Chapter 2

Observation and Analysis Techniques

“MAGNITUDE, n. Size. Magnitude being purely relative, nothing is large and nothing
small. If everything in the universe were increased in bulk one thousand diameters nothing
would be any larger than it was before, but if one thing remains unchanged all the others
would be larger than they had been. To an understanding familiar with the relativity of
magnitude and distance the spaces and masses of the astronomer would be no more
impressive than those of the microscopist. For anything we know to the contrary, the
visible universe may be a small part of an atom, with its component ions, floating in the
life-fluid (luminiferous ether) of some animal. Possibly the wee creatures peopling the
corpuscles of our own blood are overcome with the proper emotion when contemplating
the unthinkable distance from one of these to another.”

Ambrose Bierce
2.1 Photometric Observations

The rapidly increasing sophistication in the optics and electronics has made small telescope very efficient and intelligent. With state of art back end instruments like photoelectric photometers interfaced with computer and small format CCD camera, the conventional photometric monitoring has become a highly potential tool for the study of variable stars. When working with on sufficiently bright stars, small telescopes are capable of the same photometric accuracy as large telescopes. There exist a large number of bright variable stars that need continuous, systematic observation over a long time span to determine their short-term and long-term light variation and unusual stellar activity, and for this kind of work small telescope is a natural choice. The main objective of thesis is to collect good quality of photometric data using small telescopes and analyze them to obtain a better understanding of the photometric light variation of chromospherically active stars.

All the photometric data analyzed in this thesis were obtained using small telescopes; 12” Meade LX 200 Schmidt- Cassegrain reflector telescope mounted at J.E.S. College, Jalna, India. Optical tube assembly with EMC super multi-coating (D=305 mm, F = 3048mm – F/10); heavy duty fork mount, with 4” dia. Sealed polar ball bearing, quartz- microprocessor controlled 5.75” worm gear on both axes and multi function power panel display on drive base; manual and electric slow-motion controls on both axes, setting circle in RA and Dec. On the other hand, Meade LX 200 telescope can be controlled either by keypad or through computer.

J.E.S. Meade telescope is equipped with SSP-3A Gen 2 (OPTEC Inc. USA) Photoelectric Photometer and Standard Johnson BVRI filters. The detector used in the SSP-3A is a model S1087-01 manufactured by Hamamatsu Corporation, (Japan) made silicon
PN-Photodiode. It has silicon PN-Photodiode which allows detection from the UV to the near infrared with a single detector. Light enters the photometer through the 1.25-inch telescope adapter and is directed either to the focusing eyepiece or the detector by means of a flip-mirror. The focusing eyepiece consists of a 1 inch focal length Ramsden and a reticle with a precisely scribed ring that defines the detector field of view. A green LED illuminates the reticle from the side. After a star is centered in the ring, the flip mirror is rotated to expose the detector. A six positional filter slider is mounted between the flip & the detector. The user through software can select any filter. The response of the B, V, R & I filters with the detector closely matches the Johnson standard B, V, R & I response function. From the electrometer amplifier, the signal is then routed to the voltage to frequency converter for final processing into counts based on a 1, 10 or 100-second gain time interval. The resulting count is then read from the four–digit readout on the front panel or from an external counter/ computer connected to the pulse output connector.

**Chris Marriott’s SkyMap Pro 7:**

SkyMap Pro 7 is a powerful and flexible astronomical companion of the Computer, giving easy access to information which was only available to a select group of professional astronomers. SkyMap combines sophisticated planetarium and map drawing capabilities with the ability to easily display detailed information on many millions of different objects - stars, planets, galaxies, nebulae and so on.

At its most basic level SkyMap can act as a simple planetarium, showing the appearance of the sky as seen from anywhere on Earth, for any date between 4000 BC and 8000 AD. With a click of the mouse, you can select exactly which features to display-constellation lines, names and boundaries, planets, comets, ‘deep sky objects’, various
scale lines and coordinate grids & much more of rather more practical use to modern astronomers is sky map’s ability, to act as a sophisticated mapping test for more flexible and containing hugely more information than the best printed star atlas. You can zoom into a practically unlimited extent, displaying stars as faint as magnitude 16, and deep sky objects from dozens of different catalogs. Sophisticated “filtering” capabilities allow you to select exactly which object to display. Once you have map just the way you want it, it can be printed as a high quality chart an only printer supported by windows. Full support is provided for printing in both black and white and in color.

