Chapter 9

Entropy, Light Cluster Production and Thermalization in Heavy-Ion Collisions

9.1 Introduction

Various experimental and theoretical studies in the recent past have indicated a clear demarcation of colliding matter into participant and spectator zones especially at relativistic bombarding energies [1–4]. This is characterized by the formation of hot & dense fireball and relatively cold spectator zones. In earlier chapters, we have discussed, in detail, the characteristics of spectator matter fragmentation and universality behavior observed in the emission of intermediate mass fragments. Highly dense fireball is, however, formed for a very short duration (< 10^{-22}s) where temperature reached can be as high as 70-80 AMeV depending upon colliding geometry and bombarding energy. The bulk condition of thermal equilibrium may not be fully attained except for central collisions. This is partly because of rapid evolution of reaction from hot and dense excited zone to expansion and cooling thereafter. In addition, participant volume is also linked with the emission of composite particles and ultimately with the mechanism of the production of entropy [5–10]. Entropy produced in HI reactions is one of the thermodynamic observables that preserves the signature of violent phase of the reactions. Based upon hydrodynamics, it has been argued that at certain freeze-out time, the phase space density stays constant and entropy determines the abundance of clusters produced. The extraction of entropy from cluster abundances is, therefore, helpful in understanding the EoS of hot and dense nuclear matter. Entropy is observed to decline with increase in the strength of repulsive
forces between baryons, which in turn depend upon maximum compression achieved in
the reaction [5,11].

As a matter of fact, pion production also contributes significantly [12-15] towards the
entropy generation at SPS [16,17] and higher incident energies. At SPS energies (~160
AGeV fixed target), the pion number increases with beam energy to about ten times the
number of the nucleons [17]. There, an increase in the entropy production was observed
with beam energy as one moves from AGS energies towards SPS and higher energies [15].
This enhancement may also be conjectured as manifestation of change in the collision
dynamics at such high energy [18]. For the incident energy range considered (400-1050
AMeV) in the present work, the inclusion of the pion production is not going to affect the
entropy production appreciably [13,14,19]. For instance, at SIS energies (upto 2 AGeV),
the total number of pions is only 10% of the nucleons [20].

In the present chapter, we shall address the thermal properties of participant zone
via evolution of mean density and temperature reached in central region, the degree of
equilibration and modeling composite particle formation. We would also compare cluster
distribution in coordinate space and cluster-to-proton yield ratios obtained using soft (S)
and soft momentum dependent (SM) equations of state. In the following section, we shall
describe the method used to extract the baryonic entropy $S_N$ from the yields of composite
particles (i.e. $p$, $n$, $d$, $t$, $^3$He, and $\alpha$-particles).

### 9.2 Tracking the entropy

It has been conjectured that entropy information may be obtained from a classical
charge symmetric gas of nucleons and deuterons in thermal and chemical equilibrium
using the relation suggested by Siemens and Kapusta [5,6]:

$$S_N = 3.945 - \ell n(d/p),$$  \hspace{1cm} (9.1)

with $d/p$ as deuteron-to-proton yield ratio established during early stages of fireball for-
mation. One source of error arises due to neglect of other light composite particles viz. $t$,
$^3$He and $\alpha$-particles. Bertsch and Cugnon [21] proposed to take into account these light
clusters as well generalizing Eq. (9.1) as:

$$S_N = 3.945 - \ell n \tilde{R}_{dp},$$  \hspace{1cm} (9.2)
with

\[
\hat{R}_{dp} = \frac{d_{like}/p_{like}}{d + \frac{3}{2}t + \frac{3}{2}^3He + 3\alpha} = \frac{p + d + t + 2\,^3He + 2\alpha}{N_p}
\]  \quad (9.3)

In the above equation (9.3), denominator term depicts the participant proton multiplicity \(N_p\) that solely depends upon the size of the reaction volume. This neglects the contribution coming from low-excitation chunks of spectator matter. As can be seen from Eq.(9.3), \(\hat{R}_{dp}\) is the yield ratio of deuteron-like to proton-like fragments that takes into account these light clusters as well. It has been further established in experiments that highest proton multiplicity accounts for most of the charges in HI system, thus leaving no room for heavier clusters. Since in QMD model, there is no scope of isospin identification of nucleons, we can’t distinguish between different isobars such as \(t\) and \(^3He\). For this reason and to compare our model predictions with experimental data, we define the yield ratio of deuteronlike \(d_{like}\) to protonlike \(p_{like}\) clusters in the following way [11]:

