Chapter 4

Onset of Multifragmentation in Low Energy Heavy-Ion Collisions: System Size Effects and Comparison with MSU-NSCL Data

4.1 Introduction

The dynamical calculations have shown that fragment-emission is the result of compressional energy stored in early phase of the reaction in central collisions [1-3]. The fragment emission from hot and compressed nuclear system is believed to occur due to fluctuations in the mean field and instability generated by the Coulomb forces. Longer the system stays in the instability region, more probable it is bound to break into smaller pieces. If system is light, break-up is expected to occur at incident energies as low as \( \sim 40-60 \) AMeV. For heavier systems naturally, the demand for compression energy and hence beam energy is more to cause the effective break-up. QMD calculations by Puri et al [4] have shown that maximal fragment production in central \(^{40}\text{Ca} + ^{40}\text{Ca}\) collisions takes place around 60 AMeV. An earlier attempt by Bauer et al [5] showed that for the collisions of two heavy-ions, maximum multiplicity of intermediate mass fragments is observed around 100 AMeV in central collisions. This existence of peak IMF emission was in accord with similar study by Peilert et al [6] using Au-nuclei. The multi-fragment emission in central collisions at low incident energies, therefore, reveals a complex picture with energy deposition and system mass as the controlling factors.

However, very few attempts have been reported in literature that are concerned with beam energy dependence and system size effects [7-11]. In the past, percolation ap-
proaches have been successfully applied to study fragment emission pattern, kinetic energy of emitted fragments, IMF yields from the decay of heavy projectiles and beam energy dependence [8–10, 12]. Li et al [8] based upon percolation model calculated the fit parameter $\lambda$ in the power law: $\sigma(Z) \propto Z^{-\lambda}$ for different lattice sizes. Interestingly the critical parameter $\lambda$ was observed to scale with size of the fragmenting system. Similarly, the ‘LATINO1’ model simulations were performed for central collisions of $^{58}\text{Ni} + ^{58}\text{Ni}$ at different incident energies to extract the entropy produced [13]. The maximum entropy generated is observed to decrease with size of the asymptotic source. This analysis showed that lighter sources tend to produce more entropy.

Another important phenomenon such as flow effects also witness similar system size effects. The elliptical flow is found to shift from the positive value (in-plane emission) to negative value (out-of-plane emission) at certain beam energy called as transition energy. Recently transition energies in the reaction systems with masses between $^{58}\text{Ni} + ^{58}\text{Ni}$ and $^{197}\text{Au} + ^{197}\text{Au}$ were calculated within improved quantum molecular dynamics (ImQMD) model [14]. The transition energy was observed to follow the power law: $E_{\text{trans}} \propto (A_{\text{tot}})^{-\tau}$ with exponent $\tau \approx 0.22$. Similar mass dependence is reported for transition energy of light charged particles studied within isospin dependent quantum molecular dynamics (IQMD) model [15].

As far as fragment production is concerned, the recent experiments performed on MSU 4π-Array set up indicated a rise and fall pattern in the multiplicity of intermediate mass fragments (IMFs) as a function of beam energy in the center-of-mass frame [9]. The calculations based on percolation theory, however, could not accurately predict the system size dependence of peak $E_{\text{c.m.}}$ (at which maximal IMF production occurs). Even maximal multiplicity of IMFs was also overestimated. This led Sisan et al [9] to the conclusion that perhaps phase space model could correctly interpret this dependence. We plan to address this situation by employing the QMD model, in which reaction dynamics can be followed from the start to the end where matter is cold and fragmented. We have used a soft EoS along with Cugnon parametrization of $n-n$ cross section [16]. The choice of soft EoS has been advocated in many theoretical studies. Recently, Magestro et al [17] tried to pin down the nuclear incompressibility using balance energy. Their detailed study pointed towards a softer equation of state. Another study concerning the linear momentum transfer occurring in central HI collisions also showed that a soft compressibility modulus is needed to explain the experimental data [18]. The cluster yield
is obtained through standard minimum spanning tree (MST) procedure with additional binding energy check. In the following section, we shall give details of MST clusterization algorithm along with its extensions.

