of an individual. Self concept is the sum total of person’s perceived and desired mental and physical characteristics, as well as the person’s perceived worthiness form this summation, is often referred to as the (Lawrence, 1996) the relative measure between self-image and the ideal-self reflects the individual’s self-esteem or self worth).

Self-efficacy versus self-efficacy beliefs or expectations

Self-efficacy as a notion of one’s complete concept of his or her ability to perform a type of task related to a particular context and domain. Self-efficacy beliefs or expectations, however, are the item-specific tasks and measurements of one’s beliefs that such tasks can be performed. Self-efficacy beliefs or expectations combine together to form one’s overall concept of self-efficacy.

Self-efficacy versus efficacy

Unlike efficacy, which is the power to produce an effect—in essence, competence Self-efficacy is the belief (whether or not accurate) that one has the power to produce that effect by completing a given task or activity related to that competency. For example, a person with high self-efficacy may engage in a more health-related activity when an illness occurs, whereas a person with low self-efficacy would harbor feelings of hopelessness.

Self-Efficacy versus Self-Esteem

Self-efficacy relates to a person’s perception of their ability to reach a goal whereas self-esteem is a measure of feeling proud about a certain trait, in comparison with others (Klassen, 2004). For example, a person who is a terrible rock climber would probably have poor self-efficacy with regard to rock climbing, but this need not affect that person's self-esteem since most people don’t invest much of their self-esteem in this activity. On the other hand, one might have enormous skill at rock climbing, yet set such a high standard for oneself that self-esteem is low (Bandura, 2008). At the same time, a person who has high self-efficacy in general but is poor at
rock climbing might think that he/she is good at rock climbing, or might still believe that he/she could quickly learn.

Self-Efficacy versus Self-Confidence

Confidence is a nondescript term that refers to strength of belief but does not necessarily specify what the certainty is about “I can be supremely confident that I will fail at an endeavour”. Perceived self efficacy refers to belief in one’s power to produce given levels of attainment. A self-efficacy assessment, therefore, includes both the affirmation of a capability and the strength of that belief (Bandura, 1997).

Self confidence is having the confidence that you can do this or that thing. It is about your skills and performance and is conditional. Confidence is conditional upon you being good at the said skill. If you are not good at it, there is no use being confident whereas Self-efficacy is understanding and belief in you that comes from becoming confident at a number of skills.

1.4.2 Sources of Self-Efficacy

Bandura (1994, 1997b) cited four sources of self-efficacy: Mastery experiences, vicarious experiences provided by social models, social persuasion and reliance on somatic and emotional states. Pajares (2002) called the four sources mastery experiences, vicarious experiences, social persuasion and physiological states. According to Bandura, four major sources of self-efficacy are:

Mastery experience: "The most effective way of developing a strong sense of efficacy is through mastery experiences," Bandura (1994). Known also as "performance accomplishments" (Brown, 1999) or “enactive attainment” (Zimmerman, 2000), refers to the way people assess their own personal attainment in a given arena. Students who judge their own past academic results as being successful often develop a high sense of confidence about their abilities while those who view their academic outcomes as unsuccessful are likely to experience feelings of doubts and uncertainty about their own effectiveness.

Vicarious experience (observational): According to Bandura, “Seeing people similar to oneself succeed by sustained effort raises observers’ beliefs that they too possess the capabilities master comparable activities to succeed” (1994). It relates
to the self-evaluation that individuals derive from observing and comparing
themselves with a given “social model” (classmate, a friend etc). When students
observe a given model- that they view as compatible with them- in terms of traits and
skills – succeed at handling a certain situation or solving a given task they are likely
to feel, able to meet a similar challenge. By the same token, watching a similar model
fail in accomplishing the task at hand might undermine their self-confidence.

Verbal persuasions: The conceptions that people develop about their
capacities in a given field are likely to be influenced by the verbal and “tacit” output
they receive from others. Note yet, those verbal and non-verbal messages (like a facial
expression, for instance) become particularly influential when they are emitted by
persons that are regarded as “credible persuaders” (Zimmerman, 2000) and
“believable evaluators” in their own environment such as parents, teachers,
experts...etc. Bandura(1994) also asserted that people could be persuaded to belief
that they have the skills and capabilities to succeed. Consider a time when someone
said something positive and encouraging that helped you achieve a goal. Getting
verbal encouragement from others helps people overcome self-doubt and instead focus
on giving their best effort to the task at hand.