\[
\hat{R}_{dp} = \frac{Y(N_f = 2) + \frac{3}{2}Y(N_f = 3) + 3Y(N_f = 4)}{N_p}
\]  \quad (9.1)

where \(Y(N_f)\) stands for the number of fragments with mass \(N_f\) in one event. Analogous to experimental results, we calculate the total participant multiplicity \(N_p\) as:

\[
N_p = \frac{Z_p + Z_T}{A_p + A_T} \left[ Y(N_f = 1) + 2Y(N_f = 2) + 3Y(N_f = 3) + 4Y(N_f = 4) \right], \quad (9.5)
\]

where \(Z_p + Z_T \text{ and } A_p + A_T\) define the total charge and mass of the colliding system, respectively. This procedure allows us to estimate the baryonic entropy produced in a reaction. The phase space of nucleons is clusterized employing minimum spanning tree (MST) procedure [22] as discussed in chapter 4.

9.3 Results and discussion

9.3.1 Time evolution of \(Nb + Nb\) reactions and thermalization

First of all we analyze the evolution of thermal properties such as average central density \(\rho^\text{avg}\), local temperature \(T\) reached in central zone, and degree of equilibrium for central \(^{93}Nb + ^{93}Nb\) collisions at bombarding energies of 400 and 650 AMeV. Figure 9.1 (a)-(b) compares the average density and temperature reached in the central region at these
bombarding energies. Temperature ‘T’ is calculated in the central zone of 2 fm radius using the hot Thomas Fermi formalism for two overlapping Fermi spheres [23]. To find, the evolution of temperature at each point \((r, t)\) in a HI reaction, in addition to nuclear matter densities of target and projectile:

\[
\rho_T(r, t) = \sum_{i=1}^{\Lambda_T} \rho_i(r, t); \quad \rho_P(r, t) = \sum_{i=1}^{\Lambda_P} \rho_i(r, t),
\]

one needs to evaluate kinetic energy densities at this point as:

\[
\Gamma_T(r, t) = \sum_{i=1}^{\Lambda_T} \frac{\mathbf{p}_i^2(t)}{2m} \rho_i(r, t); \quad \Gamma_P(r, t) = \sum_{i=1}^{\Lambda_P} \frac{\mathbf{p}_i^2(t)}{2m} \rho_i(r, t).
\]

These quantities are calculated in QMD simulations in the nucleus-nucleus c.m. frame. These are used then, in a generalized local density approximation to extract the temperature. Further details of this procedure can be found in Ref. [24]. The shaded area in Fig.9.1 corresponds to the time zone for highly excited nuclear matter which is followed by the decompression and cooling phase. With increase in incident energy, more compression and thus higher temperature is attained in the participant zone. Beyond this region, the nucleon density saturates and hard n-n collisions cease almost. In third row, we show the momentum anisotropy ratio \(\langle R_{iso} \rangle\) which measures the degree of thermalization achieved by the heavy-ion system. The anisotropy ratio is defined as follows [25]:

\[
\langle R_{iso} \rangle = \frac{\sqrt{\langle p_\perp^2 \rangle} + \sqrt{\langle p_\parallel^2 \rangle}}{2 \sqrt{\langle p^2 \rangle}}.
\]

From Fig. 9.1(c), one can see that participant zone is already equilibrated around 40-45 fm/c just after the violent phase is over. One can, therefore, measure yields of composite particles and baryonic entropy at this time. It may be mentioned that full equilibrium with \(\langle R_{iso} \rangle \approx 1\) is not possible in HI reactions even for the central geometry. This is due to surface effects that are more pronounced in lighter reaction systems. In the last two panels, we display the evolution of \(d_{like}\) and \(p_{like}\) cluster abundances. The slight enhancement in these yields at later times is due to de-excitation of and secondary emission from heavier clusters.

### 9.3.2 Coordinate space distribution of light clusters

Since it is well established that the production of light charged particles and clusters, and ultimately the entropy is related to the fireball, it is of interest to see their distribution.
Figure 9.1: The time evolution of central collisions of $^{93}$Ar$+^{93}$Nb at incident energies of 400 and 650 AMeV. Results are shown here for: (a) average central nucleon density $\rho_{av}/\rho_0$; (b) temperature $T$ (in MeV); (c) anisotropy ratio $R_{iso}$; (d)-(e) the yields of $d_{like}$ and $p_{like}$ clusters, respectively.

