Chapter 3

Propagation of plane elastic waves at a plane interface between two electro-microelastic solid half-spaces

3.1 Introduction

The study of transmission of elastic waves through an interface between two semi-infinite elastic solid half-spaces in perfect contact has been the subject of continued interest since long. Tomar and Gogna (1995a, 95b) investigated the reflection and refraction of a longitudinal microrotational wave and a longitudinal displacement wave impinging at a plane interface between two distinct micropolar elastic solid half-spaces. They obtained the amplitude ratios of reflected and transmitted waves and studied their variation with respect to angle of incidence and frequency of the incident wave for a specific model. Later, Tomar and Garg (2005) explored the possibility of propagation of plane waves in an infinite microstretch elastic solid medium and investigated the reflection phenomena of plane waves at a plane interface between two different microstretch elastic solid half-spaces. In the present chapter, we have investigated the transmission of elastic waves through the plane interface between two distinct electro-microelastic solid half-spaces. This problem is an extension of the problem considered in the previous chapter to a plane interface. Amplitude and energy ratios of various

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3.2 Field equations and relations

Reflected and transmitted waves are presented when (i) a set of coupled longitudinal waves is made incident and (ii) a set of coupled transverse waves is made incident. Results of Tomar and Gogna (1995a, 95b) are recovered from the present formulation by ignoring the electric and microstretch effects. In the absence of electric effects from the present problem, we have obtained the results that exactly match with the results of Tomar and Garg (2005).

3.2 Field equations and relations

The field equations and constitutive relations for an electro-microelastic solid medium are same as given through equations (2.1)-(2.5) and (2.6)-(2.9), respectively. It can be seen that the coefficients $b_0$ and $\lambda_3$ appearing in the expressions of $m_{kl}$, $m_k$ and $D_k$ given in (2.7)-(2.9), do not contribute to the field equations (2.1)-(2.4). Therefore, in the subsequent analysis, we shall drop the terms corresponding to these coefficients from the expressions of $m_{kl}$, $m_k$ and $D_k$. The reason of dropping the coefficient $b_0$ can be seen in Eringen (1990).

3.3 Reflection and transmission of elastic waves

Here, we shall discuss the reflection and transmission phenomena of incidence of a plane wave at a plane interface between two distinct electro-microelastic solid half-spaces in perfect contact. Let the plane interface be along $x-$axis and $z-$axis is taken along the direction pointing vertically downward. We take the lower and upper half-spaces as

\[ M = \{(x, z) : -\infty < x < \infty, 0 < z < \infty\} \]

and

\[ M' = \{(x, z) : -\infty < x < \infty, -\infty < z < 0\}, \]

respectively. We denote the elastic constants and density in medium $M$ by $\lambda$, $\mu$, $K$, $\alpha$, $\beta$, $\gamma$, $\alpha_0$, $\lambda_0$, $\lambda_1$, $\lambda_2$, $\chi^E$ and $\rho$, and the corresponding elastic parameters in medium $M'$ by $\lambda'$, $\mu'$, $K'$, $\alpha'$, $\beta'$, $\gamma'$, $\alpha'_0$, $\lambda'_0$, $\lambda'_1$, $\lambda'_2$, $\chi'^E$ and $\rho'$. The complete geometry of the problem is shown in Figure 3.1. We shall consider a two-dimensional problem in $x-z$ plane, so we shall take the components of $u$ and $\phi$ as given in (2.34).
3.3.1 Incidence of a coupled longitudinal waves with speed $V_1$

Let a set of coupled longitudinal waves of amplitude $A_0$ propagating with phase speed $V_1$ through the lower half-space be incident at the plane interface and makes an angle $\theta_0$ with the $z-$ axis. To satisfy the boundary conditions at the interface, we take the following reflected and transmitted waves

**Reflected waves:**

(i) two sets of coupled longitudinal waves of amplitudes $A_{1,2}$ propagating with phase speeds $V_{1,2}$ in medium $M$ and making angles $\theta_{1,2}$ with the normal,

(ii) two sets of coupled transverse waves of amplitudes $A_{3,4}$ propagating with phase speeds $V_{3,4}$ in medium $M$ and making angles $\theta_{3,4}$ with the normal.

**Transmitted waves:**

(i) two sets of coupled longitudinal waves of amplitudes $A'_{1,2}$ propagating with phase speeds $V'_{1,2}$ in medium $M'$ and making angles $\theta'_{1,2}$ with the normal,

(ii) two sets of coupled transverse waves of amplitudes $A'_{3,4}$ propagating with phase speeds $V'_{3,4}$ in medium $M'$. 

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**Figure 3.1:** The geometry of the problem
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speeds \( V_{3,4} \) in medium \( M' \) and making angles \( \theta_{3,4} \) with the normal.

The expressions of the phase speeds \( V_{1,2} \) and \( V_{3,4} \) are defined earlier through (2.22) and (2.25), respectively. The expressions of the phase speeds \( V_{1,2}' \) and \( V_{3,4}' \) are similar to the expressions of phase speeds \( V_{1,2} \) and \( V_{3,4} \), respectively, with appropriate dashes.

The potentials of various reflected waves in the half-space \( M \) are the same as given in Section 2.4.1 through (2.35) and (2.36). They are reproduced here as

\[
\{q', \psi'\} = \left\{1, \zeta_1 p\right\} A_0 P^R_0 + \sum_{p=1,2} \left\{1, \zeta_p p\right\} A_p P^R_p, \quad (3.1)
\]

\[
\{U_2, \phi_2\} = \sum_{p=3,4} \left\{1, \eta_p p\right\} A_p P^T_p, \quad (3.2)
\]

while the potentials of various transmitted waves in the half-space \( M' \) are given by

\[
\{q', \psi'\} = \sum_{p=1,2} \left\{1, \zeta_1 p\right\} A'_p P'^R_p, \quad (3.3)
\]

\[
\{U'_2, \phi'_2\} = \sum_{p=3,4} \left\{1, \eta'_p p\right\} A'_p P'^T_p, \quad (3.4)
\]

where \( P^R_0 = \exp[i k_3 (\sin \theta_0 x - \cos \theta_0 z) - \omega_0 t] \), \( P^T_p = \exp[i k_p (\sin \theta_p x + \cos \theta_p z) - \omega_p t] \) and \( P'^R_p = \exp[i k'_p (\sin \theta'_p x - \cos \theta'_p z) - \omega'_p t] \). The coefficients \( \zeta_{1,2} \) are the coupling parameters between \( q' \) and \( \psi' \), while the coefficients \( \eta_{3,4} \) are the coupling parameters between \( U'_2 \) and \( \phi'_2 \). The expressions of \( \zeta_{1,2} \) and \( \eta_{3,4} \) are similar to the expressions \( \zeta_{1,2} \) and \( \eta_{3,4} \) given through (2.37) and (2.38), respectively, and can be written by putting appropriate dashes.

Using the expressions of \( u, \phi \) and \( E \) from (2.11)-(2.13) into (2.6)-(2.9) after dropping the coefficient \( b_0 \) and from the expression of \( u \) given through (2.11), the requisite components of stress, microrotation, microstretch and displacement are given by

\[
t_{zz} = (\lambda + 2\mu + K) q_{zz} + (2\mu + K) U_{2,zz} + \lambda q_{xz} + \lambda_0 \psi.
\]
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\begin{align}
t_{zz} &= (2\mu + K) q_{zz} - (\mu + K) U_{2,zz} + \mu U_{2,zz} - K \phi_2, \\
m_{yz} &= \gamma \phi_{2,z}, \\
\psi &= \left( a_0 - \frac{\lambda^2}{1 + \chi} \right) \psi_{2,z}, \\
u_1 &= q_{,x} - U_{2,z}, \\
u_3 &= q_{,x} + U_{2,z}. 
\end{align}

Similar expressions can be written for \( t'_{zz}, t'_{zx}, m'_{zy}, m'_z, u'_1 \) and \( u'_3 \) with appropriate dashes, e.g., \( m'_{zy} = \gamma' \phi'_{2,z} \) and \( u'_1 = q'_{,x} - U'_{2,z}, \) etc.