SkyMap’s real power comes from the fact that a vast wealth of information lies behind the symbols you see on the map. Click on any object on a map and sky map can display all the information available for that object from many different catalogs. The information can be copied to the windows clipboard, from where it can easily be incorporated into a word processor, or other windows application. The quantity of information available is huge for a bright star.

SkyMap offers the powerful search facilities to make that task very simple. The search engine allows you to either draw a map showing the location of an object or simply to display all the information available about the object. Although sky map is supplied with a large number of different data catalogs, you are not limited to those supplied with a program. An easy to use data preparation tool makes it easy to add virtually any data catalog to sky map and once added. Those “external” catalogs are fully integrated into the program, allowing the objects they contain to be displayed on a map and searched, exactly as can be done with the “built in” catalogs. This is extremely
powerful facility for astronomers with special interests who wish to use some of the more “obscure” data catalogs which are available.

Another facility offered by sky map is the ability to associate images with objects in the sky map database. Click on any objects on a map and sky map will search for a picture of that objects, giving the option to display it if found. Sky map can be used with almost any picture viewing program, and can support any image format which that viewer can handle. Sky map is supplied with a large collection of astronomical pictures, generously contributed by amateur astronomers and it’s very easy to add your even picture too. Sky map isn’t supplied with a picture viewer-you can either use the simple viewers supplied with Windows.
Fig 2.1 12” Meade LX 200 Schmidt-Cassegrain reflector telescope at J. E. S. College, Jalna
Fig. 2.2 SSP-3A Solid-State Photometer
Figure 2.3: The SSP-3A Gen 2 Photometer System and Accessories
2.2 Atmospheric Extinction

The light from stars and other celestial objects are dimmed and reddened significantly by absorption and scattering while passing through earth’s atmosphere. The amount of loss of star light is characterized by extinction and it depends on:

1. Height of the star above the horizon
2. Altitude of the Observing place
3. The wavelength of incoming light, and
4. The current atmospheric and meteorological condition

For a given night the loss of star light mainly depends on the path length (airmass) in the earth’s atmosphere through which the light travels. Extinction is smallest for stars near zenith and greatest for the stars near the horizon. For a precise photometric observations, extinction correction to measure or instrumental magnitude is necessary.

For this purpose following equations can be used (Henden & Kaitchuck 1982):

\[ m_{\lambda,o} = m_\lambda - k'\lambda X \]  

Where \( k'\lambda \) is called first order extinction coefficient for a given wavelength\( \lambda \), \( X \) is airmass, \( m_\lambda \) is measured or instrumental magnitude, \( m_{\lambda,o} \) is extinction corrected magnitude measured above earth’s atmosphere.
**Single/Two star photometry:** In order to obtain atmospheric extinction we observed two standard stars having different color and RA through a large range of air mass.

Following equations were used to obtain extinction coefficient:

\[
\begin{align*}
    v_o &= v - k'X \quad (2) \\
    (b - v)_o &= (b - v) - k'_{bv}X \quad (3) \\
    (v - r)_o &= (v - r) - k'_{vr}X \quad (4) \\
    (v - i)_o &= (v - i) - k'_{vi}X \quad (5)
\end{align*}
\]

Where \( b, v, r \) and \( i \) are the instrumental magnitudes and \( b_o, v_o, r_o \) and \( i_o \) are the instrumental magnitudes corrected for extinction and \( k'_{v}, k'_{bv}, k'_{vr} \) and \( k'_{vi} \) are the principal extinction coefficients.

**All sky Photometry:** Several standard stars at various airmass were observed within a short time to measure the atmospheric extinction. Equation 3 to Equation 5 were used to obtain the extinction coefficients for each pass band and color.
Table 2.1 Observed Average Extinction Coefficients for J.E.S. Jalna, Raipur, Pune and Guwahati

<table>
<thead>
<tr>
<th>Observatory</th>
<th>Years</th>
<th>$K_v$</th>
<th>$K_{bv}$</th>
<th>$K_{vr}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) J. E. S. Jalna</td>
<td>2006-07</td>
<td>0.2519 ± 0.066</td>
<td>0.0785 ± 0.099</td>
<td>0.1463 ± 0.0244</td>
</tr>
<tr>
<td>2) Raipur</td>
<td>2000-01</td>
<td>0.434 ± 0.142</td>
<td>0.214 ± 0.067</td>
<td>0.100 ± 0.0068</td>
</tr>
<tr>
<td>3) IUCAA, Pune</td>
<td>1998-99</td>
<td>0.440 ± 0.180</td>
<td>0.180 ± 0.05</td>
<td>0.110 ± 0.0500</td>
</tr>
<tr>
<td>4) GUO, Guwahati</td>
<td>1999-2000</td>
<td>0.433 ± 0.142</td>
<td>0.611 ± 0.063</td>
<td>0.294 ± 0.0340</td>
</tr>
</tbody>
</table>