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in coordinate ($R_3$) space. In Fig. 9.2, we display the 2-D snapshots of $^{93}$Nb+$^{93}$Nb collision at 650 AMeV and at a ‘reduced’ impact parameter $b/b_{\text{max}}=0.6$. Left and right panels depict the results for soft momentum dependent (SM) and static soft (S) equations of state, respectively. First of all, consistent with earlier attempts, we see that heavier fragments belong to the residue of either projectile or target, whereas lighter entities such as free nucleons and light charged particles LCPs [$2 \leq A \leq 4$] are produced due to the coalescence and emerge from the mid-rapidity region. As shown in Ref. [25], these light charged particles carry vital information about the stopping as well as thermalization of the nuclear matter, therefore, are also good candidates for estimating production of entropy in HI reactions. A very little influence can be seen of momentum dependent interactions.

Figure 9.2: The 2-D snapshots of the distribution of nucleons in Z-X and Z-Y planes for a single event of Nb(650 AMeV)+Nb collisions at a ‘reduced’ impact parameter $b/b_{\text{max}}=0.6$. Time taken here corresponds to the case when $n-n$ collisions cease almost just after the violent phase of the reaction. Different colors depict the free particles, nucleons bound in light charged particles as well as in heavier fragments.
9.3.3 Participant proton multiplicity dependence of composite particle yield ratios

Next we study the final state composite particle yield ratios $X/p$ for the soft (S) and soft momentum dependent (SM) interactions. This is shown in Fig. 9.3 for the collisions of $^{93}$Nb+$^{93}$Nb at incident energy of 650 AMeV as a function of impact parameter. Here ‘X’ stands for $A=2, 3$ and 4 clusters. From these curves, one can observe several interesting points:

1. The $X/p$ ratio decreases with impact parameter (alternately, increases with $N_p$) indicating more production in central collisions compared to peripheral collisions. As shown by many authors [9–11], $N_p$ remains same for nearly central collisions and decrease sharply for semi-central collisions and peripheral collisions.

2. For central impact parameters (or, higher $N_p$ values), $X/p$ ratios reach an asymptotic value indicating that for central events, small variation in impact parameter does not give different results.

3. Role of momentum dependent interactions is nearly marginal justifying the earlier attempts [26] and use of soft equation of state.

Since entropy production is mostly measured for central collisions, the use of momentum dependent interactions will not give different results compared to static soft (S) equation of state. These different yield ratios $X/p$ also implied that one obtains different behavior of density reached, collision rate and multiplicity of various light mass fragments for S and SM interactions.

In Fig. 9.4, we extend the above study by including the ratio of deuteronlike ($d_{like}$) to protonlike ($p_{like}$) clusters. The calculations for $d_{like}/p_{like}$ are done using a soft equation of state for the collisions of $^{40}$Ca+$^{40}$Ca (at 400 and 1050 AMeV) and $^{93}$Nb+$^{93}$Nb (at 400 and 650 AMeV) as a function of participant proton multiplicity. The results from Plastic Ball data [9] are also displayed for comparison. The Plastic ball data takes into account the overlap region for the yield of deuteronlike and protonlike clusters, while our ratios are calculated for the unfiltered events using MST procedure. The yield ratios...
Figure 9.3: The yield ratio of light clusters to protons ($X/p$) as a function of impact parameter $b$ using minimum spanning tree procedure. The term 'global' for the ratios $X/p$ signifies that particle yield is calculated taking full ensemble into account and not the limited region only.

are calculated typically after $40 \text{ fm/c}$, when average nucleonic density saturates and $n-n$ collisions practically cease. At this time, yield of composite particles is well established and may be compared with experimental data. One can clearly see that our model describes well the functional form of experimental $d_{like}/p_{like}$ ratio which is found to increase with $N_p$ (or centrality of the collision) and saturates at higher multiplicity end. At low $N_P$, there is a large drop in the yield ratios for the model calculations as also observed for individual cluster-to-proton ($X/p$) ratios (See Fig. 9.3). These trends are closely related with nuclear matter stopping and flow effects in the formation of hot & dense fireball. Recently, Dhawan et al [25] studied impact parameter dependence of light charged particles (LCPs)
Figure 9.4: The $d_{\text{like}}/p_{\text{like}}$ ratio as a function of baryon charge multiplicity $N_p$. The model calculations (open symbols) at the time of freeze out are compared with experimental data (solid symbols). The results are shown here for the reactions of $^{40}\text{Ca} + ^{40}\text{Ca}$ (l.h.s.) and $^{93}\text{Nb} + ^{93}\text{Nb}$ (r.h.s.)

