Chapter 7

On the Nuclear Incompressibility via Au+Au Fragmentation at 35 MeV/nucleon

7.1 Introduction

One of the important goals of heavy-ion (HI) physics is to gain information about the nuclear equation of state (EoS) by modeling interactions among nucleons and nucleon-nucleon scattering. An empirical property that is used to characterize the equation of state of nuclear matter is the nuclear incompressibility $K$, which is a measure of the stiffness of nuclear matter in ground state against change in the density $\rho$ [1]. Response of nuclear matter towards external pressure (or, change in the density) can be studied via change in the compression energy stored in cold nuclear matter as shown in Fig. 1. This relationship is what commonly known as equation of state (EoS). For a given nuclear interaction employed, it should be able to reproduce the well-known ground state properties of nuclear matter in ground state i.e. binding energy of about -16 MeV/nucleon at $\rho_0 = 0.17$ fm$^{-3}$ [2]. The labels $[1]$, $[2]$, & $[3]$ in the Fig. 7.1 depict the stages arrived during the compression of nuclear matter in heavy-ion collisions. Red arrows in Fig. 7.1 indicate how the total center-of-mass energy per baryon $E_{cm}$ is divided into compressional part $E_c$ and thermal part $E_{th}$. It is worthwhile to see that the curvature of the binding energy is a measure of the stiffness of nuclear matter near the saturation point ($\rho_0,T = 0$). A larger curvature implies that more energy is needed to compress the nucleus and makes the nuclear matter stiffer. The key question is how to measure the density, compression energy $E_c$ and/or temperature reached in such highly excited nuclear matter.
Figure 7.1: A sketch of the nuclear EoS relating the energy per baryon $E/A$ ($\rho,T=0$) to the density of nuclear matter. The labels [1], [2], and [3] indicate various stages arrived in a reaction.

7.2 Nuclear equation of state and heavy-ion collisions

Earliest attempts to pin down the nuclear EoS (or, incompressibility) were based upon the study of giant monopole resonance (GMR) [3]. The scattering of $\alpha$-particles off the nucleus induces volume oscillations with $L=0$, which can be used to determine the incompressibility $\kappa$ of that nucleus. These results generally yield incompressibility in the range $\kappa \sim 250-270$ MeV indicating the matter to be softer. A recent GMR study in the $^{208}$Pb and $^{90}$Zn nuclei showed that softening of nuclear matter is needed to explain the collective modes with different neutron-to-proton ratios [4]. Another study on the fusion reported linear momentum transfer to be sensitive to both the EoS and n-n cross...
Within the quantum molecular dynamics (QMD) model, an incompressibility of $K = 200$ MeV (i.e., soft EoS) was reported to explain the experimental data on energy transfer in a compound nucleus formation [5].

The study of heavy-ion collisions at intermediate energies can be of importance to probe the compressibility of nuclear matter and/or nuclear EoS. Various attempts have been made to find the observables which are sensitive to the nuclear EoS [6-14]. These microscopic transport models have the convenience to use EoS as input directly and study different quantitative aspects. The main problem in dealing with heavy-ion collisions is that the equilibrium can not be guaranteed always even at later stages of the reaction. The concept of equation of state is, however, valid for the system under equilibrium. This picture is very well realized in neutron stars (NS). In fact, internal structure and composition of neutron stars and supernovae depend strongly on the high density behavior of nuclear EoS.