Note: a – Sudhanshu Barway, 2005, Ph. D. thesis  
     b – Padmakar, 2000, Ph. D. thesis  
     c – Biman Jyoti Medhi 2004, Ph. D. thesis
2.3 Calibration of the photometric system

2.3.1 Transformation to the BVRI System

The BVRI system is the most commonly used broadband photometric system device by Johnson in 1950 based on a set of glass filters and a 1P21 photomultiplier tube (Johnson & Morgan 1951; Johnson 1955). The bandpass of the BVRI system are tabulated in Bessel (1990) and are illustrated graphically in Fig 2.4. The largest changes in effective wavelength of a stellar object occur for B and, because large difference in spectral energy distribution and for R band because the long red tail in the R band response (see table).

Table 2.2: Effective wavelengths in different bands & spectral class

<table>
<thead>
<tr>
<th>Filter</th>
<th>B</th>
<th>V</th>
<th>R</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>A0V</td>
<td>4363</td>
<td>5448</td>
<td>6407</td>
<td>7982</td>
</tr>
<tr>
<td>K0III</td>
<td>4529</td>
<td>5524</td>
<td>6535</td>
<td>8028</td>
</tr>
</tbody>
</table>

![Transmission curves for Johnson UBVRI filters](attachment:image.png)

Fig 2.4: Transmission curves for Johnson UBVRI filters
2.3.2 Transformation Coefficient

A standard photometric system is defined by a list of stars called standard stars, each having a magnitude (or a number of magnitudes, if it is a multifilter photometric system) assigned to it. Although a photometric system is usually set up by means of observations made at one observatory site with a specific telescope, photomultiplier, filters etc, it is not the equipment which defines the standard system; rather it is the published magnitudes, thus called standard magnitudes, assigned to the certain stars, thus called standard stars. Today there are over a dozen photometric system in use for astronomical photometry, however, most widely used photometric system is the Johnson & Morgan (1953) broad UBV ($\delta\lambda \approx 700 - 900 \text{ Å}$) photometric system. Most astronomers working with small telescopes prefer UBV photometric system because the broad bandpass of the filters allow more photons to reach the detector. Johnson UBV photometric system has been extended to red R ($\lambda_{\text{eff}} \approx 7000 \text{ Å}$) and near infrared I ($\lambda_{\text{eff}} \approx 9000 \text{ Å}$) bandpass to increase spectral sensitivity, and these taken together constitute the standard UBVRI photometric system. Most valuable aspect of the standard photometric system, especially for variable star photometry, is that observations of the same star taken by different observers with different photometric equipments can be compared and, later can be combined on the same light curve to reveal the nature of the variability in greater detail or to study the long-term variability of the star.

Magnitudes determined by any observer with a given telescope and filter-detector (Photometric equipment) system are called instrumental magnitude. For the universal use of astronomical data each instrumental magnitude and thus color requires to be transformed to a standard system.
We used following transformation equations to transform instrumental magnitudes to Johnson’s BVRI system:

\[ V - v_0 = \alpha_v + \beta_v (B - V), \]  
\[ B - V = \alpha_{bv} + \beta_{bv} (b_0 - v_0), \]  
\[ V - R = \alpha_{vr} + \beta_{vr} (v_0 - r_0), \]  
\[ V - I = \alpha_{vi} + \beta_{vi} (v_0 - i_0) \]

Where B, V, R and I are standard magnitudes, b_0, v_0, r_0 and i_0 are extinction corrected instrumental magnitudes, \( \alpha_v, \alpha_{bv}, \alpha_{vr} \) and \( \alpha_{vi} \) are the zero points and \( \beta_v, \beta_{bv}, \beta_{vr} \) and \( \beta_{vi} \) give the color transformation coefficients in bands B, V, R and I, respectively. We observed a number of UBVRI standard stars having wide range of colors in \( (B-V) \) and \( (V-R) \) to obtain transformation coefficients and zero points using linear least square fitting. Resulting values of transformation coefficients and zero points are given in Table 2.3. The transformation coefficients of J. E. S. observatory system are to be satisfactory. Mayya et al. 1991 and Burki et al. 1995 suggest that large variations in zero points can arise due to:

(I) Observed night might not have been photometric,

(II) Inaccurate measurements of the extinction coefficients,

(III) Very high dust deposition rate Schmidt corrector plate or

(IV) Deterioration of the telescope-filter-detector system.