4.2 Minimum Spanning Tree (MST) method

The variational approach employed in QMD model reduced the complication of following time evolution of an A-body wave function by resolving to 6A coupled differential equations for the centroids of the coherent wave functions in configuration and momentum space. This approach allows us to define clusters in a very convenient manner. At the end of the reaction, phase space occupancy is quite low, so nucleons forming a cluster will be one closer in coordinate space. This simplest approach of identifying clusters is well known as minimum spanning tree (MST) method [6.19-26]. In this method, two nucleons share the same cluster if their centroids in coordinate space are closer than a given clusterization radius $R_{clus}$, that is

$$|\mathbf{r}_i - \mathbf{r}_j| \leq R_{clus},$$

where $\mathbf{r}_i, \mathbf{r}_j$ are the spatial coordinates of the two nucleons. The clusterization radius $R_{clus}$ is used as a free parameter which may lie between 2-4 fm. The variation in the clusterization radius is found to have negligible influence on the final state fragment pattern at the end of reaction ($\sim 300$ fm/c), since nucleons belonging to different clusters are well separated in spatial coordinates [6, 27].

An improvisation over MST approach has also been tried which look for bound two-nucleon structure in the same cluster. This algorithm labeled as MSTE [28, 29] recognizes two nucleons $i$ and $j$ bound in a cluster if

$$e_{ij} \leq 0,$$

with $e_{ij} = V_{ij} + \frac{(p_i - p_j)^2}{4\mu}$; $\mu$ being the reduced mass.
4.2.1 Minimum Spanning Tree with Momentum cut (MSTM) method

In addition to spatial cut of Eq. (4.1), we introduce a cut on relative momentum of two nucleons. That is, we demand [25,30]:
\[
|r_i - r_j| \leq R_{clus},
\]
\[
|p_i - p_j| \leq P_F.
\] (4.3)

Here, $P_F$ is the average Fermi momentum of nucleons bound in a nucleus ($\approx 150$ MeV/c) in its ground state. This improvisation checks the formation of artificial and unbound fragments by excluding those nucleons having relative momenta larger than $P_F$. Eventually, MST and MSTM methods give different results at the start of a reaction. The MST method gives one largest cluster of size $(=A_F + A_T)$, while MSTM method gives two distinct clusters of masses $A_F$ and $A_T$ having very large relative momenta. This algorithm identifies the largest fragment $A^{max}$ as early as 50-60 fm/c [23]. As a result, fragment emission starts earlier with MSTM, when MST method just detects a single biggest cluster.

4.2.2 Minimum Spanning Tree with Binding energy check (MSTB) method

In this modified version, pre-clusters obtained with conventional MST approach are subjected to the binding energy check:
\[
\zeta = \frac{1}{N_f} \sum_{i=1}^{N_f} \left[ \frac{(p_i - P_{cm})^2}{2m_i} + \frac{1}{2} \sum_{j \neq i}^{N_f} V_{ij}(r_i, r_j) \right] < -E_{bind}. 
\] (4.4)

We take $E_{bind} = 4.0$ MeV, if $N_f \geq 3$ else, $E_{bind} = 0$. $N_f$ is the number of nucleons in a fragment and $P_{cm}^{N_f}$ is the center-of-mass momentum of a fragment. This criterion forbids the formation of loosely bound clusters. The role of MSTB method is quite important in central symmetric reactions [23,26]. The MSTB method doesn’t recognize the largest fragment $A^{max}$ during early violent phase. Therefore, it shows the nucleus as unbound group of nucleons. A properly bound $A^{max}$ is identified only around 120 fm/c [24,31]. In addition, binding energy check helps to obtain the stable fragment configuration quite early and reduces the computation time for multifragmentation.
4.3 The time evolution of fragments using MST and MSTB approaches