The appropriate mechanical boundary conditions at the interface between two electro-microelastic solid half-spaces are

(i) the continuity of stresses,

(ii) the continuity of microstretch,

(iii) the continuity of displacement and microrotation.

Mathematically, these boundary conditions can be written as: At \( z = 0, \) we have

\begin{align}
t_{zz} &= t'_{zz}, \\
t_{zx} &= t'_{zx}, \\
m_{yz} &= m'_{zy}, \\
m_z &= m'_z, \\
\psi &= \psi', \\
u_1 &= u'_1, \\
u_3 &= u'_3, \\
\phi_2 &= \phi'_2. 
\end{align}

Owing to the expressions of potentials given in (3.1)-(3.4) and assuming

\[ k_i \sin \theta_i = k'_i \sin \theta'_i, \quad \omega_i = \omega'_i \quad (i = 1, 2, 3, 4), \quad \text{at} \quad z = 0, \]

the boundary conditions given in (3.6) are identically satisfied if

\begin{align}
&\left[ \lambda + (2\mu + K) \cos^2 \theta_0 - \frac{\lambda_0 \kappa_1}{k_1^2} \right] k_1^2 A_0 + \sum_{p=1,2} \left[ \lambda + (2\mu + K) \cos^2 \theta_p - \frac{\lambda_0 \kappa_p}{k_p^2} \right] k_p^2 A_p \\
&- \sum_{p=1,2} \left[ \lambda' + (2\mu' + K') \cos^2 \theta'_p - \frac{\lambda_0' \kappa'_p}{k'_p^2} \right] k'_p A'_p + (2\mu + K) \sum_{p=3,4} \sin \theta_p \cos \theta_p k_p^2 A'_p = 0.
\end{align}

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\[ + (2\mu' + K') \sum_{p=3,4} \sin \theta'_p \cos \theta'_p k'_p^2 A'_p = 0, \quad (3.7) \]

\[ (2\mu + K) \sin \theta_0 \cos \theta_0 k_0^2 A_0 - (2\mu + K) \sum_{p=1,2} \sin \theta_p \cos \theta_p k_p^2 A_p \]

\[-(2\mu' + K') \sum_{p=1,2} \sin \theta'_p \cos \theta'_p k'_p^2 A'_p + \sum_{p=3,4} \left[ \mu \cos 2\theta_p + K \cos^2 \theta_p - \frac{K' \eta_p}{k_p^2} \right] k_p^2 A'_p = 0, \quad (3.8) \]

\[ \gamma \sum_{p=3,4} \eta_p \cos \theta_p k_p A_p + \gamma' \sum_{p=3,4} \eta'_p \cos \theta'_p k'_p A'_p = 0, \quad (3.9) \]

\[ \left( \alpha_0 - \frac{\lambda^2_1}{1 + \chi^2 E} \right) \zeta_1 \cos \theta_0 k_1 A_0 - \left( \alpha'_0 - \frac{\lambda^2_1}{1 + \chi^2 E} \right) \sum_{p=1,2} \zeta_p \cos \theta_p k_p A_p \]

\[- \left( \alpha'_0 - \frac{\lambda^2_1}{1 + \chi^2 E} \right) \sum_{p=1,2} \zeta'_p \cos \theta'_p k'_p A'_p = 0, \quad (3.10) \]

\[ \sin \theta_0 k_1 A_0 + \sum_{p=1,2} \sin \theta_p k_p A_p - \sum_{p=1,2} \sin \theta'_p k'_p A'_p \]

\[- \sum_{p=3,4} \cos \theta_p k_p A_p - \sum_{p=3,4} \cos \theta'_p k'_p A'_p = 0, \quad (3.11) \]

\[ \cos \theta_0 k_1 A_0 - \sum_{p=1,2} \cos \theta_p k_p A_p - \sum_{p=1,2} \cos \theta'_p k'_p A'_p \]

\[- \sum_{p=3,4} \sin \theta_p k_p A_p + \sum_{p=3,4} \sin \theta'_p k'_p A'_p = 0, \quad (3.12) \]

\[ \sum_{p=3,4} \eta_p A_p - \sum_{p=3,4} \eta'_p A'_p = 0, \quad (3.13) \]

\[ \zeta_1 A_0 + \sum_{p=1,2} \zeta_p A_p - \sum_{p=1,2} \zeta'_p A'_p = 0. \quad (3.14) \]
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The equations (3.7)-(3.14) enable us to determine the amplitude ratios of various reflected and transmitted waves. The matrix form of these equations is written as

$$\begin{bmatrix} a_{ij} \end{bmatrix} \begin{bmatrix} Z \end{bmatrix} = \begin{bmatrix} Y \end{bmatrix},$$

(3.15)

where $[a_{ij}]$ is a 8 x 8 matrix, $[Y]$ is a 8 x 1 matrix, $[Z] = [Z_1, Z_2, ..., Z_8]^T$ is a column matrix, $Z_r = \Delta_r / A_0$ and $Z_{r+4} = \Delta'_r / A_0 \ (r = 1, 2, 3, 4)$ are the amplitude ratios corresponding to the reflected and transmitted waves, respectively, propagating with respective phase speeds $V_r$ and $V'_r$, for an incident set of coupled longitudinal waves traveling with phase speed $V_1$.

The non-zero entries of the matrix $[a_{ij}]$ together with the column matrix $[Y]$ are given as

$$a_{11} = 1,$$

$$a_{12} = \left[ \lambda + \Delta_2 (1 - v_{21}^2 \sin^2 \theta_0) - \frac{\lambda \sigma_2}{k_2^2} \right] / \Delta_1 v_{21}^2,$$

$$a_{1p} = \Delta_2 \sin \theta_0 \sqrt{1 - v_{p1}^2 \sin^2 \theta_0} / \Delta_1 v_{p1},$$

$$a_{15} = \left[ \lambda + \Delta'_2 (1 - v_{21}^2 \sin^2 \theta_0) - \frac{\lambda \sigma'_2}{k_2^2} \right] / \Delta_1 v_{11}^2,$$

$$a_{16} = \left[ \lambda + \Delta'_2 (1 - v_{21}^2 \sin^2 \theta_0) - \frac{\lambda \sigma'_2}{k_2^2} \right] / \Delta_1 v_{21}^2,$$

$$a_{1s} = \Delta'_2 \sin \theta_0 \sqrt{1 - V_{s1}^2 \sin^2 \theta_0} / \Delta_1 V'_s,$$

$$a_{21} = \sin \theta_0 \cos \theta_0,$$

$$a_{22} = \sin \theta_0 \sqrt{1 - v_{21}^2 \sin^2 \theta_0} / v_{21},$$

$$a_{23} = -\left[ \mu (1 - 2 \alpha^2 \sin^2 \theta_0) + K (1 - \alpha^2 \sin^2 \theta_0) - \frac{K \eta_3}{k_3^2} \right] / \Delta_2 v_{31}^2,$$

$$a_{24} = -\left[ \mu (1 - 2 \alpha^2 \sin^2 \theta_0) + K (1 - \alpha^2 \sin^2 \theta_0) - \frac{K \eta_4}{k_4^2} \right] / \Delta_2 v_{41}^2.$$
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\[ a_{27} = \frac{\mu'(1 - 2\nu_{31}^2 \sin^2 \theta_0) + K'(1 - \nu_{31}^2 \sin^2 \theta_0) - \frac{K'\nu_{31}^2}{k_0^2}}{\Delta_2 \nu_{31}^2}, \]

\[ a_{28} = \frac{\mu'(1 - 2\nu_{41}^2 \sin^2 \theta_0) + K'(1 - \nu_{41}^2 \sin^2 \theta_0) - \frac{K'\nu_{41}^2}{k_0^2}}{\Delta_2 \nu_{41}^2}, \]

\[ a_{33} = \frac{\sqrt{1 - \nu_{31}^2 \sin^2 \theta_0}}{\nu_{31}}, \]

\[ a_{34} = \eta_4 \sqrt{1 - \nu_{41}^2 \sin^2 \theta_0} / \eta_3 \nu_{41}, \]

