CHAPTER -8
PARTIAL ORDERING ON C- ALGEBRA

In this chapter we give the definition of a C-Algebra $A$ and different properties. We define a partial ordering $\leq$ on a C-Algebra $A$ by $x \leq y$ if and only if $x \lor y = y$ and give some examples. In general, in a C-Algebra $A$ the infimum of $\{x, y\}$ need not exist under this partial ordering. In fact, we prove that if $\text{Inf} \{x, y\}$ exist for all $x, y$ in the poset $(A, \leq)$ then $<A, \wedge, \lor, '>$ is a Boolean algebra. We also give a particular cases when the supremum and infimum of $\{x, y\}$ exist in a C-Algebra. We give a number of equivalent conditions for a C-Algebra to become a Boolean algebra, in terms of $\leq$.

S.C. Kleene [6] introduced the notion of a regular extension of two element classical logic with truth values T and F. The algebraic form of two valued logic is the two element Boolean algebra $B = \{T, F\}$ with binary operations $\wedge, \lor$ and unary operation $'$. A regular extension of an algebra $A$ is obtained by adjoining a new element $U$ (for “unknown”, undetermined”, or “undefined”) to $B$ and extending the operations on $B$ to $B \cup \{U\}$. If $B$ is the two element Boolean algebra $B = \{T, F\}$, then $B \cup \{U\} = \{T, F, U\}$ is denoted by $A$.

the concept of the centre $B(A)$ of a C-algebra $A$ and proved that the centre of a C-algebra is a Boolean algebra.

8.1. Definition: C-algebra is algebra of type $(2, 2, 1)$ with binary operations $\land, \lor$ and unary operation $'$ satisfying the following identities. For all $x, y, z \in A$

(a) $x' = x$

(b) $(x \land y)' = x' \lor y'$

(c) $(x \land y) \land z = x \land (y \land z)$

(d) $x \land (y \lor z) = (x \land y) \lor (x \land z)$

(e) $(x \lor y) \land z = (x \land z) \lor (x' \land y \land z)$

(f) $x \lor (x \land y) = x$

(g) $(x \land y) \lor (y \land x) = (y \land x) \lor (x \land y)$

Now we give two examples for a C-algebra, which play very important role in the variety of C-algebras.

8.2. Example: The two element algebra $B = \{T, F\}$ is a C-algebra with operations $\land, \lor$ and $'$ defined in the following table.

<table>
<thead>
<tr>
<th></th>
<th>$\land$</th>
<th>$\lor$</th>
<th>$x$</th>
<th>$x'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>$T$</td>
<td>$T$</td>
<td>$T$</td>
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</tr>
<tr>
<td>$F$</td>
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<td>$T$</td>
</tr>
</tbody>
</table>

8.3. Example: The 3-element algebra $A = \{T, F, U\}$ is a C-algebra with operations $\land, \lor$ and $'$ defined as in the following tables.
We can observe that in a C-algebra $A$,

(i) Identities 8.1(a), 8.1(b) imply that the variety of C-algebras satisfies all the dual statements of 8.1(b) to 8.1(g).

(ii) $\wedge$ and $\vee$ are not commutative in $A$.

(iii) The ordinary right distributive law of $\wedge$ over $\vee$ fail in $A$.

(iv) Every Boolean algebra is a C-algebra.

8.4. Lemma[28]: Every C-algebra satisfies the following identities:

(a) $x \wedge x = x$

(b) $x \wedge y = x \wedge (x' \vee y)$

(c) $x \wedge (x' \vee x) = (x' \vee x) \wedge x = x \vee (x' \wedge x) = (x' \wedge x) \vee x = x$

(d) $(x \vee x') \wedge y = (x \wedge y) \vee (x' \wedge y)$

(e) $x \vee x' = x' \vee x$

(f) $x \vee y \vee x = x \vee y$

(g) $x \wedge x' \wedge y = x \wedge x'$

(h) $x \vee x' = x' \vee x$

(i) $x \wedge y = (x' \wedge y) \vee x$

(j) $x \wedge y = (x' \vee y) \wedge x$

(k) $x \vee y = F$ if and only if $x = y = F$

(l) If $x \vee y = T$ then $x \vee x' = T$

(m) $x \vee T = x \vee x'$

(n) $T \vee x = T$ and $F \wedge x = F$
8.5. Lemma [28]: Let $A$ be a C-algebra if $a \in A$ then

(i) $a$ is left identity for $\wedge$ if and only if $a$ is right identity for $\wedge$
(ii) $a$ is left identity for $\vee$ if and only if $a$ is right identity for $\vee$

8.6. Definition: If a C-algebra $A$ has an identity element $T$ for $\wedge$, then we say that $A$ is a C-algebra with $T$. In this case we write $T$ for the identity for $\wedge$ and $F$ in the place of $T'$.