Our calculations within QMD model are done employing minimum spanning tree procedure with additional binding energy check. Additional condition of minimum fragment binding energy rules out formation of improper and unbound clusters and speeds up the fragment recognition procedure [23, 24, 31]. One has to also keep in the mind that semi-classical models like QMD can not keep nuclei stable for long time. A typical stability of nuclei can be seen till 200 fm/c. If one analyzes the fragment formation with MST alone, then one may not achieve true fragment structure at 200 fm/c. To support this fact, we display in Fig. 4.1, the multiplicities of various fragment species, size of heaviest fragment $A_{\text{max}}$ as well as binding energy of light charged particles (LCPs) and intermediate mass fragments (IMFs) as a function of time. In the beginning, MST could detect only a bigger $A_{\text{max}}$ and small yield of free nucleons. On other hand, MSTB approach detects all nucleons to be free entities in the violent phase of reaction. Free nucleons begin to clusterize into lighter fragments and heavier residues as the time goes on and saturate around $t \sim 150$ fm/c. It is only after 150 fm/c that MSTB is capable of identifying the properly bound $A_{\text{max}}$. The evolution of fragments’ binding energy suggests that fragment structure with MSTB approach is earlier decided. A time scale of 200 fm/c is enough to pin down the IMF yield since all fragments would be having binding energy $\leq -4.0$ AMeV.

It may mentioned that for asymptotic times, both MST and MSTB approaches yield same fragment multiplicities [23, 24]. The binding energy check i.e. Eq.(4.4) helps to a greater extent in identifying the fragments quite earlier. The normal MST method would, however, takes a long time to identify stable fragmentation pattern.

4.4 Beam energy dependence of fragmentation in central collisions

Here, we simulate the central heavy-ion collisions of $^{20}\text{Ne} + ^{20}\text{Ne}$ ($E_{\text{lab}}=10-55$ AMeV), $^{40}\text{Ar} + ^{45}\text{Sc}$ ($E_{\text{lab}}=35-115$ AMeV), $^{58}\text{Ni} + ^{58}\text{Ni}$ ($E_{\text{lab}}=35-95$ AMeV), $^{86}\text{Kr} + ^{90}\text{Nb}$ ($E_{\text{lab}}=35-95$ AMeV), $^{129}\text{Xe} + ^{124}\text{Sn}$ ($E_{\text{lab}}=45-130$ AMeV) and $^{197}\text{Au} + ^{197}\text{Au}$ ($E_{\text{lab}}=70-130$ AMeV). The systematic study over a wide range of beam energies and system masses allows one to confront the theoretical predictions with experimental findings and search for the
mass dependence. Note that only symmetric reactions are taken for present analysis. The choice of symmetric systems allows us to neglect asymmetry affects in the fragment production. Our calculations were performed at fixed impact parameter of \( b = 0 \) fm and employing a soft equation of state. We simulated the reactions at fixed incident energies and then calculated corresponding center-of-mass energies. For each such set, 500 events were simulated that minimizes the fluctuations to greater extent. The choice of central
Figure 4.2: The mean IMF multiplicity $\langle M_{\text{imf}} \rangle$ versus beam energy $E_{\text{c.m.}}$ for the reactions of $^{20}\text{Ne} + ^{20}\text{Ne}$, $^{40}\text{Ar} + ^{40}\text{Sc}$, and $^{58}\text{Ni} + ^{58}\text{Ni}$. Open circles depict the calculations employing QMD + MSTB approach for unfiltered events. The quadratic fits (solid curves) to the model calculations are drawn to estimate the peak energy at which the maximal IMF emission occurs.

