Introduction

Algebra evolved from the rules and operations of arithmetic, which begins, with the four operations: addition, subtraction, multiplication and division of numbers. Operations in algebra follow the same rules as those in arithmetic. Algebra uses variables, which are symbols that represent a number and expressions, which are Mathematical statements that use numbers, and or variables.

Abstract algebra is the subject area of Mathematics that studies algebraic structures, such as Groups, Rings, Fields, Modules, Vector Spaces, and Algebras. The phrase Abstract Algebra was coined at the turn of the 20th century to distinguish this area from what was normally referred to as Algebra, the study of the rules for manipulating formulae and algebraic expressions involving unknowns and real or complex numbers, often now called elementary algebra. The distinction is rarely made in more writings that are recent.

Two Mathematical subject areas that study the properties of algebraic structures viewed as a whole are Universal Algebra and Category Theory. Algebraic structures, together with the associated homomorphisms, form categories. Category theory is a powerful formalism for studying and comparing different algebraic structures.

The end of 19th and the beginning of the 20th century saw a tremendous shift in methodology of Mathematics. Abstract algebra emerged around the start of the 20th century, under the name Modern Algebra. Its study was part of the drive for more intellectual rigor in Mathematics. Initially, the assumptions in classical algebra, on which the whole of mathematics (and major parts of the natural sciences) depend, took the form of axiomatic systems. No longer satisfied with establishing properties of concrete objects, mathematicians started to turn their attention to general theory. Formal definitions of certain algebraic structures began to emerge in the 19th century. For example, results about various groups of permutations came to be seen as instances of general theorems that concern a general notion of an abstract group. Questions of structure and classification of various mathematical objects came to forefront. These processes were occurring throughout all of mathematics, but became especially pronounced in algebra. Formal definition through primitive operations and axioms were proposed for many basic algebraic structures, such as groups, rings and fields. Hence, such things as group theory and ring theory took their places in pure mathematics.

Universal algebra has enjoyed a particularly explosive growth in the last twenty years, and a student entering the subject now will find an incomprehensible amount of material to digest. One of the aims of universal algebra is to extract, whenever possible, the common elements of several different types of algebraic structures. In achieving this, one discovers general concepts, constructions and results, which not only generalize but also unify the known special situations, thus leading to an economy of presentation. Being at a higher level of abstraction, it can also be applied to entirely new situations, yielding significant information and giving rise to new directions.

In the study of the properties common to all algebraic structures (such as groups, rings, etc.) and even some of the properties that distinguish one class of algebras from another, lattices enter in an essential and natural way. In particular, congruence lattices play an important role. The origin of the lattice concept can be traced back to Boole's analysis of thought and Dedekind's study of divisibility. Schroeder and Pierce were also pioneers at the end of the last century. The subject started to gain momentum in 1930's and was greatly promoted by Birkhoff's book *Lattice Theory* in1940's. Lattice theory entered the foreground of mathematical interest and its rate of development increased rapidly. Despite the fact that up to now it has yielded less profound theorems than other algebraic theorems. Today it is already one of the important branches of algebra. Its concepts and methods have found fundamental applications in various areas of mathematics (e.g. diverse disciplines of abstract algebra, mathematical logic, projective and affine geometry, set and measure theory, topology and ergodic theory) and theoretical physics (e.g. quantum and wave mechanics and the theory of relativity).

Logic is much like mathematics in this respect: the so-called "Laws" of logic depend on how we define what a proposition is. The Greek philosopher Aristotle founded a system of logic based on only two types of propositions: true and false. The English mathematician George Boole (1815-1864) sought to give symbolic form to Aristotle's system of logic.

Boolean algebras, essentially introduced by Boole in 1850's to codify the laws of thought, have been a popular topic of research since then. A major breakthrough was the duality of Boolean algebras and Boolean spaces as discovered by Stone in 1930's. Stone also proved that Boolean algebras and Boolean rings are essentially the same in the sense that one can convert via terms from one to the other. Since every Boolean algebra can be represented as a field of sets, the class of Boolean algebras is sometimes regarded as being rather uncomplicated. However, when one starts to look at basic questions concerning decidability, rigidity, direct products etc., they are associated with some of the most challenging results.