The stability of the zero points and transformation coefficients on a given night provide a good measure of the transformations. To check this on each photometric night when observations for transformation coefficients were performed, 10 to 15 other standard stars were also observed. Instrumental magnitudes and colors of these stars were transformed
into the standard system using the derived transformation coefficients and compared with their standard values.

**Table 2.3: Transformation coefficient of J.E.S. Observational System**

<table>
<thead>
<tr>
<th>Observation Night</th>
<th>1st Transformation Coefficient ε_v</th>
<th>Zero Point of the System for V Filter ζ_v</th>
<th>2nd Transformation Coefficient μ_{bv}</th>
<th>Zero Point of the System for B,V Filter ζ_{bv}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nov 16, 2006</td>
<td>-0.0460</td>
<td>7.723</td>
<td>1.3374</td>
<td>-0.0988</td>
</tr>
<tr>
<td></td>
<td>± 0.015</td>
<td>± 0.028</td>
<td>± 0.009</td>
<td>± 0.258</td>
</tr>
</tbody>
</table>
2.4 Differential Photometry

Differential photometry involves determining the magnitude difference between two stars: the chosen variable star and the standard star taken as the comparison star of same color very close to each other (preferably within 1 degree) in the sky. Differential photometry is the simplest and the most accurate technique and highly recommended for carrying out optical photometry of variable stars. This technique has capability to measure very small variations in brightness of the star even under very adverse sky conditions. In order to minimize errors introduced by inaccurate extinction measurement and rapid variation in the transparency of the local sky, comparison star should be as near as possible to the variable star, preferably within a degree. Apart from errors introduced by atmospheric conditions, differential photometry can overcome typical instrumental errors, for example arising from mismatch of the filter-detector combination with standard system, variation in instrumental gain and zero points etc.

We followed the observation sequence for differential photometry during all observation runs as SkyNext, Com, V, V, S, Com SkyLast where Com, V, and S represents Comparison, Variable star respectively. Especially for large airmass, effect of the inaccurate extinction measurements on differential photometry is usually large, and therefore to achieve higher accuracy we planned our observation schedule in such way that each variable star along with its comparison was observed quite close to zenith.

Stars having period of more than a day were observed four to five times on each observation nights, and corrected for the atmospheric extinction using formula given below

\[
\Delta v_v = \Delta v - k_v \Delta X \\
\Delta (b - v)_o = \Delta (b - v) - A k_{bv} \Delta X
\]

(10)  (11)
\[\Delta (\nu - r)_o = \Delta (\nu - r) - \Delta k_{\nu r} \Delta X \quad (12)\]

For the nights on which extinction were not available, we used interpolated value of the extinction from extinction data base for the observing season. To obtain standard differential magnitude and color in the standard photometric Johnson system following relations were used

\[\Delta V = \Delta v_o + \varepsilon \Delta (B - V) \quad (13)\]

\[\Delta (B - V) = \mu_{\nu r} \Delta (b - \nu)_o \quad (14)\]

\[\Delta (V - R) = \mu_{\nu r} \Delta (v - r)_o \quad (15)\]

\[\Delta (V - I) = \mu_{\nu i} \Delta (v - i)_o \quad (16)\]

After averaging and excluding data above 3\sigma level of the average value, one data left for each night for every program star. Typical uncertainty in differential magnitude \(\Delta V\), and differential colors \(\Delta (B-V)\), \(\Delta (V-R)\) and \(\Delta (V-I)\) are found to be 0.01, 0.012, 0.015 and 0.013, respectively. These standard differential magnitudes can be plotted against Julian days (JD) to study the light variation of variable stars. If variation of light is periodic for a given star then nature of each cycle is more or less identical. Progress through each cycle is described by phase; a number which is directly proportional to time and goes from 0 to 1.

For computation of phase following relation is used:

\[\text{Fractional part of Phase} = \frac{JD - t_o}{P}\]

Or \[\text{Phase} = \frac{JD - \text{Epoch}}{\text{Period}}\]

Where \(JD\) is the Julian Date of observation, \(t_o\) is \(JD\) at minimum light and \(P\) is Period of star. Fractional part of phase against differential magnitude is used to obtain light curve for a given Variable star.
2.5 Calibration of System for testing nonvariability of Standard Star

In differential photometry, a second star of nearly the same color and brightness as the variable star is used as a comparison star. This star should be as near to the variable as possible, preferably within one degree. This allows the observer to switch rapidly between the two stars. All changes in the variable star are determined as magnitude differences between it and the comparison star. It is important that the comparison star be measured frequently because the altitude of these objects is continuously changing throughout the night. This type of photometry can be extremely accurate (0.005 magnitude) and is highly recommended where atmospheric conditions can be quite variable. Any star that meets the criteria can be comparison star. However, it is a good idea to pick a second star, called check star, as a test of the nonvariability of the comparison star. Differential measurements between the comparison star and check star should remain constant to within the nightly errors, usually \(0^m.02\) or less with good skies. If the variations larger than this consistently occur, either the comparison or the check star is variable, a fairly frequent occurrence.