8.7. Theorem [28]: The following are equivalent in a C-algebra $A$.

(a) $A$ is a Boolean algebra
(b) $x \vee (y \wedge x) = x$ for all $x, y \in A$
(c) $x \wedge y = y \wedge x$ for all $x, y \in A$
(d) $(x \vee y) \wedge y = y$ for all $x, y \in A$
(e) $x \vee x'$ is identity for $\wedge$ for all $x, y \in A$
(f) $x \vee x' = y \vee y'$ for all $x, y \in A$
(g) $A$ has a right zero for $\wedge$
(h) For any $x, y \in A$, there exists $a \in A$ such that $x \wedge a = y \wedge a = a$
(i) For any $x, y \in A$ if $x \vee y = y$ then $y \wedge x = x$

8.8. Definition: Let $A$ be a C-algebra with $T$. An element $x \in A$ is called a central element of $A$ if $x \vee x' = T$. The set $\{x \in A / x \vee x' = T\}$ of all central elements of $A$ is called the centre of $A$ and is denoted by $B(A)$.

8.9. Theorem [28]: Let $A$ be a C-algebra with $T$, then $B(A)$ is a Boolean algebra with induced operations $\wedge, \vee$ and $'$.

8.10. Lemma: Let $A$ be a C-algebra for any $x, y \in A$.

(a) $x \vee (x' \wedge y) = (x' \wedge y) \vee x$
(b) $x \wedge (x' \vee y) = (x' \vee y) \wedge x$

Proof: By 8.4 (i), we have, $x \vee y = (x' \wedge y) \vee x$
Similarly we can prove that \( x \land (x' \lor y) = (x' \lor y) \land x \) (by dual of 8.4(b)).

8.11. Definition: Let \( A \) be a C-algebra. Define \( \leq \) on \( A \) by \( x \leq y \) if and only if \( x \lor y = y \).

8.12. Theorem: If \( A \) be a C-algebra, then \((A, \leq)\) is a poset.

Proof: Since \( x \lor x = x, \ x \leq x \) for all \( x \in A \).
Therefore \( \leq \) is a reflexive.

Suppose that \( x, y, z \in A, \ x \leq y \) and \( y \leq z \), that is \( x \lor y = y \) and \( y \lor z = z \).

Now \( z = y \lor z = x \lor y \lor z = x \lor z \).

This shows that \( x \leq z \)

Therefore \( \leq \) is transitive.

Suppose that \( x \leq y \) and \( y \leq x \), that is \( x \lor y = y \) and \( y \lor x = x \).

We have
\[
\begin{align*}
x \land y &= x \land (x \lor y) = x \\
y \land x &= y \land (y \lor x) = y \\
\text{Now} \quad y &= x \lor y = (x \land y) \lor (y \land x) = (y \land x) \lor (x \land y) = y \lor x = x
\end{align*}
\]

Therefore \( \leq \) is antisymmetric. Hence \((A, \leq)\) is a poset.

8.13. Note: If a C-algebra \( A \) has \( F \), then \( F \lor x = x \), for all \( x \in A \). That is \( F \leq x \). This shows that \( F \) is the least element of the poset.