In a draft paper [5], *The Equational theory of Disjoint Alternatives*, around 1989, E.G.Manes introduced the concept of Ada (Algebra of disjoint alternatives) ($A, \wedge, V, (-)^I, (-)_\pi, 0, 1, 2$)(Where $\wedge$,V are binary operations on A, $(-)^I$, $(-)_\pi$ are unary operations and 0,1,2 are distinguished elements on A)which is however differ from the definition of the Ada of his later paper[6] *Adas and the equational theory of if-then-else* in 1993. While the Ada of the earlier draft seems to be based on extending the If-Then-Else concept more on the basis of Boolean algebras and the later concept is based on C-algebras ($A, \wedge, V, (-)^\sim$)(where $\wedge$,V are binary operations on A,$(-)^\sim$ is a unary operation) introduced by Fernando Guzman and Craig C. Squir[2]. In 1994, P.Koteswara Rao[3] first introduced the concept of A*-algebra ($A, \wedge, V, *, (-)^\sim, (-)_\pi, 0, 1, 2$))(where $\wedge$,V,$*$ are binary operations on A, $(-)^\sim$,$(-)_\pi$ are unary operations and 0,1,2 are distinguished elements on A) not only studied the equivalence with Ada, C-algebra, Ada's connection with 3-Ring, Stone type representation but also introduced the concept of A*-clone, the

If-Then-Else structure over A*-algebra and Ideal of A*-algebra. In 2000, J.Venkateswara Rao[10] introduced the concept Pre A*-algebra (A, ∨, ∧, (-)$^{\sim}$)(where ∧,V are binary operations on A,(-)$^{\sim}$ is a unary operation on A)analogous to C-algebra as a reduct of A*-algebra, studied their subdirect representations, obtained the results that **2**= {0, 1} and **3**= {0, 1, 2} are the subdirectly irreducible Pre-A*-algebras and every Pre-A*-algebra can be imbedded in $\mathbf{3}^X$ **(** where $\mathbf{3}^X$ is the set of all mappings from a nonempty set X into **3**= {0, 1, 2}**).**

Boolean algebra depends on two-element logic. C-algebra, Ada, A*- algebra and our Pre A*-algebra are regular extensions of Boolean logic to 3 truth-values, where the third truth-value stands for an undefined truth-value. The Pre A*- algebra structure is denoted by (A, ∨, ∧, (-)$^{\sim}$)where A is non-empty set, ∧ , ∨ are binary operations and $^{\sim}$ is a unary operation.

The thesis *FUNDAMENTAL STUDY ON SPECIFIC CONCEPTS ON PRE A*-ALGEBRAS:* consists of six Chapters.

The Chapter-1 deals with the concept of *Boolean algebras and Pre A*-algebras* This Chapter commences with the concept of Boolean algebra and some basic fundamental results of Boolean algebra. It also includes the useful properties of Boolean algebra. We introduce the concept of Pre A*-algebra and obtain the useful characterizations. We obtain the various methods of generation of Pre A*-algebras from Boolean algebra. The main content of this Chapter was embodied in “Boolean algebras and Pre A*-algebras” [11].and was published in the journal Acta Ciencia Indica (Mathematics), (ISSN: 0970-0455**),** (2006) 32: pp71-76.)

The Chapter-2 describes the concept of *Pre A*-algebra as a semilattice.*In this Chapter, we define semilattice on a Pre A*-algebra with respect to ∧(meet) and as well as ∨ (join) and obtain the properties of semilattice on a Pre A*-algebra We establish Pre A*-algebra as a semilattice. We prove necessary conditions for a semilattice to become a lattice with respect to meet and as well, as join. We obtain semi-*-complemented semilattices on a Pre A*-algebra. We establish theorems on these semi-*-complemented semilattices on a Pre A*-algebra. We define atoms,dual atoms, irreducible elements with respect to meet as well as join on Pre A*-algebra. We obtain various theorems on these atoms,dual atoms, irreducible elements on Pre A*-algebra We establish the atomic,dual atomic semilattices *on* Pre A*-algebra .