Physiological states: Our own responses and emotional reactions to situations
also play an important role in self-efficacy. Moods, emotional states, physical
reactions, and stress levels can all impact how a person feels about their personal
abilities in a particular situation. A person who becomes extremely nervous before
speaking in public may develop a weak sense of self-efficacy in these situations.
However, Bandura also notes "it is not the sheer intensity of emotional and physical
reactions that is important but rather how they are perceived and interpreted" (1994).
By learning how to minimize stress and elevate mood when facing difficult or
challenging tasks, people can improve their sense of self-efficacy.

1.4.3 Effects of Self-Efficacy

Bandura’s theory emphasizes the role of observational learning, social
experience, and reciprocal determinism in the development of personality. According
to the Social Cognitive Perspective, the environment/learning and cognition are the
determining factors in behavior. Bandura (1994) stated that self-efficacy has
affected human functioning (academic attainment) through "four major psychological processes": Cognitive, motivational, and affective and selection processes:

**At the cognitive level:** the nature of beliefs students hold about their abilities in relation to a given task influences the way they perceive their prospective future academic results. Students who believe in their abilities visualize successful positive outcomes while those who do not trust their capacities are likely to suffer from what Bandura (1997 b) names “cognitive negativity” (A state where they become somewhat “obsessed” by their shortcomings and too skeptic about their capacity to succeed in the face of challenging learning situations)

**At the motivational level:** a high sense of self-efficacy increases students’ readiness to invest efforts in their learning, serves them well to persist when facing difficulties and helps them to recover more quickly after a negative attainment. Conversely, a perceived sense of inefficacy diminishes students’ interest in their learning, lessens from their capacity to resist when facing impediments and undermines their commitment to achieving their goals.

**At the affective level:** a strong perceived sense of competence is likely to reduce the amount of stress students might experience in the course of their learning whereas a low self-estimation of capacity might result in high levels of anxiety and agitation that often lead to in “irrational” thinking that ultimately impair their cognitive and intellectual effectiveness.

**At the selection level:** the conceptions that students develop about their academic abilities are likely to influence the type of decisions they take, the environment they opt for and the kind of choices they select. It is often the case that students often engage in activities in which they feel efficacious while they avoid those in which they feel less competent.

To summarize, self-efficacy refers to the confidence people have in their abilities that they will be successful at a given task. It is determined by enactive mastery experiences, various experience verbal persuasion and physiological & emotional states of these factors, enactive mastery experience has the most influence .Self-Efficacy beliefs vary between individuals, fluctuate under different
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circumstances, and can change over time. Self-Efficacy also contributes to performance connections’ between self-efficacy and academic performance is especially of interest to educator.

1.4.4 Mathematics Self-Efficacy

In the 1970’s, Bandura, saw a gap in the theory of social learning and identified self-efficacy as a key element that fits this gap. Self-efficacy beliefs refer to a person’s belief about their own capability to perform certain actions or tasks effectively to a designated level of performance. According to Bandura (1986) individuals engage in personal ability judgements based on their own unique system of appraisal, the task at hand and the situation at that point in time. This in turn informs their behaviour. Self-efficacy beliefs in the Maths class refer to learner beliefs in their capability to do Maths and to perform at a designated level of performance. Self-efficacy beliefs are context specific which means that a learner may have high self-efficacy beliefs in some aspects of Maths, but not in others.

Kiamanesh, Hejazi, & Esfahani, (2005); Hackett & Betz, (1989) and Pajares & Kranzler, (1995) Mathematics self-efficacy is defined as —a situational assessment of an individual's confidence in her or his ability to successfully perform or accomplish a particular mathematical task or problem”.

Pajares and Miller (1997) self-efficacy for performance in mathematics tasks has been operationalised in mathematics as a score in the measure of an individual’s self-beliefs about the capacity of performance in some particular aspect of mathematics.

Kennedy (1996) math self efficacy is a “critical factor” in career choice.

Pajares (1996) Self-efficacy refers to a set of beliefs regarding a person’s competence to formulate and carry out a particular course of action. Most important, self-efficacy is task-specific and is not conceptualized as global personality characteristic. For example, an individual may have high self-efficacy at solving math problems but low self-efficacy at giving public speeches.