8.14. Example: Let \( A = \{T, F, U\} \) be a C-algebra then the Hasse diagram of the poset \((A, \leq)\) is

```
F
  |
  |
  |
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We know that $A \times A$ is a $C$-algebra under point wise operations.

The Hasse diagram of the poset $(A \times A, \leq)$ is given below.

$A \times A = \{a_1 = (T,T), a_2 = (T,F), a_3 = (T,U), a_4 = (F,T), a_5 = (F,F), a_6 = (F,U),$
$a_7 = (U,T), a_8 = (U,U), a_9 = (U,F)\}.$

![Hasse diagram](image)

**8.15. Theorem:** In a poset $(A, \leq)$ for any $x \in A$, supremum of $\{x, x'\} = x \lor x'$,
infinum of $\{x, x'\} = x \land x'$.

**Proof:** We have $x \lor (x \lor x') = (x \lor x) \lor x' = x \lor x'$.

This shows that $x \leq x \lor x'$.

And also $x' \lor (x \lor x') = (x' \lor x) \lor x' = (x \lor x') \lor x'$ (by 8.4(e))

$$= x \lor (x' \lor x') = x \lor x'.$$

This shows that $x' \leq x \lor x'$.

Hence $x \lor x'$ is an upper bound of $\{x, x'\}$.

Let $n$ be an upper bound of $\{x, x'\}$. Then $x \leq n, x' \leq n \Rightarrow x \lor n = n, x' \lor n = n$

Consider $(x \lor x') \lor n = x \lor (x' \lor n) = x \lor n = n$

This shows that $x \lor x' \leq n$.

Therefore supremum of $\{x, x'\} = x \lor x'$. 
Similarly we can prove that, infimum of \( \{x, x'\} = x \wedge x' \).

**8.16. Theorem:** Let \( x, y \in A \) and \( x \lor y = y \lor x \) then supremum of \( \{x, y\} = x \lor y \).

**Proof:** Let \( x, y \in A \) and \( x \lor y = y \lor x \)

\[
x \lor (x \lor y) = (x \lor x) \lor y = x \lor y
\]

This shows that \( x \leq x \lor y \)

And also \( y \lor (x \lor y) = y \lor (y \lor x) = y \lor x = x \lor y \)

This shows that \( y \leq x \lor y \)

Therefore \( x \lor y \) is an upper bound of \( \{x, y\} \)

Let \( m \) be an upper bound of \( \{x, y\} \). Then \( x \leq m, \ y \leq m \Rightarrow x \lor m = m, \ y \lor m = m \)

Now \( (x \lor y) \lor m = x \lor (y \lor m) = x \lor m = m \).

This shows that \( x \lor y \leq m \).

Therefore \( x \lor y \) is a least upper bound of \( \{x, y\} \).

That is supremum of \( \{x, y\} = x \lor y \) .

In general for a C-algebra with \( T \), \( x \wedge y \) need not be greatest lower bound of \( \{x, y\} \). For example in \( A = \{T, F, U\} \), \( U \wedge T = U \) is not a lower bound of \( \{U, T\} \). However we have the following theorem.

**8.17. Theorem:** Let \( A \) be a C-algebra with \( T \) and \( x, y \in A \), if \( x \in B(A) \) and \( x \lor y = y \lor x \) then inf \( \{x, y\} = x \wedge y \).

**Proof:** Let \( x \lor y = y \lor x \) and \( x \in B(A) \)

\[
(x \land y) \lor x = (x \lor x) \land (x \lor y \lor x) \quad \text{(by 8.1(e))}
\]

\[
= x \land (x \lor y) \quad \text{(since } x \lor y = y \lor x) \]

\[
= x \land (x \lor x') \quad \text{(by 8.4(g))}
\]

\[
= x \quad \text{(by 8.4(c))}
\]

Hence \( x \land y \leq x \)

Again \( (x \land y) \lor y = (x \lor y) \land (x \lor y \lor y) \) (by 8.1(e))
\[
= (x \lor y) \land (x' \lor y) = (x \land x') \lor y \quad \text{(by 8.4(d))}
\]
\[
= F \lor y = y
\]

Hence \( x \land y \leq y \).

Therefore \( x \land y \) is a lower bound of \( \{x, y\} \).

Suppose \( t \) is a lower bound of \( \{x, y\} \), so that \( t \leq x, \ t \leq y \implies t \lor x = x, \ t \lor y = y \)