To find an appropriate nuclear EoS, one should, in principle, develop a many-body theory whose parameters governing $n-n$ interactions should describe the behavior of nucleons in vacuum as well as in bulk nuclear medium. Earlier attempts using realistic $n-n$ interactions could not accurately reproduce the ground state characteristics i.e., binding energy and saturation density $\rho_0$ of the nuclear matter [15]. Even higher order correlations were also taken into account to explain the ground state characteristics of normal nuclear matter [16]. The microscopic models like Hartree-Fock theory, VUU and QMD [17-20] use phenomenological parametrization of the nuclear EoS that originates from the Skyrme-type interaction [21]. For the cold and symmetric nuclear matter, the baryonic energy $e(\rho, T = 0)$ is given as:

$$e(\rho, T = 0) = T_F(\rho, T = 0) + \frac{\alpha}{2} \rho + \frac{\beta}{\gamma + 1} \rho^\gamma.$$  

The first term in Eq.(7.1) is the kinetic energy of a non-relativistic cold Fermi gas. Remaining terms constitute the potential energy. In the vicinity of saturation density, the energy density per nucleon $e(\rho, T = 0)$ can be expanded around $\rho_0$ as:

$$e(\rho, T = 0) = e(\rho_0, T = 0) + \frac{K}{18} \left( \frac{\rho - \rho_0}{\rho_0} \right)^2 + \ldots .$$  

Clearly from Eq.(7.2), compressibility $K$ is defined as curvature of $e(\rho, 0)$ versus $\rho$ curve near $\rho_0$, i.e.

$$K = 9\rho_0^2 \left. \frac{\partial^2 e(\rho, 0)}{\partial \rho^2} \right|_{\rho = \rho_0} .$$  

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For the case of asymmetric nuclear matter at supra-normal densities, different values of nuclear compressibility have been predicted by microscopic approaches such as Brueckner-Hartree-Fock (BHF) [22], Dirac-Brueckner-Hartree-Fock (DBHF) [23] and phenomenological models like NL3 [24]. This has led to intense focus on constraining the symmetry energy component of nuclear EoS [25,26] in high density region. The isospin dependence of nuclear EoS finds immense application in understanding the structure of neutron stars (NS) [26]. Unfortunately, due to different density dependence of symmetry energy, it has remained difficult till now to constrain the exact relationship between mass and radius of NS. For instance, BHF calculations give maximum mass of NS close to two solar masses (=2M⊙), while DBHF and variational equations predicted slightly higher value \( \approx 2.2 - 2.3M_\odot \) [25]. Besides this, knowledge of symmetry energy component of the nuclear EoS is important for understanding structure of radioactive nuclei, nuclear reactions with radioisotopes as well as liquid-gas phase transition in asymmetric nuclear matter [26–30]. For asymmetric nuclear matter with asymmetry \( \delta = \frac{n_p - n_n}{\rho} \), the energy density up to second order expansion is given as:

\[
e(\rho, \delta) = e(\rho, 0) + \delta^2 e_{\text{sym}}(\rho) \tag{7.4}
\]

where \( \rho = n_n + n_p \) is the baryon density with \( n_n \) and \( n_p \) denoting neutron and proton densities, respectively. The symmetry energy term \( e_{\text{sym}} \) is, therefore, defined as:

\[
e_{\text{sym}}(\rho) = \frac{1}{2} \frac{\partial^2}{\partial \delta^2} (e(\rho, \delta)) \bigg|_{\delta=0} \tag{7.5}
\]

To illustrate the effect of isospin content and thus, asymmetry of nuclear matter on equation of state, we display in Fig. 7.2, the energy per baryon for pure neutron matter (upper curve) and symmetric nuclear matter (lower curve). From Eq.(7.4), we can now define \( e_{\text{sym}} \) for the extreme cases of pure neutron matter and symmetric matter (as shown in Fig. 7.2) as:

\[
e_{\text{sym}}(\rho) = e(\rho, 1) - e(\rho, 0) \tag{7.6}
\]