Bandura (1986) introduced a theory of social learning which propagates the notion that the perceptions that people develop about their capability, affects the goals
that they pursue and the degree of control that they have over their environment. Cognition is emphasised because of its role “in people’s capabilities to construct reality, self-regulate, encode information and perform behaviours and is a strong predictor in determining how learners produce their future in Mathematics” (Pajares, 2002, p. 1). Bandura (1986) says that learners act on what they believe they can do and what they expect the ‘outcomes’ to be. There are many variables that affect learner achievement in Maths for example affective, motivational and cognitive variables. According to Singh (cited in Hammouri, 2004) the dynamic interaction between these factors is the catalyst in the formation of learner perceptions and attitude in Maths and understanding these factors is critical to understanding the challenges facing Maths learners. One of these factors is self efficacy.

According to Bandura (cited in Pajares, 2002) self-efficacy forms the basis for human motivation, perseverance and behaviour. Human action and perseverance is determined by failure or success in a task or action. Learners in the Maths class may react quite differently to the results that each receives in an assessment. Two learners, who have achieved similar results after putting in equal effort, might be affected in different ways. A learner who normally achieves well might be disillusioned by the lower performance, whereas a learner who does not normally achieve well, will no doubt be highly excited by this increased performance (Pajares, 2002).

1.4.5 Impact of Self-Efficacy on Maths Learning

Mathematics is a specialized language with its own contexts, metaphors, symbol systems and purposes (Pimm, 1995). From an epistemological point of view there is a basic difference between mathematics and other domains of scientific knowledge as the only way to access mathematical objects and deal with them is by using signs and semiotic representations (Duval, 2006). Cognitive development is related with metacognitive and affective development. One’s behavior and choices, when confronted with a task, are determined by her/his beliefs and personal theories, rather than her/his knowledge of the specifics of the task. Thus, students’ academic performance somehow depends on what they have come to believe about their capability, rather than on what they can actually accomplish.
Self-efficacy beliefs influence learner’s expectations of outcomes. Learners, who are confident in their ability to do Maths, will have a high expectation in the success that they will achieve in a given task or test. But learners whose self-efficacy indicates low confidence in Maths have a low expectation in their ability to perform the task with success. Self-efficacy beliefs therefore influence the way that learners envision performing in a task and the expected outcomes. However, as Pajares (2002) points out, the link between self-efficacy beliefs and behaviour is not always consistent. A learner, who is highly self-efficacious in Maths, may not always act in accordance with these beliefs. Some learners might look at the requirements of the task and decide that it is not worth the effort. According to Bandura (1986), highly efficacious Maths learners display qualities of (a) eagerness and commitment in tackling a Maths task and (b) perseverance and resilience in completing the task. They also regard failure as a minor setback and attribute failure to factors that they believe are within their control.

Self-efficacy plays an important role in determining the amount of effort that a learner will put into an activity, the learners perseverance and the extent of his resilience in the face of adversity. The higher a learner’s self-efficacy beliefs, the more will be the amount of effort, resilience and persistence that a learner is likely to show toward an activity (Bandura, 1986). Learners with high self-efficacy beliefs in Maths, tend to show greater effort in any activity that is given. They generally attribute failure to lack of effort and approach a task with feelings of calmness and confidence. Success or attainment of the desired level of performance encourages better performance, which in turn fuels greater confidence and higher self-efficacy. Learners with low self-efficacy, see things as being more difficult or insurmountable than they actually are (Bandura, 1986). These learners tend to be less confident and therefore more anxious and stressed when attempting a task. They would rather avoid the task seeing it as a threat than confront it. Consequently, they do not exert as much effort into an activity and this more often than not, leads to failure. Failure in learners with low self-efficacy lowers the learner’s confidence and morale (Pajares & Schunk, 2001). Consequently low self-efficacy tends to perpetuate itself.
Thus, it was found that self-efficacy is a major determinant of the choices that individuals make, the effort they expend, the perseverance they exert in the face of difficulties, and the thought patterns and emotional reactions they experience (Bandura, 1986). Furthermore, self-efficacy beliefs play an essential role in achievement motivation, interact with self-regulated learning processes, and mediate academic achievement (Pintrich, 1999).

1.4.6 How Can Self-Efficacy Beliefs be improved in Maths Learners?

Understanding that self-efficacy is an important factor in achievement in Maths, it has been suggested that self-efficacy not be left in the hope that it will develop on its own, if at all, but that it be developed in the learner. Pajares (2002) outlines four primary sources that he considers to be major contributors to the formation of self-efficacy beliefs that is (a) mastery experience, (b) vicarious experience, (c) emotional states and (d) verbal/social persuasions. He contends that it is the “integration, interpretation and recollection” of information derived from experiences with these sources that influences the development of one’s self-efficacy beliefs.