In order to obtain accurate differential photometry, we used two nearby stars HD 52071 (K2 III, \(V=7.11, B-V = 1.27\)) as comparison star and 37 Gem/HD 50692 (G0 V, \(V = 5.75, B-V= 0.597\)) as a check star. No significant light variation was detected for the magnitudes of the comparison star and check star, which is a good measure of the quality of our observation. The nature of graph is shown in Fig.
Fig. 2.5 Light curve of HD 50692 (Julian date v/s V band)

Fig. 2.6 Light curve of HD 50692 (V Band v/s B-V filter)
2.6 Software for Data Acquisition and Reduction

To make data acquisition and reduction simple and convenient for SSP 3A Gen 2 photometer was interfaced with computer through SSP interfacing card provided by OPTEC Inc. For data acquisition BV photometry.exe is any integral part of SSPDATAQ.exe and is used with SSP-3 photometer to reduce raw data to standard V magnitude and B-V index. The BV photometry program is written in Liberty Basic version 4.0. For acquiring data the user to change integration time (0.02 to 10 sec) to select any number of filter (1 to 6) as well as to set observing sequence (C_b C_v V_b V_v C_b C_v V_b V_v C_b C_v where C_b is comparison star in blue filter, C_v is Comparison star in V filter, V_b is variable star in blue filter and V_v is Variable star in V filter) in case of differential photometry. The various files that make up BV photometry.exe are installed with SSPDATAQ.exe. In addition to the files listed for SSPDATAQ; the following apply to BV photometry.

* Data editor tkn program file to enter and edit position and magnitude values for the variable comparison and transformation stars.

* Extinction. tkn program file to reduce a raw data file containing comparison star data to derive the extinction coefficient K’_v and K’_{bv}.

* Transformation.tkn program file to reduce a raw file containing transformation star data to derive the transformation coefficient epsilon and mu.

* Reduction.tkn program file to reduce a raw file containing comparison and variable star data to derive standard V magnitude and standard B-V color index.

*Ppparms.txt text file that contains observer location, extinction and transformation coefficient.
* Star Data.txt text file contains position that and magnitude information for comparison and variable stars.

* Transformation.txt text file contains position that and magnitude information for transformation stars.

* Sample Variable Stars.raw sample data file to demonstrate use of the Extinction and reduction modules.

* Sample Transformation Stars.raw sample raw data file to demonstrate use of the transformation module.

The final step in the reduction process is running the Data Reduction program module. If the proper differential photometry procedure has been used in the observation and the coefficient have been correctly obtained, the reduction to finished V magnitude and B-V color is reduced to the ultimate simplicity.
2.7 An Overview of the Photometric Spot Modeling Techniques

A study of surface inhomogeneities is one of the rapidly expanding areas of stellar physics and it is supposed to play a key role in solar-stellar connection. It is well accepted that photometric variations seen in chromospherically active stars are a manifestation of the modulation of the stellar light due to presence of cool dark spots on its surface. Over past three decades, observational astronomy is flooded with photometric observation of these stars. A major breakthrough occurred during mid-eighties after the advent of Automatic Photoelectric Telescope (APT) in the field of photometric light curves using geometrical starspot models and extracts as much physical information of starspots as it can provide. And the accumulated information would be very useful in exploring the issue of origin and evolution of the solar-stellar activities. Modeling of multi-bandpass light curves of spotted star started since 1973, when Bopp & Evans (1973) applied a very simplified two spot model on the distorted light curve of BY Dra star. Thereafter, with an objective to analyze quantitatively unusual light variation of spotted star more than two dozen spot models have been developed so far. Most of the starspot models have chosen single/multiple discrete starspots having circular or rectangular spots (Budding 1977; Bopp & Noah 1980; Vogt 1981; Poe & Eaton 1985; doreen 1987; Kjurkchieva 1987; Strassmeier 1988; Kang & Wilson 1989; Mohin & Raveendran 1992; Ekar 1994; Olah et al. 1997 and references therein), while a few of them adopted entirely different technique to treat light curves of spotted stars (Eaton & Hall 1979; Eaton et al. 1996; Lanza et al. 1998; Crews et al. 1988 and references therein) to investigate short and long term variation in the light curves of these active stars. With assumption that spots are circular in shape, Budding (1977), developed a spot model, which enabled him to solve the integration analytically to compute
net flux coming from the projected star. This well formulated spot model was widely accepted by different contemporary star spot researcher and currently in use too (Vivekannanda Rao et al. 1991, Budding et al. 1997; Kovari and Bartus 1997; Olah et al. 1997 etc.). Subsequently after Budding noteworthy model a very different spot modeling scheme was introduced by Eaton & Hall (1979) in which they assumed a continuous distribution of small spots in two symmetric bands with respect to the equator. The spot distribution within each band was assumed to be represented by sine function and such rotating spotted star was supposed to be responsible for sinusoidal shape of the observed light curves.