yield and anisotropy ratio. It was found that LCPs production was maximum at central collisions where maximum stopping of nuclear matter is also achieved. Thus, production of light clusters can act as an indicator of global stopping achieved in the nuclear matter. Interestingly, $d_{\text{like}}/p_{\text{like}}$ ratios calculated using dynamical approach are in good agreement with experimental data. This shows that one can reliably explore the applicability of dynamical approach such as the QMD model to further investigate the formation of fireball at intermediate energies.
9.3.4 Participant proton multiplicity dependence of baryonic entropy

We have estimated the entropy produced in the fireball using Eq.(9.2). Figure 9.5 shows the baryonic entropy $S_N$ calculated as a function of participant proton multiplicity $N_p$ for the reactions of $^{40}\text{Ca} + ^{40}\text{Ca}$, $^{93}\text{Nb} + ^{93}\text{Nb}$, and $^{197}\text{Au} + ^{197}\text{Au}$ at incident energies of 400 and 650 AMeV. Note that mass dependence has also been used to understand physics behind a particular phenomenon [27]. In central collisions (i.e. high $N_p$), entropy produced in highly dense matter is expected to be smaller. This happens due to the inhibition of translational expansion of hot nuclear matter. Contrary to this, lower density is reached in peripheral collisions leading to larger entropy production. It clearly brings out the participant-spectator picture of HI collisions at relativistic beam energies. One can clearly see that at given beam energy, it is the volume of participant nucleons (that is, $N_p$) which governs the entropy production rather than the total number of nucleons in the phase space. It means that participant volume solely determines entropy production, independent of the system size. These results are in agreement with the experimental data and theoretical approaches. In the next section, we try to understand beam energy dependence of baryonic entropy.

9.3.5 Beam energy dependence of baryonic entropy

Using the calculated yield ratios $d_{\text{like}}/p_{\text{like}}$, we tried to understand the beam energy dependence of $S_N$ for the collisions of $^{40}\text{Ca} + ^{40}\text{Ca}$ (at 400 and 1050 AMeV) and $^{93}\text{Nb} + ^{93}\text{Nb}$ (at 400 and 650 AMeV). One of the earlier experiments on entropy measurement were done by Nagamiya et al [28]. Entropy was observed in the range $\sim 5 - 6$ and almost constant as a function of bombarding energy [29]. However, experimentalists have also used the QSM approach [8] to estimate entropy produced in the heavy-ion reactions. For Ca+Ca and Nb+Nb systems, it was found to be around $S_N \sim 4$ based upon Kapusta's prescription [5]. This points towards quite unusual mechanism for entropy formation.

In Fig. 9.6, we display our model predictions for baryonic entropy $S_N$ along with experimental data taken with Plastic Ball detector [9]. The calculated $S_N$ values in the range $\sim 4 - 5$ depict weak dependence on beam energy as is expected of an infinite nuclear matter source. Further, these trends are quite close to experimental values [9, 28]. Nearly no effect of beam energy on the baryonic entropy is visible in calculations as well as in
Figure 9.5: Entropy per nucleon $S_N$, as a function of baryon charge multiplicity $N_p$ for the reactions of $^{40}\text{Ca}+^{40}\text{Ca}$ (open circles), $^{93}\text{Nb}+^{93}\text{Nb}$ (half filled circles), and $^{197}\text{Au}+^{197}\text{Au}$ (open squares). Calculations shown here are at incident energies of 100 (top) and 650 (bottom) AMeV.
Figure 9.6: The relationship between baryonic entropy $S_N$ and beam energy ‘E’ in lab. frame for the central collisions of $^{40}\text{Ca} + ^{40}\text{Ca}$ and $^{93}\text{Nb} + ^{93}\text{Nb}$. Also shown are entropy values extracted by the Plastic Ball group [9].

9.4 Summary

In this chapter, we have discussed, in detail, the thermal properties of hot and dense nuclear matter produced at different bombarding energies and colliding geometries. Our calculations indicate that compression and temperature achieved in the participant zone are directly linked with bombarding energy chosen. The degree of thermalization achieved, however, shows little sensitivity towards the bombarding energy chosen. Momentum dependent interactions are found to have negligible influence on the cluster distribution in coordinate space and cluster-to-proton ($X/p$) ratios. The $d_{like}/p_{like}$ yield ratios calculated
as a function of participant proton multiplicity for unfiltered events accurately reproduce
the experimental trends. The baryonic entropy calculated as a function of participant
proton multiplicity is found to show system mass independence and depends solely on the
volume of participants. Further, entropy calculated for central $Ca + Ca$ and $Nb + Nb$
collisions is largely independent of beam energy in accordance with experimental data
based upon Siemens and Kapusta’s formalism.
Bibliography