Now \( t \lor (x \land y) = (t \lor x) \land (t \lor y) = x \land y \).

This shows that \( t \leq x \land y \)

Therefore \( \inf \{x, y\} = x \land y \).

8.18. Theorem: In a C-algebra

(i) \( x \leq y \iff (x' \land y) \lor x = y \iff x \lor (x' \land y) = y \)

(ii) \( x \leq y \implies x \land y = x \)

Proof: (i) It is clear 8.4 (i), and 8.10

(ii) \( x \leq y \) then \( x \lor y = y \)

Now \( x \land y = x \land (x \lor y) \)

\[
= x \quad \text{(by 8.1(f))}
\]

The converse is not true. For example in a C-algebra \( U \land F = U \), but \( U \not\leq F \).

8.19. Theorem: In a C-algebra

(1) \( x \land x' \leq x \land y \)

(2) If \( x \leq y \) then, for any \( z \in A \), (i) \( z \land x \leq z \land y \) (ii) \( z \lor x \leq z \lor y \)

(3) \( x \leq x \lor y \)

(4) If \( A \) is a C-algebra with \( T \) and \( x \in B(A) \) then \( x \land y \leq y \)

Proof: (1) \((x \land x') \lor (x \land y) = x \land (x' \lor y) = x \land y \) (by 8.4 (b))

This shows that \( x \land x' \leq x \land y \).

(2) (i) \((z \land x) \lor (z \land y) = z \land (x \lor y) = z \land y \) \quad \text{(since \( x \leq y \implies x \lor y = y \))}

Hence \( z \land x \leq z \land y \)
(ii) \((z \lor x) \lor (z \lor y) = (z \lor x \lor z) \lor y\)
\[= (z \lor x) \lor y \quad \text{(by 8.4(f))}\]
\[= z \lor (x \lor y) \]
\[= z \lor y \quad \text{(since } x \leq y \Rightarrow x \lor y = y).\]

Hence \(z \lor x \leq z \lor y\).

(3) \(x \lor (x \lor y) = (x \lor x) \lor y = x \lor y\).

Hence \(x \leq x \lor y\).

(4) \((x \land y) \lor y = (x \lor y) \land (x' \lor y \lor y)\)  \quad \text{(by 8.1(e))}
\[= (x \lor y) \land (x' \lor y) \]
\[= (x \land x') \lor y \quad \text{(by 8.4(d))}\]
\[= F \lor y \quad \text{(since } x \in B(A))\]
\[= y \]

Therefore \(x \land y \leq y\).

**8.20. Note:** (i) We observe that \(x \land x' \leq x \land y\), but \(y \land y' \leq x \land y\) not hold in a C-algebra, for example \(U \land U' \leq F \land U\).

(ii) We observe that \(x \leq x \lor y\), but \(y \leq x \lor y\) not hold in a C-algebra, for example, \(T \leq U \lor T\).

Now we prove modular type results in the following.

**8.21. Lemma:** In the poset \((A, \leq)\)
\[x \leq y \Rightarrow x \lor (y \land z) = y \land (x \lor z).\]

Proof: Suppose \(x \leq y \Rightarrow x \lor y = y\)

Now \(x \lor (y \land z) = (x \lor y) \land (x \lor z) = y \land (x \lor z)\)
8.22. Theorem: For any $x, y, z \in A$, $x \leq y$, $x \lor z = y \lor z$ and $x \land z = y \land z$ then $x = y$.

Proof: $x = x \lor (x \land z)$ (by 8.1(f))

$= x \lor (y \land z)$

$= y \land (x \lor z)$ (by 8.21)

$= y \land (y \lor z) = y$

If $x \in B(A)$ then we have proved (8.19) that $x \land y \leq y$ and also if $x \in B(A)$ and $x \lor y = y \lor x$ then $\inf \{x, y\} = x \land y$ (8.17). In general for a C-algebra with $T$, $x \land y$ need not be lower bound of $\{x, y\}$. If $x \land y$ is a lower bound of $\{x, y\}$ in $(A, \leq)$ then $A$ becomes a Boolean algebra. Also it can be observed that, for $x, y \in A$ then $x \leq x \lor y$ is true (8.19.(3)). But $y \leq x \lor y$ need not be hold in $A$. If $y \leq x \lor y$ for all $x, y \in A$ then $A$ becomes Boolean algebra. In the following theorem we prove a number of such equivalent conditions for a C-algebra to become a Boolean algebra.