Collisions for the present study guarantees the formation of highly excited systems that may break into a large number of pieces. Further, the emission from such events is almost isotropic, which may represent a 'single source' emission. In Fig. 4.2, we display the average multiplicity of intermediate mass fragments $\langle M_{\text{IMF}} \rangle$ calculated as a function of beam energy $E_{\text{c.m.}}$ available in the center-of-mass frame for $^{20}\text{Ne} + ^{20}\text{Ne}$, $^{40}\text{Ar} + ^{40}\text{Sc}$, and $^{58}\text{Ni} + ^{58}\text{Ni}$ reactions. Similarly, calculated IMF multiplicities for $^{86}\text{Kr} + ^{53}\text{Nb}$,
$^{129}\text{Xe} + ^{124}\text{Sn}$ and $^{197}\text{Au} + ^{197}\text{Au}$ reactions are displayed in Fig. 4.3. The mean IMF multiplicity first increases with beam energy, reaches a peak value and then decreases. This trend is visible in all the entrance channels as shown in Figs. 4.2 and 4.3. This trend is less clearly visible for lighter $^{40}\text{Ar} + ^{48}\text{Sc}$ system, whereas it is more clearly visible for the heavier systems. It is quite interesting to see that similar dependence of $\langle M_{IMF}\rangle$ on center-of-mass energy is also observed in experimental data taken with the MSU 4π-Array [9]. This behavior can be understood in terms of compression energy of the system. With the rise in beam energy, compression energy breaks the IMFs into smaller fragments.
leading to smaller number of IMFs. The maximal \( E_{\text{c.m.}} \) and corresponding peak \( \langle M_{\text{IMF}} \rangle \) was obtained through a quadratic fit to the model calculations. One should also note that the shape of the beam energy dependence of IMF production is quite close to one reported in the experimental data [9]. As reported by Sisan et al [9], the peak \( E_{\text{c.m.}} \) extracted for different entrance channels scales with the size of the system. Such scaling is also visible in our present calculations (see Figs. 4.2 & 4.3). In the next section, we report the fragmentation characteristics observed at peak \( E_{\text{c.m.}} \).

4.5 Fragmentation dynamics at the peak IMF emission and system size effects

We now study the fragment emission pattern at peak \( E_{\text{c.m.}} \), i.e. energy of maximal IMF emission. Figure 4.1 displays the time evolution of average nucleon density \( \rho^{\text{avg}} \), multiplicity of free nucleons emitted, light charged particles LCPs \([2 \leq A \leq 4]\), and intermediate mass fragments IMFs \([5 \leq A \leq 44]\) at the peak \( E_{\text{c.m.}} \). As expected, the average nucleonic density \( \rho^{\text{avg}} \) has a mass dependence, being maximal for the \( ^{197}\text{Au} + ^{197}\text{Au} \) system and minimal for the \( ^{20}\text{Ne} + ^{20}\text{Ne} \) system. This also indicates a linear density dependence on the system size. As discussed earlier, we observed an artificial emission of free nucleons in the beginning with MSTB approach which diminishes and saturates beyond 150 fm/c. One can see that final state multiplicities (at 200 fm/c) of different fragment species depict linear increase with size of the system. One can also notice that fragment emission almost saturates around 200 fm/c.

The maximal fragment production is for \( ^{197}\text{Au} + ^{197}\text{Au} \) system whereas \( ^{20}\text{Ne} + ^{20}\text{Ne} \) system results in minimum value. It may be mentioned that IMF multiplicities obtained in \( ^{20}\text{Ne} + ^{20}\text{Ne} \) and \( ^{40}\text{Ar} + ^{40}\text{Sc} \) collisions exclude the largest and second largest fragment respectively to infer the system size dependence accurately.