\[ a_{37} = \gamma' \eta_3 \sqrt{1 - \nu_{31}^2 \sin^2 \theta_0} / \gamma \eta_3 \nu_{31}, \]

\[ a_{38} = \gamma' \eta_4 \sqrt{1 - \nu_{41}^2 \sin^2 \theta_0} / \gamma \eta_3 \nu_{41}, \]

\[ a_{41} = \cos \theta_0, \]

\[ a_{42} = \zeta_2 \sqrt{1 - \nu_{21}^2 \sin^2 \theta_0} / \zeta_1 \nu_{21}, \]

\[ a_{45} = \left( \alpha_0 - \frac{\lambda_2^2}{1 + \chi^E} \right) \zeta_2 \sqrt{1 - \nu_{21}^2 \sin^2 \theta_0} / \left( \alpha_0 - \frac{\lambda_2}{1 + \chi^E} \right) \nu_{21} \zeta_2, \]

\[ a_{46} = \left( \alpha_0 - \frac{\lambda_2^2}{1 + \chi^E} \right) \zeta_2 \sqrt{1 - \nu_{21}^2 \sin^2 \theta_0} / \left( \alpha_0 - \frac{\lambda_2}{1 + \chi^E} \right) \nu_{21} \zeta_2, \]

\[ a_{51} = a_{22} = \sin \theta_0, \]

\[ a_{50} = -\sqrt{1 - \nu_{p1}^2 \sin^2 \theta_0} / \nu_{p1}, \]

\[ a_{5p} = -\sin \theta_0, \]

\[ a_{61} = \cos \theta_0, \]

\[ a_{62} = \sqrt{1 - \nu_{21}^2 \sin^2 \theta_0} / \nu_{21}, \]

\[ a_{6p} = \sin \theta_0. \]
\[ a_{er} = \sqrt{1 - \frac{V_{r1}'}{V_{r1}}} \sin^2 \theta_0 / V_{r1}', \]

\[ a_{6s} = -\sin \theta_0, \]

\[ a_{73} = 1, \]

\[ a_{74} = \eta_4 / \eta_3, \]

\[ a_{77} = -\eta'_5 / \eta_3, \]

\[ a_{78} = -\eta_4 / \eta_3, \]

\[ a_{81} = 1, \]

\[ a_{82} = \zeta_2 / \zeta_1, \]

\[ a_{85} = -\zeta'_1 / \zeta_1, \]

\[ a_{86} = -\zeta'_2 / \zeta_1, \]

\[ p = 3, 4, r = 5, 6, s = 7, 8, \]

where

\[ \Delta_1 = \lambda + \Delta_2 \cos^2 \theta_0 - \frac{\lambda \zeta_3}{k_1^2}, \quad \Delta_2 = 2\mu + K, \quad \Delta'_2 = 2\mu' + K', \]

\[ v_{m1} = \frac{V_{m1}}{V_1} \quad (m = 2, 3, 4), \quad v'_{n1} = \frac{V'_{n1}}{V_1} \quad (n = 1, 2, 3, 4), \]

\[ V'_{51} = v'_{11}, \quad V'_{61} = v'_{21}, \quad V'_{71} = v'_{31}, \quad V'_{81} = v'_{41} \]

and the column matrix

\[ [Y] = [-1, \sin \theta_0 \cos \theta_0, 0, \cos \theta_0, -\sin \theta_0, \cos \theta_0, 0, -1]' \]
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Next, we shall discuss the partitioning of incident energy between various reflected and transmitted waves at the plane interface \( z = 0 \). For this, following the same procedure as in the Section 2.4.1, the expressions of energy ratios \( E_i \) \((i = 1, 2, \ldots, 8)\) are given by

\[
E_1 = Z^2_1 P_1 \left[ \lambda + 2\mu + K - \frac{\lambda_0 c_2}{k_2^2} - \frac{\alpha_0 c_2^2}{k_2^2} + \frac{\lambda_0^2 c_2^2}{(1 + \chi E) k_2^2} \right] k_2^3 \cos \theta_2,
\]

\[
E_2 = Z^2_2 P_1 \left[ \lambda + 2\mu + K - \frac{\gamma_3 c_4}{k_3^2} - \frac{K_4 c_4}{k_3^2} \right] k_3^3 \cos \theta_{3,4},
\]

\[
E_{3,4} = Z^2_{3,4} P_1 \left[ \mu + K - \frac{\gamma_{3,4} c_4^2}{k_3^2} + \frac{K_{3,4} c_4^2}{k_3^2} \right] k_3^3 \cos \theta_{3,4},
\]

\[
E_{5,6} = Z^2_{5,6} P_1 \left[ \lambda + 2\mu + K + \gamma_{3,4} c_4^2 \right] k_{3,4}^3 \cos \theta_{3,4},
\]

\[
E_{7,8} = Z^2_{7,8} P_1 \left[ \mu + K - \frac{\gamma_{3,4} c_4^2}{k_3^2} + \frac{K_{3,4} c_4^2}{k_3^2} \right] k_{3,4}^3 \cos \theta_{3,4},
\]

where

\[
P_1 = \left[ \left( \lambda + 2\mu + K - \frac{\lambda_0 c_1}{k_1^2} - \frac{\alpha_0 c_1^2}{k_1^2} + \frac{\lambda_0^2 c_1^2}{(1 + \chi E) k_1^2} \right) k_1^3 \cos \theta_1 \right]^{-1}.
\]

Each energy ratio \( E_i \) \((i = 1, 2, \ldots, 8)\) gives the rate of energy transmission at the interface for the respective reflected and transmitted wave to the rate of energy transmission for the incident set of coupled longitudinal waves with speed \( V_1 \).

3.3.2 Incidence of a coupled transverse waves with speed \( V_3 \)

We shall now study the phenomena of reflection and transmission at the interface due to incidence of a set of coupled transverse waves propagating with phase speed \( V_3 \) having amplitude \( A_0 \) and making an angle \( \theta_0 \) with the normal. The geometry of the problem and the sets of reflected and transmitted waves will remain same as considered in the Section 3.3.1 of an incidence of a set of coupled longitudinal waves. Also, the boundary conditions will be the same as given in the equation (3.6).

We take the potentials in the medium \( M \), given by

\[
\{U_2, \phi_2\} = \{1, \eta_1\} A_0 Q_0^- + \sum_{p=3,4} \{1, \eta_p\} A_p Q_p^+,
\]

(3.16)
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\( \{q, \psi\} = \sum_{p=1,2} \{1, \zeta_p\} A_p Q_p^+ \), \hspace{1cm} (3.17)

and the potentials in the medium \( M' \) are the same as given in Section 3.3.1. These are reproduced here

\( \{q', \psi'\} = \sum_{p=1,2} \{1, \zeta'_p\} A'_p Q'^+_p \), \hspace{1cm} (3.18)

\( \{U'_2, \phi'_2\} = \sum_{p=3,4} \{1, \eta'_p\} A'_p Q'^-_p \), \hspace{1cm} (3.19)

where \( Q^+_p = \exp[i k_3 (\sin \theta_0 x - \cos \theta_0 z) - \omega_3 t] \), \( Q^+_p = \exp[i k_p (\sin \theta_p x + \cos \theta_p z) - \omega_p t] \)

and \( Q'^-_p = \exp[i k'_p (\sin \theta'_p x - \cos \theta'_p z) - \omega'_p t] \).