Frankfurt-Heidelberg-Chandigarh Collaboration has made extensive efforts to investigate observables which are sensitive towards nuclear EoS. Hahn and Stöcker have proposed a thermal model which could reproduce the measured pion multiplicity over incident energy 30 AMeV up to 4 AGeV. In the comparison of thermal energies with experimental values, a surplus of about 60-70% was found in the experimental data [32]. Their investigation have shown that in-medium effects strongly influence the observables related to
particle production for instance, π, K, λ yields, and deuteron-to-proton yield ratios etc [7, 14, 20, 33]. The collective flow observed in HI collisions is another observable which is very sensitive towards the stiffness of nuclear EoS [7, 8, 10, 11]. The collective transverse in-plane flow and balance energy (the energy at which flow becomes zero) have been studied extensively over the past two decades so as to constrain the EoS, but still the uncertainties are very large. For example, a stiff EoS with $\kappa=380$ MeV reproduces the transverse flow data equally well as obtained with soft momentum dependent EoS with $\kappa=210$ MeV [6, 12, 20, 34]. Similarly, comparison of transport model calculations with data of EOS Collaboration for the energy dependence of collective flow favored neither the ‘soft’ nor the ‘hard’ equation of state [10]. Another study by Pan and Danielewicz estimated the value of $\kappa$ in range $\approx 160-220$ MeV for the multiplicity dependence of sideward flow [34]. In recent comparison of elliptic flow data with microscopic transport model calculations of the Refs. [35, 36], no consistent agreement between data and calculations could be obtained [37] for two different models of Refs. [35] & [36].

It is clear from the above review that an appropriate choice of nuclear equation of state is still far from settlement. The task of deriving quantitative information about the EoS requires detailed comparison of theoretical calculations assuming different equations of state with data.
state with experimental data. At lower beam energies, mean-field effects and long range Coulomb force govern the reaction dynamics. The phenomena such as fusion-fission, cluster decay, and deep-inelastic scattering dominate the heavy-ion physics in low energy regime [38-41]. At incident energies above 20 AMeV, non-fusion events like production of intermediate mass fragments (IMFs), projectile-like and target-like fragments (PLFs and TLFS) dominate the exit channel. The phenomenon of multi-fragment emission in low energy domain is, however, least exploited to infer the nuclear EoS. Naturally, the study of fragment-emission in low energy domain may be of importance in probing the nuclear incompressibility, where the role of different $n-n$ cross sections is minimal.

7.3 Au+Au collision as a probe to determine the incompressibility

To explore the possibility of extracting information on the nuclear incompressibility via low energy nuclear collisions, we proposed to study the peripheral reactions of $^{197}$Au + $^{197}$Au at 35 AMeV and at different peripheral geometries where accurate data has been measured recently [42]. We shall perform the QMD simulation of Au+Au reactions using a ‘soft’(S) and a ‘hard’(H) equations of state. Parameters corresponding to the two equations of state employed in the QMD model can be found in chapter 3. If propagating nucleons come too close, these can scatter elastically or inelastically depending upon available center-of-mass energy. The influence of different $n-n$ scattering cross sections ($\sigma_{nn}$) will be determined by employing a set of different cross sections varying from energy-dependent cross section [43] to constant and isotropic cross sections of 40 and 55 mb strengths.

For the present study Au+Au with mass $A_{tot}$=394 is the system of choice. This is because, Au nucleus is nearly the largest drop of nuclear matter that can be created on the earth. Further, it also approximates infinite nuclear matter in some traits; most encompassing of which is the nuclear equation of state. It has been shown previously that for Au+Au system, the balance energy $E_{bal}$ (at which attractive and repulsive parts of nuclear interaction balance each other) has weaker dependence on the impact parameter as well as on the $n-n$ cross section in mass-impact parameter plane [11]. For the lighter systems such as Ar+Sc, the balance energy is found to show little sensitivity towards nuclear incompressibility $K$ [11]. Magestro et al have [11] studied the balance energy
for different values of compressibility: $\kappa = 180$, 200, 235, and 380 MeV. Their calculations based on the BUU approach showed that only $\kappa = 200$ MeV can explain the data accurately. This value of incompressibility corresponds to softer nuclear matter.