In Fig. 4.5, we further extend the above study for various fragments consisting of free nucleons, fragments with mass \( A=2 \), light charged particles LCPs \([2 \leq A \leq 4]\), medium mass fragments MMFs \([5 \leq A \leq 9]\) as well as heavy mass fragments HMFs \([10 \leq A \leq 44]\). Interestingly, in all the above cases, a clear system size dependence can be seen in a manner similar to that for \( \langle M_{\text{IMF}} \rangle \). We observe a power law of the form \( c A_{\text{tot}}^\tau \), where \( A_{\text{tot}} \) is the mass of composite system. In all the cases, exponent \( \tau \) is very close to unity. As noted in Ref. [9], the percolation model failed badly to reproduce the power law dependence.
Figure 4.4: The time evolution of average nucleon density \( \rho_{\text{avg}} \), multiplicity of free nucleons, light charged particles LCPs \( 2 < A < 4 \), intermediate mass fragments IMFs \( 5 < A < 44 \) for the systems indicated. Calculations shown here are done at the energy for peak IMF production.
Figure 4.5: The final state multiplicities of (a) free nucleons, (b) fragments with mass $A=2$, (c) light charged particles LCPs, (d) medium mass fragments MMFs, and (e) heavy mass fragments HMFs as a function of total mass of the system $A_{\text{tot}} (= A_T + A_P$; $A_T$ and $A_P$ being mass of the target and projectile, respectively). Model calculations done at peak $E_{c.m.}$ (open circles) are fitted with power law of the form $c A_{\text{tot}}^\tau$. 

55
A linear mass dependence observed with value of $\tau \sim 1$ depicts the picture of vanishing surface-Coulomb effects. Experiments are called for to verify this new prediction.

4.6 Confrontation with MSU 4$\pi$-Array data

Finally we confront our model calculations for the mass dependence of peak $E_{\text{c.m.}}$, as well as peak $\langle M_{\text{IMF}} \rangle$ with experimental data [9]. The measurements of mean IMF multiplicity in different symmetric reactions were carried out at the National Superconducting Cyclotron Laboratory (NSCL) at Michigan State University. Central events were selected to be 10% of all events with largest transverse energy. MSU 4$\pi$-Array detector is comprised of the main ball and the High Rate Array (HRA) with geometrical coverage of nearly 4$\pi$ solid angle. We have, therefore, compared our unfiltered calculations with the data. From Fig. 4.6, one can see that our model calculations employing MSTB approach (open circles) are in good agreement with the experimental data (solid squares) of MSU 4$\pi$-Array for peak $E_{\text{c.m.}}$. For peak $\langle M_{\text{IMF}} \rangle$, some deviation can be seen for heavier masses. This could also be due to fact that our calculations are not filtered for experimental acceptance. One can also see that the predictions of percolation model fail to explain the sharp dependence of peak $E_{\text{c.m.}}$. These observations suggest that peak $E_{\text{c.m.}}$, thus, acts as a measure of finite size effect. It is worth mentioning that the critical excitation energy was estimated from the cluster size distribution fitted to power law: $\sigma(A) \propto A^{-\alpha}$ at different beam energies for which the exponent $\alpha$ reaches a minimum. Based on the percolation calculations, the critical excitation energy is also found to increase with initial lattice size [8]. Interestingly, the mass scaling of peak $\langle M_{\text{IMF}} \rangle$ can be reproduced using a power law: $cA_n^{\alpha/2}$ with exponent $\tau$ close to unity.

4.7 Summary

We present in this chapter, the quantum molecular dynamics description of beam energy dependence of fragment production. Reactions were simulated selecting a wide range of symmetric systems at zero impact parameter. To obtain the final state fragment multiplicity, we employ minimum spanning tree method with additional binding energy check to recognize the final fragment pattern. This approach ensures that fragments obtained at the end of reaction are properly bound and stable. Our calculations remarkably
reproduce the trend of rise and fall in IMF multiplicity with beam energy $E_{c.m.}$ in the center-of-mass frame as observed experimentally. Peak energy $E_{c.m.}$ is found to scale linearly with system size. Average nucleon density as expected also showed systematic mass dependence. Mean multiplicities of IMFs as well as other fragment species are found to follow the power law $cA_{tot}^\tau$; $A_{tot}$ being the total mass of the system. Interestingly, exponent $\tau$ is close to unity in all cases, indicating vanishing of surface-Coulomb effects. Experiments are called to verify this new prediction.
Bibliography