Inserting the above expressions of the potentials into the boundary conditions given in the equation (3.6), we see that the boundary conditions are satisfied if

\[
(2\mu + K) \sin \theta_0 \cos \theta_0 k_3^2 A_0 - \sum_{p=1,2} \left[ \lambda + (2\mu + K) \cos^2 \theta_p - \frac{\lambda_0 \zeta_p}{k_p^2} \right] k_p^2 A_p \\
+ \sum_{p=1,2} \left[ \lambda' + (2\mu' + K') \cos^2 \theta'_p - \frac{\lambda'_0 \zeta'_p}{k'_p^2} \right] k'_p^2 A'_p - (2\mu + K) \sum_{p=3,4} \sin \theta_p \cos \theta_p k_p^2 A_p \\
- (2\mu' + K') \sum_{p=3,4} \sin \theta'_p \cos \theta'_p k'_p^2 A'_p = 0, \hspace{1cm} (3.20)
\]

\[
\left[ \mu \cos 2\theta_0 + K \cos^2 \theta_0 - \frac{K \eta_3}{k_3^2} \right] k_3^2 A_0 - (2\mu + K) \sum_{p=1,2} \sin \theta_p \cos \theta_p k_p^2 A_p \\
- (2\mu' + K') \sum_{p=1,2} \sin \theta'_p \cos \theta'_p k'_p^2 A'_p + \sum_{p=3,4} \left[ \mu \cos 2\theta_p + K \cos^2 \theta_p - \frac{K \eta_p}{k_p^2} \right] k_p^2 A_p \\
- \sum_{p=3,4} \left[ \mu' \cos 2\theta'_p + K' \cos^2 \theta'_p - \frac{K' \eta'_p}{k'_p^2} \right] k'_p^2 A'_p = 0, \hspace{1cm} (3.21)
\]

\[
\gamma \eta_3 \cos \theta_0 k_3 A_0 - \gamma \sum_{p=3,4} \eta_p \cos \theta_p k_p A_p - \gamma' \sum_{p=3,4} \eta'_p \cos \theta'_p k'_p A'_p = 0, \hspace{1cm} (3.22)
\]
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\[
\left(\alpha_0 - \frac{\lambda_0^2}{1 + \chi^2}\right) \sum_{p=1,2} \zeta_p \cos \theta_p k_p A_p + \left(\alpha_0' - \frac{\lambda_0'^2}{1 + \chi'^2}\right) \sum_{p=1,2} \zeta_p' \cos \theta_p' k_p' A_p' = 0, \quad (3.23)
\]

\[
\cos \theta_0 k_3 A_0 + \sum_{p=1,2} \sin \theta_p k_p A_p - \sum_{p=1,2} \sin \theta_p' k_p' A_p' = 0,
\]

\[\cdots \]

\[\cdots \]

\[
\sin \theta_0 k_3 A_0 + \sum_{p=1,2} \cos \theta_p k_p A_p + \sum_{p=1,2} \cos \theta_p' k_p' A_p' = 0,
\]

\[
\cos \theta_0 k_3 A_0 + \sum_{p=1,2} \sin \theta_p k_p A_p - \sum_{p=1,2} \sin \theta_p' k_p' A_p' = 0,
\]

\[
\eta_3 A_0 + \sum_{p=3,4} \eta_p A_p - \sum_{p=3,4} \eta_p' A_p' = 0,
\]

\[
\sum_{p=1,2} \zeta_p A_p - \sum_{p=1,2} \zeta_p' A_p' = 0.
\]

The system of equations (3.20)-(3.27) can be written in matrix form as

\[
[b_{ij}][Z'] = [N],
\]

where \([b_{ij}]\) is a 8 x 8 matrix, \([N]\) is a 8 x 1 matrix, \([Z'] = [Z'_1, Z'_2, ..., Z'_4]\) is a column matrix, \(Z'_r = A_r/A_0\) and \(Z'_{r+4} = A'_r/A_0\) \((r = 1, 2, 3, 4)\) are now the reflection and transmission coefficients, respectively, corresponding to the waves propagating with phase speeds \(V_r\) and \(V'_r\), respectively, for the incidence of a set of coupled transverse waves traveling with speed \(V_3\).

The non-vanishing elements of the coefficient matrix \([b_{ij}]\) and the column matrix \([N]\) are given as

\[
b_{11} = \left[ \lambda + \Delta_2 (1 - v^2_{13} \sin^2 \theta_0) - \frac{\lambda_0 \zeta_1}{k_1^2} \right] / \Delta_2 v_{13}^2,
\]

\[
b_{12} = \left[ \lambda + \Delta_2 (1 - v^2_{23} \sin^2 \theta_0) - \frac{\lambda_0 \zeta_2}{k_2^2} \right] / \Delta_2 v_{23}^2.
\]
\[ b_{13} = \sin \theta_0 \cos \theta_0, \]

\[ b_{14} = \sin \theta_0 \sqrt{1 - v_{43}^2 \sin^2 \theta_0} / v_{43}. \]

\[ b_{15} = -\left[ \lambda + \Delta'_{21} (1 - v_{13}^2 \sin^2 \theta_0) - \frac{\lambda \eta_{1}}{k_2^2} \right] / \Delta_{21} v_{13}^2, \]

\[ b_{16} = -\left[ \lambda + \Delta'_{22} (1 - v_{23}^2 \sin^2 \theta_0) - \frac{\lambda \eta_{2}}{k_2^2} \right] / \Delta_{22} v_{23}^2, \]

\[ b_{17} = \Delta^2_{22} \sin \theta_0 \sqrt{1 - v_{23}^2 \sin^2 \theta_0} / V_{43} \Delta_{2}, \]

\[ b_{21} = \Delta_{21} \sin \theta_0 \sqrt{1 - v_{13}^2 \sin^2 \theta_0} / \Delta_{4} v_{13}, \]

\[ b_{22} = \Delta_{22} \sin \theta_0 \sqrt{1 - v_{23}^2 \sin^2 \theta_0} / \Delta_{4} v_{23}, \]

\[ b_{23} = -1, \]

\[ b_{24} = -\left[ \mu (1 - 2v_{43}^2 \sin^2 \theta_0) + K (1 - v_{43}^2 \sin^2 \theta_0) - \frac{\mu \eta_{4}}{k_4^2} \right] / \Delta_{4} v_{43}^2, \]

\[ b_{25} = \Delta^2_{22} \sin \theta_0 \sqrt{1 - V_{43}^2 \sin^2 \theta_0} / \Delta_{4} \Delta_{4} \]

\[ b_{27} = \left[ \mu' (1 - 2v_{43}^2 \sin^2 \theta_0) + K' (1 - v_{43}^2 \sin^2 \theta_0) - \frac{\mu' \eta_{4}}{k_4^2} \right] / \Delta_{4} v_{43}^2, \]

\[ b_{28} = \left[ \mu' (1 - 2v_{43}^2 \sin^2 \theta_0) + K' (1 - v_{43}^2 \sin^2 \theta_0) - \frac{\mu' \eta_{4}}{k_4^2} \right] / \Delta_{4} v_{43}^2, \]

\[ b_{33} = \cos \theta_0, \]

\[ b_{34} = \eta_{4} \sqrt{1 - v_{43}^2 \sin^2 \theta_0} / \eta_{3} v_{43}, \]

\[ b_{37} = \gamma' \eta_{3} \sqrt{1 - v_{33}^2 \sin^2 \theta_0} / \gamma \eta_{3} v_{33}, \]

\[ b_{38} = \gamma' \eta_{4} \sqrt{1 - v_{43}^2 \sin^2 \theta_0} / \gamma \eta_{4} v_{43}. \]
3.3. Reflection and transmission of elastic waves

\[ b_{41} = \sqrt{1 - v_{13}^2 \sin^2 \theta_0} / v_{13}, \]

\[ b_{42} = \zeta_2 \sqrt{1 - v_{23}^2 \sin^2 \theta_0} / (\zeta_1 v_{23}), \]

\[ b_{45} = \left( \alpha_0' - \frac{\lambda_0'^2}{1 + \chi^E} \right) \zeta_1' \sqrt{1 - v_{13}^2 \sin^2 \theta_0} / \left( \alpha_0 - \frac{\lambda_0^2}{1 + \chi^E} \right) v_{13}' \zeta_1, \]

\[ b_{46} = \left( \alpha_0' - \frac{\lambda_0'^2}{1 + \chi^E} \right) \zeta_1' \sqrt{1 - v_{23}^2 \sin^2 \theta_0} / \left( \alpha_0 - \frac{\lambda_0^2}{1 + \chi^E} \right) v_{23}' \zeta_1, \]

\[ b_{51} = \sin \theta_0, \]

\[ b_{52} = \sin \theta_0, \]

\[ b_{53} = -\cos \theta_0, \]

\[ b_{54} = -\sqrt{1 - v_{23}^2 \sin^2 \theta_0} / v_{43}, \]

\[ b_{55} = b_{66} = -\sin \theta_0, \]

\[ b_{65} = -\sqrt{1 - V_{33}^2 \sin^2 \theta_0} / V_{33}', \]

\[ b_{61} = -\sqrt{1 - v_{13}^2 \sin^2 \theta_0} / v_{13}, \]

\[ b_{62} = -\sqrt{1 - v_{23}^2 \sin^2 \theta_0} / v_{23}, \]

\[ b_{66} = -\sqrt{1 - V_{33}^2 \sin^2 \theta_0} / V_{33}', \]

\[ b_{69} = \sin \theta_0, \]

\[ b_{73} = b_{63} = 1, \]

\[ b_{74} = \eta_4 / \eta_3, \]