8.23. Theorem: Let $<A, \land, \lor, >$ be a C-algebra. Then the following conditions are equivalent.

(1) $<A, \land, \lor, >$ is a Boolean algebra

(2) $x \land y \leq y$ for all $x, y \in A$

(3) $x \land y \leq x$ for all $x, y \in A$

(4) $x \land y$ is a lower bound of $\{x, y\}$ for all $x, y \in A$

(5) $\inf \{x, y\} = x \land y$ for all $x, y \in A$

(6) $y \leq x \lor y$ for all $x, y \in A$

(7) $x \lor y$ is an upper bound of $\{x, y\}$ for all $x, y \in A$

(8) $\sup \{x, y\} = x \lor y$ for all $x, y \in A$

(9) The poset $(A, \leq)$ is directed below
(10) \( x \land x' \) is the least element in the poset \((A, \leq)\) for all \( x \in A \)

(11) For any \( x, y \in A \), if \( x \leq y \), then \( y' \leq x' \)

**Proof:** (1) \( \Rightarrow \) (2) is clear.

(2) \( \Rightarrow \) (3): Assume \( x \land y \leq y \). That is \( (x \land y) \lor y = y \)

Then \( (x \land y) \lor (y \land x) = (x \land (y \land x)) \lor (y \land x) \) (by 8.4(f))

\[ = y \land x \] (by supposition)

Similarly \( (y \land x) \lor (x \land y) = x \land y \)

Therefore \( x \land y = y \land x \)

Consider \( (x \land y) \lor x = (y \land x) \lor x = x \) (by supposition)

Hence \( x \land y \leq x \) for all \( x, y \in A \)

(3) \( \Rightarrow \) (4): Assume \( x \land y \leq x \). That is \( (x \land y) \lor x = x \)

Then \( (x \lor y) \land (y \lor x) = (x \lor y) \land (y \lor (x \lor y)) \) (by 8.4(f))

\[ = [(x \lor y) \land y] \lor (x \lor y) \] (by left distributive)

\[ = x \lor y \] (by supposition)

Similarly \( (y \lor x) \land (x \lor y) = y \lor x \)

Therefore \( x \lor y = y \lor x \) for all \( x, y \in A \) and hence \( x \land y = y \land x \) for all \( x, y \in A \).

Consider \( (x \land y) \lor y = (y \land x) \lor y = y \) (by supposition)

Thus \( x \land y \leq y \). Hence \( x \land y \) is a lower bound of \( \{ x, y \} \).

(4) \( \Rightarrow \) (5): Assume(4). Suppose \( l \) is the lower bound of \( \{ x, y \} \) and \( x, y \in A \).

That is \( l \leq x \) and \( l \leq y \), so that \( l \lor x = x \) and \( l \lor y = y \).

Now \( l \lor (x \land y) = (l \lor x) \land (l \lor y) = x \land y \)

Thus \( l \leq x \land y \). Therefore Infimum \( \{ x, y \} = x \land y \)

(5) \( \Rightarrow \) (6): Assume (5). That is \( x \land y = \inf \{ x, y \} = \inf \{ y, x \} = y \land x \).
Therefore $x \land y = y \land x$ for all $x, y \in A$ and hence $x \lor y = y \lor x$ for all $x, y \in A$.

Now $y \lor (x \lor y) = y \lor x = x \lor y$

Therefore $y \leq x \lor y$.

(6) $\Rightarrow$ (7): Assume (6). That is $y \lor (x \lor y) = x \lor y$

Now $x \lor (x \lor y) = x \lor (y \lor x)$ (by above $x \lor y = y \lor x$)

$= y \lor x$ (by supposition)

$= x \lor y$

Therefore $x \leq x \lor y$.

Hence $x \lor y$ is an upper bound of $\{x, y\}$.

(7) $\Rightarrow$ (8): Assume (7). Suppose $x, y \in A$ and $u$ is an upper bound of $\{x, y\}$

That is $x \leq u$ and $y \leq u$, so that $x \lor u = u$ and $y \lor u = u$.

Consider $(x \lor y) \lor u = x \lor (y \lor u) = x \lor u = u$

Therefore $x \lor y \leq u$.

Hence $\text{Sup} \{x, y\} = x \lor y$.

(8) $\Rightarrow$ (1): Assume (8). For any $x, y \in A$,

we have $x \lor y = \text{Sup} \{x, y\} = \text{Sup} \{y, x\} = y \lor x$.