7.4 Different nucleon-nucleon cross sections

The choice of different nucleon-nucleon cross sections $\sigma_{nn}$ tends to influence the reaction observables such as fragment emission [7] and collective transverse expansion [44]. Peilert et al. had earlier shown that the effective in-medium cross section led to appreciable transparency in the reaction system [7]. One may expect that cross sections have more or less same strength at low incident energies. However, it may not be the case always. A parametrization of the in-medium cross section was proposed based upon Bonn meson exchange potential and Dirac-Brueckner theory for nuclear matter [15]. This cross section is observed to deviate substantially from the energy dependent cross section parameterized by Cugnon [43]. In the following subsections, we shall elaborate various kinds of n-n cross section employed in the QMD approach.

7.4.1 In-medium cross section

It has been shown that medium-dependent cross section strongly affects the reaction observables such as density, temperature [6], fragment and flow variables [46–48]. Not only static properties of hadrons (e.g., rest mass) but also the dynamical ones (e.g., n-n scattering) differ from the corresponding counterparts in free space. In this direction, Tübingen Collaboration led by Prof. Faessler has extensively studied the in-medium scattering [49] based upon non-relativistic Brueckner theory and Reid soft-core optical potential [50]. Using this approach, mean field, and in-medium total cross section as well as differential n-n cross section are calculated self-consistently in the QMD approach. We call this as self-consistent quantum molecular dynamics model since we now have consistency between mean field and n-n cross section which together govern the dynamical evolution of nucleus-nucleus collisions. The essential ingredient to investigate the microscopic optical potential between two nucleons is the Bethe-Goldstone (BG) equation [49]:

$$G(\omega) = V + V \frac{Q^2}{\omega^2 - H^2 + i\epsilon} G(\omega),$$

(7.7)
where \( V \) is bare \( n-n \) interaction (Reid soft-core interaction, in our case), \( \omega \) is the starting energy and \( Q_F \) is the Pauli operator. To solve the BG equation in momentum space, matrix inversion method proposed by Haftel and Tabakin [51] is employed. Here, we introduce the c.m. momentum \( K = \frac{1}{2}(k_1 + k_2) \) and relative momentum \( k = \frac{1}{2}(k_1 - k_2) \) of the two nucleons with momenta \( k_1 \) and \( k_2 \). The Hamiltonian operator \( H_0 \) acting on a two-nucleon state gives:

\[
H_0|k_1k_2\rangle = (\varepsilon(k_1) + \varepsilon(k_2))|k_1k_2\rangle.
\]

The single-particle potential \( U(k) \) is calculated in a self-consistent manner as:

\[
U(k) = \frac{1}{4} \sum_{\text{spin, isospin}} \int_F \frac{d^3k}{(2\pi)^3} \langle k, \hat{\mathbf{k}} | G(\omega = \varepsilon(k_1) + \varepsilon(k_2)) | k, \hat{\mathbf{k}} \rangle,
\]

where integration is done over all occupied states in Fermi sea \( F \) occupied by two colliding Fermi spheres \( F_1 \) and \( F_2 \). The numerical calculations of G-matrix is done using standard averaging for Pauli operator and single particle potential in the energy denominator. We, thus obtain decoupled partial wave BG equations:

\[
\langle \hat{k}|G_{ji}(\omega, K)|k\rangle = \langle \hat{k}|V_{ji}|k\rangle + \frac{2}{\pi} \int d\mathbf{k}\,' k''k''\langle \hat{k}|V_{ji}|k''\rangle \times \frac{Q_F(k'', K)}{\omega - E(k'', K) + i\epsilon} \langle k''|G_{ji}(\omega, K)|k\rangle,
\]

where \( i, j, \) and \( l \) represent partial wave (LSJT) and \( Q_F \) and \( E \) are averaged quantities. The differential cross section \( \frac{d\sigma}{d\Omega} \) for \( n-n \) scattering is calculated as a function of relative momentum \( k \) for two nucleons with \( K=0 \). Solution of Eq.(7.10) i.e. \( \langle k|G_{ji}(\omega = 2 \varepsilon(k), K = 0)|k\rangle \) then corresponds to the partial wave scattering amplitude in the c.m. system. The differential cross section is given by:

\[
\frac{d\sigma}{d\Omega} = \frac{1}{4} \sum_{m_s,m_{s'}} |T_{m_s,m_{s'}}^{S=1}(\theta)|^2 + |T_{m_s,m_{s}}^{S=0}(\theta)|^2,
\]

with

\[
T_{m_s,m_{s'}}^{S}= \sum_{LLJ} \left[ \frac{2L+1}{4\pi} \right] \langle L \circ m_s | J m_s | L \circ m_{s'} \rangle \langle \hat{L} m_s - m_{s'} | S m_s | J m_s \rangle \times \langle k, \hat{L} S J | G | k, L S J \rangle.
\]

Equation (7.12) contains an appropriate combination of \( T=0 \) and \( T=1 \) parts depending on \( nn, pp, \) or \( pn \) scattering. As an example, we display the G-matrix cross section.
Figure 7.3: The in-medium total cross section $\sigma_{nn}$ calculated using non-relativistic Brueckner theory as a function of incident energy per nucleon $E_{N,N,\text{lab}}$ in the laboratory frame (Figure is taken from Ref. [52]).
\[ \sigma_{nn} = \int \frac{d \sigma}{d t} d \Omega \] in Fig. 7.3 as a function of incident energy \( E_{NN, \text{lab}} \) for the average relativistic momentum per nucleon \( K_r = 2.1 \text{ fm}^{-1} \). Also shown in the figure is free nucleon-nucleon cross section for comparison. At low incident energies, G-matrix cross section drops to zero as expected due to Pauli blocking of final state. A horizontal dashed line represents constant cross section of 40 mb.

### 7.4.2 Energy-dependent cross section

For energy dependent cross section, parametrization proposed by Cugnon et al is employed in the QMD model. In this parametrization, \( \Delta \) resonance is included in elastic and inelastic channels. For \( \Delta \)-excitation channel \((nn \rightarrow n\Delta)\), the total inelastic cross-section is calculated as [43]:

\[
\begin{cases}
\sigma_{nn \rightarrow n\Delta}(\sqrt{s}) = 0 & ; \sqrt{s} \leq 2.015 \\
= \frac{20(\sqrt{s} - 2.015)^2}{0.015 + (\sqrt{s} - 2.015)^2} & ; \sqrt{s} > 2.015
\end{cases}
\]

angular distribution : isotropic.  \((7.13)\)

The cross section for \( \Delta \)-absorption channel \((n\Delta \rightarrow nn)\) can be obtained from Eq. \((7.13)\) with the use of detailed balance principle:

\[
\begin{cases}
\sigma_{n\Delta \rightarrow nn}(\sqrt{s}) = \frac{1}{2}(p_f^2/p_i^2)\sigma_{nn \rightarrow n\Delta}(\sqrt{s}). \\
\text{angular distribution : isotropic.}
\end{cases}
\]

Here \( p_f \) is the momentum in final \( n-n \) state given as:

\[
p_f = \left. s^2 - 2s(m_1^2 + m_2^2) + (m_1^2 - m_2^2)^2 \right]^{1/2} 
\]

\((7.14)\)

Angular distribution has been studied intensively by Puri and Collaborators and no influence of this has been seen on fragment formation and rapidity distribution [53].
7.4.3 Constant and isotropic cross section