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\[ b_7s = -\eta''_{s-4}/\eta_3, \]
\[ b_8r = \zeta_2/\zeta_1, \]
\[ b_{sr} = -\zeta''_{s-4}/\zeta_1, \]
\[ p = 3, 4, \ r = 5, 6, \ s = 7, 8, \]

where

\[ \Delta_4 = \mu \cos 2\theta_0 + K \cos^2 \theta_0 - \frac{K\eta_2}{k_3^3}, \ V'_{53} = V'_{13}, \ V'_{63} = V'_{23}, \]
\[ V'_{33} = V'_{33}, \ V'_{83} = V'_{43}, \ v_{m3} = \frac{V_{m3}}{V_3} (m = 1, 2, 4), \ v'_{n3} = \frac{V'_{n3}}{V_3} (n = 1, 2, 3, 4) \]

and the column matrix

\[ [N] = [\sin \theta_0 \cos \theta_0, 1, \cos \theta_0, 0, -\cos \theta_0, \sin \theta_0, -1, 0]^T. \]

The matrix equation (3.28) will enable us to provide the expressions of reflection and transmission coefficients in the present case.

The expressions for energy ratios \( E_i \) \((i = 1, 2, \ldots, 8)\) of various reflected and transmitted waves in this case, are given by

\[ E_{1,2} = Z^2_{12} P_2 \left[ \lambda + 2\mu + K - \frac{\lambda_2 c_{1,2}}{k_{1,2}^3} - \frac{\alpha_2 c_{1,2}}{k_{1,2}^3} + \frac{\lambda_2^2 c_{1,2}^2}{(1 + \chi^2)k_{1,2}^6} \right] k_{1,2}^3 \cos \theta_{1,2}, \]
\[ E_3 = Z^2_{3} P_2, \]
\[ E_4 = Z^2_{4} P_2 \left[ \mu + K - \frac{\gamma \eta_4}{k_4^3} - \frac{K\eta_4}{k_4^3} \right] k_4^3 \cos \theta_4, \]
\[ E_{5,6} = Z^2_{5,6} P_2 \left[ \lambda' + 2\mu' + K' - \frac{\lambda' c_{1,2}}{k_{1,2}^3} - \frac{\alpha' c_{1,2}}{k_{1,2}^3} + \frac{\lambda_2^2 c_{1,2}^2}{(1 + \chi^2)k_{1,2}^6} \right] k_{1,2}^3 \cos \theta_{1,2}', \]
\[ E_{7,8} = Z^2_{7,8} P_2 \left[ \mu' + K' - \frac{\gamma' \eta_{3,4}}{k_{3,4}^3} - \frac{K'\eta_{3,4}}{k_{3,4}^3} \right] k_{3,4}^3 \cos \theta_{3,4}'. \]

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3.4. Limiting cases

where

\[ P_2 = \left( \mu + K - \frac{\gamma \eta^2}{k_f^2} - \frac{K \eta A}{k_f^2} \right) k_2^3 \cos \theta_0 \right]^{-1}. \]

Here, each energy ratio \( E_i \) (\( i = 1, 2, \ldots, 8 \)) gives the rate of energy transmitted at the interface for the respective reflected and transmitted wave to the rate of energy transmitted for the incident set of coupled transverse waves propagating with phase speed \( V_3 \).

3.4 Limiting cases

(i) If we assume that both the half-spaces \( M \) and \( M' \) are free from the electric and microstretch effects, then we shall be left with the relevant problem in micropolar elastic solid half-spaces. In this case, the waves propagating with speeds \( V_2 \) and \( V'_2 \) will not appear in the mediums \( M \) and \( M' \), respectively. Therefore, \( A_2 = A'_2 = 0 \). Thus, we note that the equations (3.10) and (3.14) are identically satisfied and the remaining equations, i.e., equations (3.7)-(3.9) and (3.11)-(3.13), reduce to

\[
[\lambda + (2\mu + K) \cos^2 \theta_0] k_1^2 A_0 + [\lambda + (2\mu + K) \cos^2 \theta_1] k_1^2 A_1 - [\lambda' + (2\mu' + K') \cos^2 \theta'_1] k_1^2 A'_1 \\
+ (2\mu + K) \sum_{p=3,4} \sin \theta_p \cos \theta_p k_p^2 A_p \left[ (2\mu' + K') \sum_{p=3,4} \sin \theta'_p \cos \theta'_p k'_p^2 A'_p \right] = 0, \quad (3.29)
\]

\[
(2\mu + K) \sin \theta_0 \cos \theta_0 k_1^2 A_0 - (2\mu + K) \sin \theta_1 k_1^2 A_1 \\
- (2\mu' + K') \sin \theta'_1 \cos \theta'_1 k_1^2 A'_1 + \sum_{p=3,4} \left[ \mu \cos \theta_p k_p^2 + K \cos^2 \theta_p \frac{K \eta A}{k_p^2} \right] k_p^2 A_p \\
- \sum_{p=3,4} \left[ \mu' \cos \theta'_p k_p^2 + K' \cos^2 \theta'_p \frac{K' \eta A'}{k_p^2} \right] k'_p A'_p = 0, \quad (3.30)
\]

\[
\gamma \sum_{p=3,4} \eta_p \cos \theta_p k_p A_p + \gamma' \sum_{p=3,4} \eta'_p \cos \theta'_p k'_p A'_p = 0, \quad (3.31)
\]

\[
\sin \theta_0 k_1 A_0 + \sin \theta_1 k_1 A_1 - \sin \theta'_1 k'_1 A'_1 - \sum_{p=3,4} \cos \theta_p k_p A_p - \sum_{p=3,4} \cos \theta'_p k'_p A'_p = 0, \quad (3.32)
\]
After converting the angle of incidence to the angle of emergence, one can verify that the above equations exactly match with those obtained by Tomar and Gogna (1995b) for the case of incidence of a longitudinal displacement wave at the interface between two distinct micropolar elastic solid half-spaces.

Similarly, applying this limiting case to the problem when a set of coupled transverse waves propagating through the medium $M$ with phase speed $V_3$ becomes incident at a plane interface between two electro-microelastic solid half-spaces, we see that equations (3.23) and (3.27) are satisfied identically and the remaining equations exactly match with those obtained by Tomar and Gogna (1995a) after converting the angle of incidence into the angle of emergence.