Then $x \lor y = y \lor x$ for all $x, y \in A$.

Hence by 8.7. Theorem $A$ is a Boolean algebra.

Thus the conditions (1) and (8) are equivalent.

Condition (9) is equivalent to (h) of 8.7. Theorem and condition (10) is equivalent to (e) of 8.7. Theorem. Hence (9) and (10) are equivalent to (1).

Assume (11). Let $x \land y = y$ for any $x, y \in A$.

$\Rightarrow x' \lor y' = y'$

$\Rightarrow x' \leq y'$
\[ y^* \leq x^* \text{ (by assumption)} \]
\[ y \leq x \]
\[ y \lor x = x \]

Therefore, we have \( x \land y = y \Rightarrow y \lor x = x \) for all \( x, y \in A \). Thus by (i) of 8.7. Theorem \( <A, \land, \lor, ^\prime> \) is a Boolean algebra. Hence the Theorem.

**8.24. Theorem:** Let \(<A, \land, \lor, ^\prime>\) be a C-algebra. Then the following are equivalent.

1. \(<A, \land, \lor, ^\prime>\) is a Boolean algebra.
2. \((x \lor y) \land x \leq x\)
3. \(x \leq (x \land y) \lor x\)

**Proof:**

1. \( \Rightarrow \) (2) is clear.

2. \( \Rightarrow \) (3): Assume \((x \lor y) \land x \leq x\)

Let \( x, y \in A \) we have \((x' \lor y') \land x' \leq x'\)

\[ x' = [(x' \lor y') \land x'] \lor x' \]
\[ = [x' \lor (y' \land x')] \lor x' \]
\[ = x' \lor (y' \land x') \]
\[ = [x \land (y \lor x)]' \]
\[ = [(x \land y) \lor x]' \]

So that \((x \land y) \lor x = x\)

Therefore \( x \leq (x \land y) \lor x \).

3. \( \Rightarrow \) (1): Assume \( x \leq (x \land y) \lor x \) for all \( x, y \in A \).

\[ x \lor [(x \land y) \lor x] = (x \land y) \lor x \]
\[ x \lor (x \land y) = (x \land y) \lor x \]
\[ x = x \land (y \lor x) \text{ for all } x, y \in A. \text{ (by 8.1 (f))} \]

Therefore by 8.7. Theorem, \( A \) is a Boolean algebra.
8.25. Lemma: Let $A$ be a C-algebra with $T$. Then the following hold.

(1) For any $x \in A$, $T \leq x \iff x = T$.

(2) For any $x \in A$, $x \leq T \iff x \in B(A)$.

(3) $[F, T] = \{ x \in A / F \leq x \leq T \} = B(A)$.

Proof: (1) Let $x \in A$. Then $T \leq x \Rightarrow T \lor x = x \Rightarrow x = T$.

Converse is trivial.

(2) Let $x \leq T \iff x \lor T = T$

$\iff x \lor x' = T$ (by 8.4(m))

$\iff x \in B(A)$

(3) Let $x \in [F, T] \Rightarrow x \leq T \Rightarrow x \in B(A)$ (by (2) above)

Therefore $[F, T] \subseteq B(A)$.

Conversely, let $x \in B(A)$.

Since $B(A)$ is a Boolean algebra and $F$ is the least element, $T$ is the greatest element, we get $F \leq x \leq T$.

Therefore $B(A) \subseteq [F, T]$.

Hence $[F, T] = B(A)$.

Finally we conclude this section with the following equivalent conditions.

8.26. Theorem: Let $<A, \wedge, \vee, '> \text{ be a C-algebra. Then the following are equivalent.}$

(1) $<A, \wedge, \vee, '> \text{ is a Boolean algebra.}$

(2) $x \leq T \text{ for all } x \in A$

(3) $x \land x' \leq F, \text{ for all } x \in A$.

Proof: (1) $\Rightarrow$ (2) is clear.

(2) $\Rightarrow$ (3): Assume $x \leq T$ for all $x \in A$.

$\Rightarrow x \lor T = T$
\[ \Rightarrow x \lor x' = T \]
\[ \Rightarrow x \land x' = F \]
Thus \( x \land x' \leq F \), for all \( x \in A \).

(3) \( \Rightarrow \) (1): Assume \( x \land x' \leq F \), for all \( x \in A \).

\[ \Rightarrow (x \land x') \lor F = F \]
\[ \Rightarrow x \land x' = F \]
\[ \Rightarrow x \lor x' = T \]
Thus \( x \in B(A) \) for all \( x \in A \).

Hence \( B(A) = A \). Therefore \( A \) is a Boolean algebra.