Mastery Experience

“Failures that are overcome by determined effort, can instill robust percepts of self-efficacy through experience that one can eventually master even the most difficult obstacles” (Bandura, 1986, p. 399). He emphasises that mastery experience is the most influential source of self-efficacy beliefs in the learner. The learner’s past successes or failures are the most reliable way of assessing the self-efficacy of a learner. He contends that exposing or engaging Maths learners in authentic mastery experiences results in successful Maths experiences and that in turn enhances learners self-efficacy in that aspect of Maths. After several successful experiences, self-efficacy beliefs are quite strong and occasional setbacks are not likely to dent a learner’s self-efficacy beliefs. Learners who are highly efficacious are more likely to attribute lack of effort, poor strategies or situational factors as causes of their performance. Once achieved, enhanced self-efficacy in one aspect of Maths may be
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generalised across other aspects of Maths similar to the one in which the self-efficacy of the learner was enhanced (Bandura, 1986).

Vicarious Experience

Learner’s self-efficacy can be boosted by observing others perform (model) mathematical tasks. If the learner observes someone (who has similar attributes as the learner), succeed in an activity it contributes to creating in the learner the belief that the learner is also capable of accomplishing the task with the same degree of success (Pajares, 2002). If a learner assume a high degree of similarity with a peer, then the peer’s successful experiences are absorbed by the learner, and the learners is then persuaded that this success is possible for them as well. The converse is also true. When a learner, who identifies strongly with a model, observes that model fail despite high effort it can diminish the learner’s self-efficacy beliefs in that Maths task. It could result in doubts of the learner’s own ability to be successful in Maths (Bandura, 1994).

Verbal/Social Persuasion

Using social persuasion to build learners’ self-efficacy beliefs in Maths entails using realistic judgements about learner’s performance in Maths. It can increase learners’ beliefs in their capability to perform a task with the required level of success. Verbal persuasion must be realistic if it is to work and must be reinforced by authentic experience. Convincing learners that they are capable of the Maths task at hand enables them to overcome their doubt about their capability to take up the challenge successfully. Positive persuasion should seek to empower and motivate learners through developing their self-efficacy beliefs (Bandura, 1986; Pajares, 2002). However, persuading learners that they have the competency required to tackle a problem of extreme difficulty, only serves to diminish their already low self-efficacy in the face of failure. It then makes it more difficult to enhance the learners’ self-efficacy on subsequent attempts (Bandura, 1986).

Somatic and Emotional States

Learners who anticipate failure in a Maths task, and experience strong emotional feelings of stress and fear, have low self-efficacy beliefs in Maths.
However, learners who expect to be successful in the task have a positive emotional welfare which enhances their self-efficacy beliefs. Learners, who are convinced that they will fail at a task, most probably will because they themselves are convinced of it (Pajares, 2002).

1.4.7 Geometry Self-Efficacy

Geometry is very important branch of the mathematics. Because, everything which we can see is with relation to geometry, it can be used to describe the physical world. Geometry is a way of thinking. Geometry distinguishes for allowing students to develop their intuition. The main goal of geometry is to combine experiences in the real world with abstract knowledge.

The students’ self-efficacy beliefs towards geometry are of great importance, while the students learn geometry. Because, those with high self-efficacy are not only more likely to attempt tasks, they also work harder and persisted longer in the face of difficulties (Bandura, 1997). There are many factors affecting students’ geometry achievement such as teaching methods for the low achievement in geometry (Schoenfeld, 1983), computer programs on geometry achievement (Arcavi & Hadas, 2000; Baharvand, 2001; Choi-Koh, 1999) etc. Apart from cognitive factors that affect students’ geometry achievement; affective factors such as attitudes and self-efficacy beliefs also have profound impact on students’ geometry achievement. The affective domain is a complex structural system consisting of four main components: emotions, attitudes, beliefs and values (Goldin, 2002).

Bandura focused on self-efficacy in a variety of domains such as chemistry self-efficacy, computer self-efficacy, career self-efficacy, sport self-efficacy, Academic self-efficacy, teacher self-efficacy, mathematics self-efficacy belief, computer usage etc. But researchers like Betz and Hackett (1995) have focused specifically on self-efficacy as it related to math. Self-efficacy beliefs have already been studied in relation to a lot of aspects of mathematics learning, such as: arithmetical operations (Bandura & Schunk, 1981); Self-Efficacy towards Mathematics Scale (Umay, 2001); Statistics Self-efficacy (Finney and Schraw, 2003); problem solving and problem posing self-efficacy (Akay& Boz, 2010) and Geometry self-efficacy (Cantürk-Günhan and Baser, 2007) etc. Cantürk-Günhan
and Baser (2007) in his study the development of self-efficacy scale toward geometry reported that students have positive self-efficacy beliefs towards geometry and Bindak (2004) found out that student have positive attitudes towards geometry.