(ii) If we assume that both the half-spaces $M$ and $M'$ are free from electric effect, then we will be left with the relevant problem in linear homogeneous microstretch elastic half-spaces. In this limiting case, it can be seen that the equations of the matrix equation (3.15) reduce to

\[
\begin{align*}
\left[ \lambda + (2\mu + K) \cos^2 \theta_0 - \frac{\lambda_0 \zeta_1}{k_1^2} \right] k_1^2 A_0 &+ \sum_{p=1,2} \left[ \lambda + (2\mu + K) \cos^2 \theta_p - \frac{\lambda_0 \zeta_p}{k_p^2} \right] k_p^2 A_p \\
- \sum_{p=1,2} &\left[ \lambda' + (2\mu' + K') \cos^2 \theta_p' - \frac{\lambda_0' \zeta_p'}{k_p'^2} \right] k_p'^2 A_p' + (2\mu + K) \sum_{p=3,4} \sin \theta_p \cos \theta_p k_p^2 A_p \\
&+ (2\mu' + K') \sum_{p=3,4} \sin \theta_p' \cos \theta_p' k_p'^2 A_p' = 0, \\
(2\mu + K) &\sin \theta_0 \cos \theta_0 k_1^2 A_0 - (2\mu + K) \sum_{p=1,2} \sin \theta_p \cos \theta_p k_p^2 A_p \\
- (2\mu' + K') &\sum_{p=1,2} \sin \theta_p' \cos \theta_p' k_p'^2 A_p' + \sum_{p=3,4} \left[ \mu \cos 2\theta_p + K \cos^2 \theta_p - \frac{K \eta_p}{k_p^2} \right] k_p^2 A_p
\end{align*}
\]
3.4. Limiting cases

\[
- \sum_{p=3,4} \left[ \mu' \cos 2\theta'_p + K' \cos^2 \theta'_p - \frac{K' \eta_p}{k'_p^2} \right] k'_p A'_p = 0, \quad (3.36)
\]

\[
\gamma \sum_{p=3,4} \eta_p \cos \theta'_p k'_p A_p + \gamma' \sum_{p=3,4} \eta'_p \cos \theta'_p k'_p A'_p = 0, \quad (3.37)
\]

\[
\alpha_0 \zeta_1 \cos \theta_0 k_1 A_0 - \alpha_0 \sum_{p=1,2} \zeta_p \cos \theta_p k_p A_p + \alpha'_0 \sum_{p=1,2} \zeta'_p \cos \theta'_p k'_p A'_p = 0, \quad (3.38)
\]

\[
\sin \theta_0 k_1 A_0 + \sum_{p=1,2} \sin \theta_p k_p A_p - \sum_{p=1,2} \sin \theta'_p k'_p A'_p = 0,
\]

\[
- \sum_{p=3,4} \cos \theta_p k_p A_p - \sum_{p=3,4} \cos \theta'_p k'_p A'_p = 0, \quad (3.39)
\]

\[
\cos \theta_0 k_1 A_0 - \sum_{p=1,2} \cos \theta_p k_p A_p - \sum_{p=1,2} \cos \theta'_p k'_p A'_p
\]

\[
- \sum_{p=3,4} \sin \theta_p k_p A_p + \sum_{p=3,4} \sin \theta'_p k'_p A'_p = 0, \quad (3.40)
\]

\[
\sum_{p=3,4} \eta_p A_p - \sum_{p=3,4} \eta'_p A'_p = 0, \quad (3.41)
\]

\[
\zeta_1 A_0 + \sum_{p=1,2} \zeta_p A_p - \sum_{p=1,2} \zeta'_p A'_p = 0. \quad (3.42)
\]

These equations exactly match with the equations obtained by Tomar and Garg (2005) for the case of incidence of a set of coupled longitudinal waves at an interface between two different microstretch elastic solid half-spaces.

Similarly, when a set of coupled transverse waves propagating with speed \( V_3 \) through the medium \( M \) is incident at an interface and then making the substitutions as considered above into the equations (3.20)-(3.27), we see that the reduced equations exactly match with the equations obtained by Tomar and Garg (2005) for the incidence of a set of coupled transverse waves at an interface between two microstretch solid half-spaces.
(iii) To discuss the problem of reflection of a set of coupled longitudinal waves from free plane surface of an electro-microelastic solid half-space, we shall assume that the medium $M'$ is absent. Here, the boundary conditions will be corresponding to vanishing of mechanical loads at the free plane boundary. For this, putting the quantities having dashes equal to zero into equations (3.7)-(3.10), we obtain

$$\left[ \lambda + (2\mu + K) \cos^2 \theta_0 - \frac{\lambda_0 \zeta_1}{k_i^2} \right] k_i^2 A_0 + \sum_{p=1,2} \left[ \lambda + (2\mu + K) \cos^2 \theta_p - \frac{\lambda_0 \zeta_p}{k_p^2} \right] k_p^2 A_p$$

$$+ (2\mu + K) \sum_{p=3,4} \sin \theta_p \cos \theta_p k_p^2 A_p = 0,$$

(3.43)

$$(2\mu + K) \sin \theta_0 \cos \theta_0 k_i^2 A_0 - (2\mu + K) \sum_{p=1,2} \sin \theta_p \cos \theta_p k_p^2 A_p$$

$$+ \sum_{p=3,4} \left[ \mu \cos 2\theta_p + K \cos^2 \theta_p - \frac{K \eta_p}{k_p^2} \right] k_p^2 A_p = 0,$$

(3.44)

$$\gamma \eta_3 \cos \theta_3 k_3 A_3 + \gamma \eta_4 \cos \theta_4 k_4 A_4 = 0,$$

(3.45)

$$\left( \alpha_0 - \frac{\lambda_0^2}{1 + \chi E} \right) \zeta_1 \cos \theta_0 k_1 A_0 - \left( \alpha_0 - \frac{\lambda_0^2}{1 + \chi E} \right) \sum_{p=1,2} \zeta_p \cos \theta_p k_p A_p = 0.$$  

(3.46)

Similarly, when a set of coupled transverse waves propagating with phase speed $V_3$ is incident at the free plane boundary of an electro-microelastic solid half-space, then making the corresponding substitutions into the equations (3.20)-(3.23), we obtain the following equations

$$(2\mu + K) \sin \theta_0 \cos \theta_0 k_i^2 A_0 - \sum_{p=1,2} \left[ \lambda + (2\mu + K) \cos^2 \theta_p - \frac{\lambda_0 \zeta_p}{k_p^2} \right] k_p^2 A_p$$

$$- (2\mu + K) \sum_{p=3,4} \sin \theta_p \cos \theta_p k_p^2 A_p = 0,$$

(3.47)

$$\left[ \mu \cos 2\theta_0 + K \cos^2 \theta_0 - \frac{K \eta_p}{k_i^2} \right] k_i^2 A_0 - (2\mu + K) \sum_{p=1,2} \sin \theta_p \cos \theta_p k_p^2 A_p$$

$$+ \sum_{p=3,4} \left[ \mu \cos 2\theta_p + K \cos^2 \theta_p - \frac{K \eta_p}{k_p^2} \right] k_p^2 A_p = 0,$$

(3.48)
3.5. Numerical results and discussions

\[ \gamma \eta_3 \cos \theta_3 k_3 A_0 - \gamma \sum_{p=1,2} \eta_p \cos \theta_p k_p A_p = 0, \quad (3.49) \]

\[ \left( \alpha_0 - \frac{\lambda^2}{1 + \chi^2} \right) \sum_{p=1,2} \zeta_p \cos \theta_p k_p A_p = 0. \quad (3.50) \]

One can see that in the absence of microstretch parameter \( b_0 \), the equations (3.43)-(3.46) and (3.47)-(3.50) exactly match with the equations (2.41)-(2.44) and (2.49)-(2.52), respectively, of Sections 2.4.1 and 2.4.3 for the corresponding problems.

3.5 Numerical results and discussions

In order to examine this study in greater detail, we have computed the amplitude ratios and energy ratios of various reflected and refracted waves for a particular model. Keeping in view the restrictions on constitutive constants given through (2.10), the following values of relevant parameters are used in numerical computations:

<table>
<thead>
<tr>
<th>Medium ( M )</th>
<th>Medium ( M' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda = 7.85 \times 10^{10} ) N/m(^2)</td>
<td>( \lambda' = 7.59 \times 10^{10} ) N/m(^2)</td>
</tr>
<tr>
<td>( \mu = 6.46 \times 10^{10} ) N/m(^2)</td>
<td>( \mu' = 1.89 \times 10^{10} ) N/m(^2)</td>
</tr>
<tr>
<td>( K = 0.0139 \times 10^{10} ) N/m(^2)</td>
<td>( K' = 0.0149 \times 10^{10} ) N/m(^2)</td>
</tr>
<tr>
<td>( \alpha_0 = 0.085 \times 10^6 ) N</td>
<td>( \alpha'_0 = 0.095 \times 10^6 ) N</td>
</tr>
<tr>
<td>( \lambda_0 = 0.038 \times 10^{10} ) N/m(^2)</td>
<td>( \lambda'_0 = 0.032 \times 10^{10} ) N/m(^2)</td>
</tr>
<tr>
<td>( \lambda_1 = 0.030 \times 10^{10} ) N/m(^2)</td>
<td>( \lambda'_1 = 0.030 \times 10^{10} ) N/m(^2)</td>
</tr>
<tr>
<td>( \lambda_2 = 0.3364 \times 10^6 ) N</td>
<td>( \lambda'_2 = 0.3364 \times 10^6 ) N</td>
</tr>
<tr>
<td>( j = j_0 = 0.00000212 ) m(^2)/s</td>
<td>( j' = j'_0 = 0.00000196 ) m(^2)/s</td>
</tr>
<tr>
<td>( \rho = 1900 ) kg/m(^3)</td>
<td>( \rho' = 2200 ) kg/m(^3)</td>
</tr>
<tr>
<td>( \gamma = 0.365 \times 10^6 ) N</td>
<td>( \gamma' = 0.345 \times 10^6 ) N</td>
</tr>
<tr>
<td>( \chi^E = 318 )</td>
<td>( \chi'^E = 298 )</td>
</tr>
</tbody>
</table>

and \( \omega/\omega_0 = 10 \).