The constant and isotropic $\sigma_{nn}$ of magnitudes 40 and 55 mb have been widely used in the simulation of heavy-ion collisions [54]. The choice of constant cross section of such high values becomes essential in low energy collisions where most of the collisions are Pauli blocked and wavelength of incoming particle is comparable with nuclear size. It is worth interesting to state that a large reaction cross section was reported for the reaction of $^{22}\text{C}$ on a liquid hydrogen target at 40 AMeV. It was significantly larger than corresponding to neighboring $^{19,20}\text{C}$ isotopes [55]. A constant cross section of 40 mb has been motivated by the hard core radius of the nucleon-nucleon potential. Further, a constant cross section of 55 mb corresponds to the cut-off value in the Cugnon parametrization for invariant energy $\sqrt{s} < 1.8993$ GeV. It is the only medium effect present in this parametrization based upon free nucleon-nucleon scattering data. As mentioned above, strengths of $\sigma_{nn}$ obtained from different G-matrix calculations vary considerably. It is, therefore, more useful to employ constant cross sections of 40 and 55 mb strengths to probe the collision of Au+Au at such low energy.

7.5 Results and discussion

7.5.1 Effect of different $\sigma_{nn}$ on reaction dynamics

As mentioned above, one may not expect different nucleon-nucleon cross sections to have same strength even at lower energy regime. It is, therefore, important to understand the inter-play of different $n-n$ cross sections. We have studied characteristics of fragment emission in peripheral $^{197}\text{Au} + ^{197}\text{Au}$ collisions at 35 AMeV employing various strengths of $n-n$ cross sections. A hard EoS with energy dependent cross section is labeled as $H^{C5}$. Incorporation of isotropic and constant cross sections of 40 and 55 mb strengths have been labeled as $H^{C4}$ and $H^{C5}$, respectively. Similarly, for the soft equation of state, we have $S^{C4}$, $S^{C5}$, and $S^{C5}$, respectively. The phase space, thus obtained was subjected to $SACA(2.1)$ clusterization subroutine [56,57]. In Fig. 7.4, we display the average values of nucleon density $\rho^{avg}$, nucleon-nucleon collision rate $dN_{coll}/dt$, and size of heaviest fragment $A^{max}$ as a function of time. As expected, the choice of different cross sections $\sigma_{nn}$ has negligible role to play at such a low incident energy. However, $n-n$ collision rates differ appreciably due to medium-dependence of these cross sections. One notices several interesting points.
from these results:

(i) Maximal density is obtained nearly at same time at both impact parameters, whereas saturated value is slightly more at more peripheral geometries.

(ii) The choice of different $\sigma_{nn}$ has negligible effect on present results. This happens due to effective Pauli blocking at such a low incident energy that prohibits binary $n-n$ collisions.

(iii) However, stronger dependence can be seen on the nuclear EoS. This difference is clearly visible in the evolution of heaviest fragment $\langle A_{\text{max}} \rangle$.

The mean size of the heaviest fragment $\langle A_{\text{max}} \rangle$ reaches minimum value around 100 fm/c, where stable fragment configuration can be realized and compared with experimental results. With stiff EoS, heavier $\langle A_{\text{max}} \rangle$ is registered as shown in Fig. 7.4. Significant differences are also visible for the multiplicity of free particles, light charged particles LCPs [$2 \leq A \leq 4$], and clusters with mass $A \geq 5$ obtained using soft and stiff equations of state (See Fig. 7.5). Using the hard EoS, dissipation of energy takes place mainly via emission of free-nucleons that cools down the nuclear system in case of hard EoS. Consequently, lesser yields of LCPs and fragments with mass $A \geq 5$ are obtained with a stiff EoS. On the other hand, soft EoS favors emission of LCPs and heavier fragments ($A \geq 5$) from the spectator zone, thereby decreasing the size of $A_{\text{max}}$. One can, thus, conclude that fragment observables are least affected with the choice of different $\sigma_{nn}$. This observation would be helpful to constrain the nuclear EoS. To investigate further the role of different nuclear EoS, we have used standard energy-dependent cross section.