The main content of this Chapter was embodied in “Pre A*-algebra as a semilattice” [12]. and was published in the journal Asian Journal of Algebra, **Volume 4, Number 1, 12-22, 2011**

The Chapter-3 analyzes the concept of *Lattice on Pre A*-algebra.* We define the lattice on Pre A*-algebra and we call such a defined lattice on Pre A*-algebra as Pre A*-lattice and we derive the properties of the Lattice on Pre A*-algebra. We define greatest lower bound and least upper bound on Pre A*-algebra .We observe that 2 acts as greatest lower bound(g.l.b) with respect to meet whereas 2 acts as least upper bound(l.u.b) with respect to join. We define atoms,dual atoms, irreducible elements with respect to meet as well as join on Pre A*-algebra. We obtain various theorems on these atoms,dual atoms, irreducible elements on Pre A*-algebra We establish the atomic lattices,dual atomic lattices *on* Pre A*-algebra . The main content of this Chapter was embodied in “Lattice in Pre A*-algebra” [13]. and was published in the journal Asian Journal of Algebra, ISSN 1994-540X **Volume 4, Number 1, 1-11, 2011**

The Chapter-4 deals with the concept of *ring on a Pre A*-algebra*. In this chapter we define a ring on Pre A* - algebra .This chapter consists of four sections. In the first section we prove some basic theorems on Pre A* - algebra. In the second section, we define a ring on a Pre A* - algebra and its properties.We establish the identities to prove Pre A* - algebra as a ring and ring as a Pre A* - algebra. In the third section we define a Boolean ring on a Pre A* - algebra and we introduce the theorem Pre A* - algebra as a Boolean ring and Boolean ring as a Pre A* - algebra. . In the fourth section we define a p-ring, 3- ring and we prove Pre A* - algebra as a 3- ring and we prove 3- ring as a Pre A* - algebra. The main content of this Chapter was embodied in “Pre A*-algebras and rings”[14] and was published in the journal International Journal of Computational Science and Mathematics.ISSN 0974- 3189, Volume 3, Number 2 (2011), pp. 161-172,2011

The Chapter-5 explains the concept of *homomorphisms,ideals and Congruence relations on Pre A*-algebra.*We establish the concept of Pre A*-homomorphism and we prove some theorems on these Pre A*-homomorphisms.We define an ideal on Pre A*-algebra and establish its useful theorems. By a well known definition and some related results on Congruence relations on Pre A*-algebra we establish theorems related with these concepts of homomorphisms,ideals and Congruence relations on Pre A*-algebra. The main content of this Chapter was embodied in “homomorphisms,ideals and Congruence relations on Pre A*-algebra” [15] and was published in the journal Global Journal of Mathematical Sciences, Theory and Practical. ISSN No 0974 – 3200 Volume 3, Number 2 (2011), pp. 111-125, May, 2011.

The Chapter-6 describes the concept of *logic circuits and gates on Pre A*-algebra***.** We analyze the concept of logic on Pre A*-algebra**.** Pre A*-algebra is regular extension of Boolean logic to 3 truth-values, where 0 stands for false,1 stands for true but the third truth-value stands for an undefined truth-value. We give truth tables for 2 inputs,3 inputs on Pre A*-algebra**.** By well known definitions of logic gates,AND gate,OR gate,NOT gate,NAND gate,NOR gates in the Boolean algebra, we define logic gates,AND gate,OR gate,NOT gate,NAND gate,NOR gates in Pre A*-algebra**.**We use the operations $\wedge,\vee$ for AND gate,OR gate respectively where the complementation is used by NOT gate. We introduce the concept of logic circuits on Pre A*-algebra and we establish the logic circuits on Pre A*-algebra .We prove logic gates forms a Pre A*-algebra .In a similar way **,** we prove that logic circuits forms a Pre A*-algebra. The main content of this Chapter was embodied in “logic circuits and gates in Pre A*-algebra” [16]. and was published in the journal Asian Journal of Applied Sciences,4(1): 89-96, 2010.