The uniqueness of spot parameters has been questioned by several researchers. To this end Steve Vogt suggested to decouple geometrical effects of spots from the temperature effect on the light curve and was able to determine starspots temperature and area uniquely from the standard V and R light curves. This method, which is based on certain reasonable assumptions and approximations that size of spots is relatively small and they reside near the disks center, restricts its universal use. Later on a very efficient program for obtaining reliable spot parameters known as Information Limit Spot Optimization Technique (ILOT) was developed by Budding & Zeilik (1987). ILOT is essentially based on $\chi^2$ minimatization technique and uses theoretical maculation wave fitting function of Budding (1997), Dorren (1987) further extended analytical solution of spot model by applying direct surface integration over the spots. Although shape of spots in his model is assumed to be circular, it can incorporate the possibility of spots with concentric circular umbral and penumbral regions, analogous to sunspots. A fairly general starspots model, which can treat arbitrary spot shape comprising
umbral-penumbral morphological structure, was developed by Stassmeier (1988). This model is based on numerical integration and allows users to model BVRI light & color curves as well as deformation in the line profile simultaneously, so as to obtain a unique solution. Because of the fact that majority of stars are found to be a member of binary system, a well known binary star light curve modeling program called Wilson-Deviney (WD), incorporated starspots model (Kang & Wilson 1989) in their code. With the help of this model apart from light curve produced by rotation modulation, proximity effects in binary system such as gravity darkening, reflection, tidal and eclipse effects can be simultaneously handled. In order to account for the short-term variation due to rapid evolution in starspots a time dependent formulation of the spot model (TISMO) has been introduced by Strassmeier & Bopp (1992) and later on used by others (Strassmeier, Hall & Henry 1994; Olah et al. 1997). Another spot modeling technique based on numerical integration but in essence not very different from older existing spot model was introduced by Mohin & Raveendran (1992). It employs the method of least square different correction technique to achieve unbiased reliable best fit spot parameters, using multi-bandpass (UBVRI) light curves simultaneously. Eker (1994) introduced a new formulation of the starspot model are equivalent to other existing models, but possesses much greater transparency and is easier to use. Impressed by dominant localization of sunspots on solar surface and also realizing the shortcoming and inadequacies of hierarchical two or multiple spot models, Alekseev and Gershberg (1996) proposed a model of zonal spottedness. This model parameterized observed light curve by three independent geometrical parameters, the latitude of the spotted band, the extent of the band in latitude and the parameter determining the degree of non-uniformity of the bands in longitude.
A new approach, considering multiple random spots, has recently been introduced by Eaton et al (1996) to solve the erratic behavior of light variation in active stars. Even though this model can explain rotation modulation and long term activity very well, random spot model was promptly rejected on theoretical and observational ground by Olah et al (1997). It has already been cautioned that surface mapping of the spotted star using photometry alone is an ill-posed problem. Nevertheless, few attempts have been made to reconstruct map of stellar surface using multi-passband photometric observations. Inverting photometric data just by minimizing $\chi^2$ is usually highly unstable and photometric observational error (noise) introduces artifact. In order to avoid degradation of mapping and to obtain stable unique solution some regularization criteria, for example Maximum Entropy, Thekhonov method etc, are also used. Lanza et al. (1998) has introduced this kind of mapping scheme for spotted stars. Recently, another effort was made by crews et al. (1998) for mapping the surface intensity structure of spotted stars by Matrix Light Curve Inversion (MLI) technique, and they were able to reproduce spot latitudes and their distribution (in case multi spots) accurately on stellar surface using multi-pass band photometric observation alone.