In Figure 3.2, we have plotted the modulus values of the reflection coefficients as a function of angle of incidence, when a set of coupled longitudinal waves propagating with phase speed \( V_1 \) becomes incident obliquely at the interface between the half-spaces \( M \) and \( M' \). The reflection coefficient \( Z_1 \) begins with the value 0.1132 near the normal...
incidence, it decreases till $\theta_0 = 35^\circ$ and afterwards it increases very slowly till $\theta_0 = 50^\circ$. Beyond $\theta_0 = 50^\circ$, the reflection coefficient $Z_1$ decreases very smoothly till $\theta_0 = 61^\circ$, thereafter, it increases frequently and attains its maximum value, which is equal to unity at the grazing incidence. The reflection coefficient $Z_2$ first decreases with increase of angle $\theta_0$ in the range $1^\circ \leq \theta_0 \leq 28^\circ$, then it increases sharply to attain its maximum value at $\theta_0 = 64^\circ$, afterwards its value decreases and approaches to zero at $\theta_0 = 90^\circ$. The value of reflection coefficient $Z_3$ increases slowly with increase of the angle of incidence till $\theta_0 = 25^\circ$, then its value decreases till $\theta_0 = 45^\circ$. Beyond $\theta_0 = 45^\circ$, the reflection coefficient $Z_3$ again increases rapidly with angle of incidence to attain its maximum value, thereafter, it decreases and vanishes at $\theta_0 = 90^\circ$. The value of the reflection coefficient $Z_4$ increases very slowly with increase in the angle of incidence till $\theta_0 = 34^\circ$ and then it gradually decreases to the value zero at grazing incidence. We found that the values of the reflection coefficients $Z_2$ and $Z_3$ are very small as compared to that of the coefficient $Z_1$ in the entire range of the angle of incidence. Therefore, we have plotted their curves by magnifying $10^5$ times their original values. At the grazing incidence, all the reflected waves are found to disappear except one corresponding to the reflected set of coupled longitudinal waves propagating with speed $V_1$.

**Figure 3.2:** Variation of modulus of reflection coefficients with angle of incidence of a set of coupled longitudinal waves propagating with speed $V_1$

**Figure 3.3:** Variation of modulus of transmission coefficients with angle of incidence of a set of coupled longitudinal waves propagating with speed $V_1$
3.5. Numerical results and discussions

In Figure 3.3, we have shown the variation of modulus of transmission coefficients with the angle of incidence due to incidence of a set of coupled longitudinal waves propagating with speed $V_1$. We observed that the amplitude ratios $Z_5$ and $Z_6$ corresponding to the transmitted sets of coupled longitudinal waves attain their maximum values at normal incidence and then their values decrease gradually to zero at the grazing incidence. Both the amplitude ratios $Z_7$ and $Z_8$ increase with increase in angle of incidence and attain their maximum values near $\theta_0 = 70^\circ$. Thereafter, they decrease and vanish at grazing incidence. However, the maximum value of amplitude ratio $Z_6$ is greater than the maximum value of the amplitude ratio $Z_7$. Also, it is noted that the modulus of transmission coefficient $Z_5$ corresponding to the transmitted set of coupled longitudinal waves propagating with speed $V_1'$ is contributing significantly as compared to all other transmission coefficients which are very small. The variation of the coefficients $Z_6$ and $Z_7$ with angle of incidence are plotted after magnifying their original values with the factors $10^3$ and $10^5$, respectively.

![Figure 3.4: Variation of modulus of energy ratios of reflected waves with angle of incidence of a set of coupled longitudinal waves propagating with speed $V_1$](image1)

![Figure 3.5: Variation of modulus of energy ratios of transmitted waves with angle of incidence of a set of coupled longitudinal waves propagating with speed $V_1'$](image2)

Figures 3.4 and 3.5 depict the variation of energy ratios corresponding to the reflected and transmitted waves, respectively, with the angle of incidence of a set of coupled longitudinal waves propagating with speed $V_1$. It can be seen from Figure 3.4...
that the energy carried by reflected set of coupled longitudinal waves propagating with speed $V_1$ is maximum in comparison to the energy carried along with other reflected waves. In Figure 3.5, we note that the transmitted set of coupled longitudinal waves and the transmitted sets of coupled transverse waves propagating with speeds $V'_2$ and $V'_3$, respectively, have minimum energy in comparison to other transmitted waves. It has been verified that $\sum_{i=1}^{8} E_i = 1$ at each angle of incidence, showing that there is no dissipation of energy at the interface during transmission. In Figure 3.5, the dotted curve represents the energy ratio carried by the transmitted longitudinal displacement wave at the plane interface between two micropolar elastic solid half-spaces.

Figure 3.6: Variation of modulus of reflection coefficients with frequency ratio $(\omega/\omega_0)$ when a set of coupled longitudinal waves propagating with speed $V_1$ is incident at an angle $\theta_0 = 45^\circ$

Figure 3.7: Variation of modulus of transmission coefficients with frequency ratio $(\omega/\omega_0)$ when a set of coupled longitudinal waves propagating with speed $V_1$ is incident at an angle $\theta_0 = 45^\circ$

Figure 3.6 depicts the variation of the modulus of reflection coefficients with the non-dimensional frequency $(\omega/\omega_0)$ when a set of coupled longitudinal waves traveling with speed $V_1$ strikes the interface at $45^\circ$ angle of incidence. We observed from this figure that the reflection coefficients $Z_2$ and $Z_3$ are influenced with frequency in the range $1.9 \leq \omega/\omega_0 \leq 4.5$, while for higher values of $\omega/\omega_0$, these coefficients are almost independent of $\omega/\omega_0$ and bear different constant values. The reflection coefficients $Z_1$ and $Z_4$ are least affected by $\omega/\omega_0$ and remain almost constant in the entire range. The
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pattern of reflection coefficient $Z_2$ is similar to that of the coefficient $Z_3$. Both these coefficients decrease in the influential range and approaches to zero as $\omega/\omega_0$ takes larger and larger values. We have plotted the reflection coefficients $Z_1$, $Z_2$, $Z_3$ and $Z_4$ by magnifying their original values by the factors $10^3$, $10^3$ and 10, respectively.

![Figure 3.8: Variation of modulus of reflection coefficients with angle of incidence of a set of coupled transverse waves propagating with speed $V_3$](image1)

In Figure 3.7, we have plotted the modulus of transmission coefficients against frequency ratio ($\omega/\omega_0$) when a set of coupled longitudinal waves traveling with speed $V_1$ strikes the interface at 45° angle of incidence. The transmission coefficients $Z_5$ and $Z_6$ exhibit reverse behavior as $\omega/\omega_0$ takes values greater than 1.8 (i.e., $\omega/\omega_0 > 1.8$). The transmission coefficient $Z_7$ decreases in the range $1.9 \leq \omega/\omega_0 \leq 4.0$ and then becomes constant for higher values of $\omega/\omega_0$. The value of the transmission coefficient $Z_7$ remains almost constant in the entire range. The values of transmission coefficient $Z_7$ is found to be very small in comparison to the values of other transmission coefficients. We have plotted the transmission coefficient $Z_7$ by magnifying its original value by the factor $10^3$.