### 7.5.2 Stopping phenomenon and nuclear EoS

The phenomenon of stopping and equilibration of various fragment species is closely related with $n-n$ interactions used and thus nuclear EoS. We display in Figs. 7.6 and 7.7, the spectrum of scaled transverse (left panel) and longitudinal (right panel) rapidity distributions of free particles and intermediate mass fragments IMFs [$5 \leq A \leq 65$] at ‘reduced’ impact parameters of $b/b_{\text{max}}=0.55$ and 0.85 respectively.

We find that cluster emission is quite sensitive to nuclear incompressibility that brings significant changes in their stopping as well as transverse expansion. Using a ‘stiff’ EoS, the system seems to cool-off via abundant production of free nucleons from midrapidity.
Figure 7.4: QMD simulation of Au (35 AMeV)+Au collisions at reduced impact parameter $b/b_{\text{max}}=0.55$ (left panel) and $b/b_{\text{max}}=0.85$ (right panel) as a function of time for (a) mean nucleon density $\rho_{\text{avg}}/\rho_0$; (b) $n-n$ collision rate $dN_{\text{coll}}/dt$; (c) size of heaviest fragment $A_{\text{max}}$, respectively.

$^{197}\text{Au}^{+}\text{Au}$

$E=35\text{ AMeV}$

Figure 7.4: QMD simulation of Au (35 AMeV)+Au collisions at reduced impact parameter $b/b_{\text{max}}=0.55$ (left panel) and $b/b_{\text{max}}=0.85$ (right panel) as a function of time for (a) mean nucleon density $\rho_{\text{avg}}/\rho_0$; (b) $n-n$ collision rate $dN_{\text{coll}}/dt$; (c) size of heaviest fragment $A_{\text{max}}$, respectively.
Figure 7.5: QMD simulation of Au (35 AMeV)+Au collisions at reduced impact parameter $b/b_{max}=0.55$ (left panel) and $b/b_{max}=0.85$ (right panel) as a function of time for the multiplicities of (a) free nucleons; (b) light charged particles LCPs; (c) fragments with mass $A \geq 5$, respectively.
Figure 7.6: The rapidity distribution $dN/dy$ as a function of scaled transverse, $y^{(x)}/y_{beam}$ (left) and longitudinal, $y^{(z)}/y_{beam}$ (right) rapidities for Au(35 AMeV) + Au reaction at reduced impact parameter $b/b_{max}=0.55$. Solid and dashed curves correspond to model calculations using a ‘soft’ (S) and a ‘hard’ (H) EoS, respectively.
Figure 7.7: Same as Fig. 7.6, but at reduced impact parameter \( b/b_{\text{max}} = 0.85 \).
as well as from spectator zones, whereas a ‘soft’ EoS contributes significantly towards the
emission of IMFs at target and projectile rapidities. It means that system propagating
under the soft interactions is less equilibrated. Similar trends are also visible in the
transverse rapidity ($y$) distribution of free nucleons and IMFs. Using hard interactions,
a larger fraction of free nucleons are emitted into transverse direction. IMFs are not,
however, dispersed much into transverse directions and continue to move at target and
projectile velocities. As a result, heavier fragments leave the participant zone quite early
and suffer less binary collisions. These findings suggest that fragment emission from the
decay of spectator component is quite sensitive to the mean field and compressibility of
participant nuclear matter.