Although starspot-modeling technique has evolved and gained considerable maturity in its approach since 1973 when Bopp and Evans proposed first spot model, yet the spot models are still in their infancy in terms of finer details, uniqueness (Wilson 1994; Eker 1996)
2.8 Starspot Modeling

2.8.1 The Dorren’s Formulation

To evaluate starspot parameters of the spotted stars from observed light curves we have developed a computer program based on the analytical formulation of the starspots model given by Dorren (1987). Details of the model and its mathematical derivation can be found in Dorren’s (1987) paper. For the sake of completeness, however, a brief description of this formulation and method used to extract best fit spot parameters (inverse problem), is described here.

Suppose a star having huge cool spot on its surface is rotating about its rotation axis, then at any time (or photometric phase \( \hat{t} \)) its light variation in terms of magnitude is given as

\[
\Delta m = -2.5 \log \left( \frac{l - l_s}{l} \right)
\]  

(1)

Where \( l \) and \( l_s \) give total brightness of the unspotted and spotted star. These two quantities are defined as

\[
l = \int_{\text{star}} F \cos \gamma \sin \theta \, d\theta \, d\phi
\]  

(2)

\[
l_s = \int_{\text{spot}} \left( F - F_s \right) \cos \gamma \sin \theta \, d\theta \, d\phi
\]  

(3)

Where angle \( \theta \) and \( \phi \) are spherical polar coordinates and \( \gamma \) is angle between the surface normal to the area element and the line of sight. \( F \) and \( F_s \) are the monochromatic fluxes at any point \((\theta, \phi)\) on stellar and spot surfaces. If we assume that both stellar and spot flux distribution obeys linear limb darkening law then anywhere on the star or spot, flux can be computed as
Figure 2.7: Schematic diagram of the partially visible starspots, where O and C are the center of the visible and spot respectively. OC = \beta is angular distance of the spot center c from visible disk center O and AC = a is spot radius (Courtesy: Padmakar 2000)

\[ F_\ast = F_\ast (1 - \mu_\ast + \mu_\ast \cos \gamma) \]  
\[ F_S = F_S (1 - \mu_s + \mu_s \cos \gamma) \]

Where \( F_\ast \) and \( F_S \) are fluxes at the center of star and spot respectively. With the assumption that star and spots are black body emitters, \( F_\ast \) and \( F_S \) can be computed using Plank’s formula. Using linear limb darkening law for the star (Equation 4), total brightness of the star \( I^{\odot} \) (Equation 2) is easily evaluated to give

\[ I = \pi \left(1 - \frac{\mu}{3}\right) F . \]  

\[ \]
Equation (1) then becomes

$$\Delta m = -2.5 \log \left\{ 1 - \frac{\iint [F - F'] \cos \gamma \sin \theta d\theta d\phi}{\pi \left( 1 - \frac{\mu'}{3} \right) F'} \right\}$$  \hspace{1cm} (7)

The quantity within brackets on the right hand side of Equation (7) represents the fractional loss of light of the star due to presence of the spot. Further, we can write Equation as

$$\Delta m = -2.5 \log \left\{ 1 - \frac{aA + bB}{\pi \left( 1 - \frac{\mu'}{3} \right)} \right\}$$  \hspace{1cm} (8)

with coefficients

$$a = \left( 1 - \mu' \right) - (1 - \mu) \frac{F'}{F}.$$  

$$b = (\mu' - \mu) \frac{F'}{F}.$$  

and integrals

$$A(\alpha, \beta) = \int_{\text{spot}} \cos \gamma \sin \theta \, d\theta \, d\phi$$  

$$B(\alpha, \beta) = \int_{\text{spot}} \cos^2 \gamma \sin \theta \, d\theta \, d\phi$$  \hspace{1cm} (9)

Here it is important to note that both these two integrals are explicit functions of two independent parameters $\alpha$, the angular radius of the spot, and $\beta$, the angular distance of the spot from center of visible hemisphere of the star. The angular distance of the spot $\beta$, can be easily computed using the relation

$$\cos \beta = \cos \psi \sin \chi + \sin \psi \cos \chi \cos (\psi - \chi)$$  \hspace{1cm} (10)
where $\chi$ and $\psi$ give latitude and longitude of the spot’s center, respectively $i$ is the stellar inclination and $x$, is rotation phase of the star (see Figure). The spot longitude $\psi$ is usually measured from zero phase of the stellar rotation.