In Figure 3.8, we have shown the variation of modulus values of reflection coefficients versus angle of incidence of a set of coupled transverse waves propagating with speed $V_3$. We have observed from this figure that the behavior of reflection coefficients
$Z'_1$ and $Z'_2$ are almost similar. Both the reflection coefficients begin with certain finite values, then their values increase with increase in the angle of incidence till $\theta_0 = 26^\circ$ and $\theta_0 = 31^\circ$, respectively, thereafter they decrease till $\theta_0 = 47^\circ$ and $\theta_0 = 58^\circ$, respectively, after that both the reflection coefficients increase sharply. The reflection coefficient $Z'_1$ begins with its minimum value at $1^\circ$ angle of incidence then it increases with increase in the angle of incidence and achieves its maximum value $0.1183$ at $66^\circ$ angle of incidence. The behavior of the reflection coefficient $Z'_2$ is found to be monotonically decreasing in the range $1^\circ \leq \theta_0 \leq 35^\circ$, which approaches to zero at $\theta_0 = 35^\circ$ and then it increases afterwards.

Figure 3.9 shows the variation of modulus of amplitude ratios corresponding to transmitted waves with angle of incidence of a set of coupled transverse waves propagating with speed $V_3$. It can be seen that both the transmission coefficients $Z'_5$ and $Z'_6$ have similar pattern in the entire range. These transmission coefficients increase with increase in the angle of incidence. The value of reflection coefficient $Z'_2$ corresponding to the transmitted set of coupled transverse waves propagating with speed $V'_2$ is found to be dominant among all other transmitted waves. The transmission coefficient $Z'_6$ is very small and so, we have plotted it by magnifying its original value with the factor.
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of 10³. The transmission coefficient \( Z'_3 \) begins with its maximum value at 1° angle of incidence, then it decreases till \( \theta_0 = 64° \) and afterwards it increases slightly.

![Figure 3.12: Variation of modulus of reflection coefficients with frequency ratio \( (\omega/\omega_0) \) when a set of coupled transverse waves propagating with speed \( V_3 \) is incident at an angle \( \theta_0 = 45° \)](image)

![Figure 3.13: Variation of modulus of transmission coefficients with frequency ratio \( (\omega/\omega_0) \) when a set of coupled transverse waves propagating with speed \( V_3 \) is incident at an angle \( \theta_0 = 45° \)](image)

Figures 3.10 and 3.11 depict the variation of modulus of energy ratios of reflected and transmitted waves, respectively, against the angle of incidence of a set of coupled transverse waves propagating with speed \( V_3 \). It can be seen from these figures that the maximum amount of energy of incident set of coupled transverse waves is carried by transmitted set of coupled transverse waves traveling with speed \( V'_3 \) and only a small amount of energy is carried by all other reflected and transmitted waves in this case. Also, it has been verified that \( \sum_{i=1}^{8} E_i = 1 \) at each angle of incidence lying in the range \( 0° < \theta_0 \leq 60° \).

In Figure 3.12, we have shown the variation of modulus of reflection coefficients with frequency ratio \( (\omega/\omega_0) \) when a set of coupled transverse waves propagating with speed \( V_3 \) is made incident at \( \theta_0 = 45° \). It can be seen that the reflection coefficient \( Z'_1 \) starts with some non-zero value at \( \omega/\omega_0 = 1.5 \) and then its value increases to attain its maximum value at \( \omega/\omega_0 = 2.2 \), beyond which it decreases smoothly. Here, the reflection coefficient \( Z'_2 \) is monotonically decreasing in nature, while the reflection coefficient
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Z'₃ possesses reverse behavior to Z'₂, in the entire range. The reflection coefficient Z'₄ decreases in the range 1.5 ≤ ω/ω₀ ≤ 2 and then it increases very frequently.

Figure 3.13 shows the variation of modulus of transmission coefficients with frequency ratio when a set of coupled transverse waves traveling with speed V₃ is incident at an angle θ₀ = 45°. We observed from this figure that the value of transmission coefficient Z'₄ increases with increase in the frequency ratio (ω/ω₀). The transmission coefficient Z'₆ begins with some non-zero value and it increases sharply in the range 1.5 ≤ ω/ω₀ ≤ 2.3, then it decreases gradually with ω/ω₀. The transmission coefficients Z'₇ and Z'₈ exhibit reverse behavior to each other as ω/ω₀ takes value greater than 1.5. The values of transmission coefficients Z'₆ and Z'₈ are small in comparison to the value of transmission coefficient Z'₄. We have plotted the coefficients Z'₆ and Z'₈ after magnifying their original values 10 times.

3.6 Conclusions

In this chapter, reflection and refraction phenomena of elastic waves at a plane interface between two different electro-microelastic solid half-spaces has been investigated. By neglecting the electric and microstretch effects from both the electro-microelastic solid half-spaces in perfect contact, the problem of reflection and refraction of elastic waves at a plane interface between two different micropolar elastic solid half-spaces is recovered. It is found that the reflected and refracted sets of coupled transverse waves are not influenced by microstretch and electric properties. These waves are common at electro-microelastic/electro-microelastic interface (say I₁) and micropolar/micropolar interface (say I₂). There appears two new waves, namely a reflected set of coupled longitudinal waves propagating at speed V₂ and a refracted set of coupled longitudinal waves propagating at speed V₂ during reflection and refraction of elastic waves at interface I₁, which are not encountered in the case of interface I₂. However, the amplitude ratios of these new waves are very small in comparison to amplitude ratios of other reflected and refracted waves. The variation of the amplitude ratios corresponding to common reflected and refracted waves with angle of incidence at a fixed frequency are found to be similar, when looked in case of incidence of a set of coupled longitudinal waves and in case of incidence of a set of coupled transverse waves separately. Interestingly, the variation of amplitude ratios with angle of incidence, corresponding
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to the reflected and refracted sets of coupled longitudinal waves at speeds $V_1$ and $V_1'$, respectively, are found to be the same at the interface $I_1$ as were that of reflected and refracted longitudinal displacement waves at the interface $I_2$. A natural question arises that how does the energy balance relation is satisfied then? or in other words, when there is no significant difference in the amplitude ratios of common reflected/refracted waves and also two more amplitude ratios corresponding to the new reflected and refracted waves are appearing at the interface $I_1$ that are not appearing at the interface $I_2$, then how does the energy relation balances at the interface $I_1$ and $I_2$? To answer this question, we have then computed the energies of various reflected and refracted waves at the interface $I_1$ as well as interface $I_2$. We found that (i) there is a significant effect of electric and microstretch properties on the energy ratio of refracted longitudinal displacement wave and (ii) the energies carried by the new reflected and refracted waves are very small (of order of $10^{-6}$ and $10^{-3}$, respectively), when a set of coupled longitudinal waves is made incident at the interface $I_1$ (see Figures 3.4 and 3.5). It is seen that the microstretch and electric effects are responsible to lower down the energy of refracted longitudinal displacement wave. This difference is being covered by the energies of new waves. This is how the balance of energy relation could be satisfied. Further, it is also to be mentioned here that when a set of coupled transverse waves is made incident at the interfaces $I_1$ and $I_2$ then no significant effect of electric and microstretch properties is noticed neither on the amplitude ratios nor on the energy ratios of various reflected and refracted waves. In this case, it can be seen from Figures 3.10 and 3.11 that the energy carried by the new reflected and refracted waves are very very small (of order of $10^{-11}$ and $10^{-8}$, respectively) and consequently, the new waves have not created any problem in satisfying the energy balance relation. We conclude that

(1) When a set of coupled longitudinal (transverse) waves is incident normally, the reflection and transmission of only the set of coupled longitudinal (transverse) waves take place.

(2) At grazing incidence of a set of coupled longitudinal waves, no reflection and transmission of waves take place and the same wave propagates along the interface.

(3) The angle of incidence $\theta_0 = 67^\circ$ is found to be the critical angle when a set of coupled transverse waves is incident.