Geometry self-efficacy may be defined as a situational assessment of an individual's confidence in her or his ability to successfully perform or accomplish a particular geometry based task or problem.

Thus, it is important to note that self-efficacy beliefs may be domain specific or general. Most studies related to mathematics self-efficacy measured a very general belief in mathematics self-efficacy which did not necessarily relate to specific mathematics topics (i.e. Usher, 2009).

1.5 SIGNIFICANCE OF THE STUDY

Teaching mathematics is much like building a house (Gluck, 1991). If the foundation is weak, many difficulties will appear later. Students’ understanding of basic mathematical concepts helps them move to the next logically connected concepts. Traditional method used in most of mathematics classes does not allow students enough time to fully reach that understanding. Mathematics Educators have always been interested in how students think in mathematics and in particular in geometry given its importance in the school curriculum.

Geometry is more than a theoretical branch of mathematics in which students learn the particulars of the content area; it is an area of mathematics education that can be used to foster different ways of thinking, discovering the properties of figures, inventing geometrical patterns, or solving problems. Considering the importance of geometrical reasoning in mathematics education, there is a need for a framework of abilities that could be used in fostering students’ geometrical performance.

There are several theoretical models which have been put forward as useful frameworks for describing and understanding the geometrical reasoning such as Van Hiele model, Anderson model, Greeno’s problem solving model and Duval’s cognitive model to teach geometry. Though Van Hiele model is more research area among mathematics educator for geometry teaching but researcher chooses Duval’s cognitive model to teach geometry with a belief that students reasoning ability,
geometry figure understanding can be improved/developed through various activities presented in syntax of Duval’s cognitive model. As Duval theory insists that there should not be any specific format imposed to the students he/she is asked to write a proof. According to Duval (1998), any model of mathematics learning in which different ways of reasoning are organised according to a strict hierarchy is inappropriate.

Duval’s model has been found effective not only in identifying students difficulties in learning geometry, but also suggests ways to improve. This model is also effective in tactile children’s cognitive development in geometry; developing students’ reasoning in solving geometric tasks; the functioning and understanding of geometrical figures; geometrical thinking, observing pupils behaviours, with particular reference to registers of representation; students’ ability to solve tasks involving geometrical figures. Duval’s cognitive model develops student’s ability of geometrical reasoning, proof and geometrical figure understanding. This model of teaching involves various activities.

The teaching and learning of geometry in school helps students to develop ways of thinking in mathematics. Students can perform experiments, develop visually based reasoning, learn to search for invariants and use these to spawn constructive arguments, collectively as “Habits of Mind” (Goldenberg, Cuoco and Mark 1998). If proving is the main activity in geometry, deductive reasoning is its main source. Reasoning as endemic to mathematics is used by teachers and textbooks to explain why mathematical conclusions are correct, rather than appealing simply to authority. Duval (1998) points out that “geometry more than other areas in mathematics, can be used to discover and develop different ways of thinking” (ibid, p.51). This is why investigation into geometry teaching is of interest not only in responding to needs in improving teaching, but also to the potential for analyzing and better understanding fundamental cognitive processes.

Further, as the outcomes of education are usually characterized as the achievement of those who have been educated, the outcomes of any teaching strategy are also measured in terms of its effects on achievement. For any new teaching
strategy, in order to test the degree or level of proficiency attained by student achievement effects must be measured.

Review of related literature revealed that Self-efficacy belief affects individuals' thinking styles, performance and emotional responses. Individuals with high self-efficacy levels can feel more relaxed and be more productive. Those confident in their academic skills expect high marks on exams and expect the quality of their work to reap personal and professional benefits. Individuals with low self-efficacy levels doubt their social skills and often envision rejection or ridicule even before they establish social contact. Those who lack confidence in their academic skills envision a low grade before they begin an examination or enroll in a course. Review of literature has established that a positive link exists between self-efficacy and achievement i.e. self-efficacy helps in predicting achievement. So, another aspect proposed for study is geometry Self-efficacy.

With this understanding of research literature, the investigator decided to work on a proposal involving these variables.