With the assumption that spots or spot groups are circular in shape, Dorren (1987) evaluated the two integrals $A(\alpha, \beta)$ and $B(\alpha, \beta)$ analytically by direct surface integration over the spots. The analytical solution for these two integrals is

$$A(\alpha, \beta) = \zeta + (\pi - \delta) \cos \beta \sin^2 \alpha - \sin \zeta \sin \beta \cos \alpha$$  \hspace{1cm} (11)

$$B(\alpha, \beta) = \frac{1}{3}(\pi - \delta)(-2 \cos 3 \alpha - 3 \sin 2 \beta \cos \alpha \sin 2 \alpha) + \frac{2}{3}(\pi - T) + \frac{1}{6} \sin 2 \beta (2 - 3 \cos 2 \alpha)$$  \hspace{1cm} (12)

Where $T = \left( \arctan(\sin \zeta \tan \beta) \right)$, $\beta \leq \frac{\pi}{2}$

$$T = \left( \pi - \arctan(-\sin \zeta \tan \beta) \right), \beta \geq \frac{\pi}{2}$$

Here auxiliary angles $\delta$ and $\rho$ are function of $\alpha$ and $\beta$ only. $2\rho$ is angular separation of the two intersection points of the spot and visible disk circumference at A and B, and $2\delta$ is angle subtended by two intersection points A and B at the spot center C (Figure 2.7).

These two angles are calculated from the following relations

$$\cos \delta = \cot \alpha \cot \beta$$

$$\sin \zeta = \sin \alpha \sin \beta = (\sin^2 \beta - \cos^2 \alpha)^{1/2} / \sin \beta$$  \hspace{1cm} (13)

$$\cos \zeta = \cos \alpha \csc$$

Now the synthetic light curve of the any rotating spotted star can be easily generated by evaluating $A(\alpha, \beta)$ and $B(\alpha, \beta)$ using equations (11) and (12), and putting it back in to the Equation (8). This formulation can be also generalized for more than one circular spots
having radii \( \alpha_i \) and location \( (\chi_i, \psi_i) \), whose angular distances from stellar disk center are \( \beta_i \).

For this case one has

\[
A = \sum_{i=1}^{n} A_i(\alpha_i, \beta_i)
\]

\[
B = \sum_{i=1}^{n} B_i(\alpha_i, \beta_i)
\]

In case of multiple starspots, overlapping of spots can be avoided by putting a criterion

\[
S_{ij} = \alpha_i + \beta_j
\]

Where \( S_{ij} \) is angular separation between the centers of \( i^{th} \) and \( j^{th} \) spots and is given by

\[
\cos s_{ij} = \sin \chi_i \sin \chi_j + \cos \chi_i \cos \chi_j \cos (\psi_j - \psi_i)
\]
2.8.2 Validity of two or three spot model

During last two decades photometric light curve of the spotted stars were successfully fitted by two spot models (Budding 1977; Bopp & Noah 1980; Dorren; Poe & Eaton 1985; Kang & Wilson Zeilik et al. 1990; Mohin & Raveendran 1992; Olah et al. 1997; Padmakar & Pandey 1996; Mekkaden & Raveendran 1998 and references therein). In most cases the two spot model explains the light variation of chromospherically active stars very well. However, there were also few instances when the model could not give satisfactory fit and hence more than two spots were required to achieve better results (Strassmeier 1988; Strassmeier & Bopp 1992; Eaton et al. 1994; 1996; Eaker 1996). Furthermore Doppler images of starspots also surface brightness than expected from two or three spots only (Strassmeier 1991; Vogt & Hatzes 1995; Hatzes 1995, Weber 7 Strassmeier 1998; Berdyugina et al. 1998; Vogt et al. 1999 and references therein). Similarly although starspots hypothesis was fundamentally borrowed from sunspots and distributions of sunspots are not so much localized i.e. the sunspot distribution is rather restricted with respect to latitude but somewhat uniform along longitude. These are few reasons which compel one to raise suspicion on the reliability and validity of the simple two or three spots model (Eaton et al. 1996). If we assume that some spot parameter (spot temperature, limb-darkening coefficients etc) and stellar system parameters (inclination, star effective temperature, luminosity ratio, unspotted light level etc ) are known quantities only spot would mean increasing the number of free parameter by three and consequently can deteriorate reliability of retrieved spots parameters.

Although in principle a maximum of five spots can be used to fit any unusual photometric light curve having typical error 0.01 mag and amplitude 0.1 mag (Budding 1996), but its
reliability could always be questioned. Therefore, it would be wise to use two or three spot model on which the two spots can be anywhere on stellar surface and allowing one to fit explicit light modulation which a polar spot is used to model the variation in mean light level which may arise either due to a single polar spot or uniformly distributed spot.

Figure 2.8: Test of validity of three–spot model.