7.5.3 Fragment charge yields and comparison with experimental
data

Next, we turn to estimate the fragment charge yield $N(Z)$ from the spectator matter de­
cay in peripheral Au (35 AMeV)+Au collisions. We shall also attempt to compare our
model predictions using soft (S) and hard (H) equations of state with experimental data
of Multics-Miniball Collaboration taken at K1200-NSCL cyclotron [42]. Beams of Au ion
at $E=35$ AMeV were accelerated by K1200 cyclotron which were used to bombard Au foils
of about 5 mg/cm² arial density. The light charged products with charge $Z \leq 20$ were
detected in the angular range $23° < \theta_{lab} < 160°$ by the MSU Miniball detector [58]. Reac­
tion products with charge $Z \leq 83$ were detected in the angular range $3° < \theta_{lab} < 23°$ by
the Multics array [59]. To account for events from the decay of quasi-projectile in forward
hemisphere, the charge dispersion at six different impact parameter intervals has been
calculated using forward rapidity condition ($y > 0.5y_{beam}$) in the center-of-mass frame.
Further, it also exclude events from midrapidity and quasi-target emission. We can see
from Fig. 7.8 that QMD model can reproduce experimental trends in charge distribu­
tion quite well at all impact parameter intervals. In the last panel (for $b/b_{max} > 0.95$),
two peaks can be seen in experimental charge distribution. As one moves towards semi-
peripheral impact parameters. U-shape disappears and slopes of curves become steeper.
This also indicates more input of excitation energy into the spectator zone. Further,
systematic differences can be seen clearly in the charge distributions obtained with soft
(solid line) and hard (dashed line) equations of state. To make the picture more clear and
distinguish between the two equations of state, we calculated the integrated multiplicity
Figure 7.8: The charge distribution $N(Z)$ obtained for Au (35 AMeV) + Au reactions at different impact parameter intervals using a 'soft' (solid line) and a 'hard' (dashed line) EoS. Filled circles depict experimental data points [42].
Figure 7.9: The impact parameter dependence of multiplicity of fragments with charge $3 \leq Z \leq 80$ obtained using a ‘soft’ (solid line) and a ‘hard’ (dashed line) equations of state in Au(35 AMeV)+Au collisions. Filled circles depict the experimental data points [42].

of charged particles with $3 \leq Z \leq 80$ (i.e. $\int_3^{80} N(Z) dZ$) at 100 fm/c as a function of ‘reduced’ impact parameter $b/b_{\text{max}}$. Calculated multiplicities along with the experimental data points [42] are displayed in Fig. 7.9.

It is worth mentioning that multiplicities were calculated keeping in mind the angular range covered by the combined Multics-Miniball array. A soft incompressibility modulus is observed to explain the impact parameter dependence of charged particle multiplicity obtained from the spectator matter decay much nicely. Due to more explosive nature of hard EoS, spectator matter mainly de-excites via emission of free nucleons and therefore, decline in multiplicity of heavier clusters occurs. An increasing trend of fragment multiplicity with centrality can be understood in terms of more excitation energy deposited in spectator matter. In semi-peripheral events, a larger chunk of excitation energy gets transferred to spectator matter, thereby leading to rise in multiplicity of fragments with charge $3 \leq Z \leq 80$. A slight discrepancy between fragment charge multiplicity (using a soft EoS) and experimental data at extremely peripheral geometries may be due to lower detection.
efficiency of combined Multics-Miniball array when quasi-projectile mainly flies off at laboratory angles smaller than minimum detection angle [42,60]. Nuclear mean-field seems to be important factor governing the outcome of spectator decay, while nucleon-nucleon collisions dominate the participant matter physics. This analysis clearly illustrates the relatively softer nature of nuclear matter in accord with previous findings [3,11].

7.6 Summary

In conclusion, dynamical calculations within the framework of QMD approach are performed to probe the nuclear incompressibility in low energy domain. First of all, we have analyzed the inter-play of different $n$-$n$ cross sections using the soft and hard equations of state in $^{197}$Au +$^{197}$Au collisions at 35 AMeV. The choice of different cross sections has marginal role to play in the reaction dynamics at such low excitation energies. These findings allow us to constrain the nuclear EoS parameter $\mathcal{K}$ to a very precise level. The stopping of fragments and charge yields obtained from the spectator decay are observed to be highly sensitive towards nuclear incompressibility of the nuclear matter. The hard EoS results in enhanced emission of free nucleons and fewer heavier fragments. Model calculations using soft EoS for charge yields from the decay of quasi-projectile are in accord with experimental trends. These findings favor softer nature of nuclear matter.
Bibliography


