APPENDIX - II

LESSON PLAN I

Topic: INTRODUCTION TO RATIONAL NUMBERS

1. **Entry Behaviour:**
   
   (i) It is assumed that students know the concept of natural numbers, whole numbers integers and fractional numbers (or fractions)
   
   (ii) The students know proper important fractions and equivalent fraction

**Instructional objectives:**

After instructions are over students will able to

- define rational number in their own words
- distinguish rational number with other number systems viz. whole numbers, integers and fractional numbers
- write numerator and denominator of a rational number
- express the equivalent form of the given rational number
- convert rational number in its lowest form
- express rational number in its standard form.

**P.K Testing:**

1. If \( \frac{p}{q} \) is a fraction, \( p = -q = \) ________________
2. reduce \( \frac{36}{72} \) into lowest terms
3. write equivalent form of \( \frac{3}{5} \) having denominator 20

**Presentation of new material:**

'you have studied in the previous class the natural number, whole number, integers and fractional number. In this topic we introduce a new number system i.e the system of rational numbers. We shall build up these numbers over fractional numbers”

<table>
<thead>
<tr>
<th>‘numbers’</th>
<th>( \frac{p}{q} ) = p- numerator</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>q - dominator</td>
</tr>
<tr>
<td></td>
<td>p &amp; q - natural numbers</td>
</tr>
<tr>
<td></td>
<td>p&lt;q ( \frac{p}{q} ) - proper fraction</td>
</tr>
</tbody>
</table>

'Let us recall fractions you know that fractional number \( \frac{p}{q} \)
p is the numerator and q is the denominator and p and q are natural numbers” Its
numerator is smaller than denominator, fraction is proper. If numerator is greater then fraction is improper. Improper fraction can be expressed in mixed form

"If \( \frac{p}{q} \) has common factor then \( \frac{p}{q} \) is in its simplest form.
If it has common factor say t, then it can be reduced

Need for rational no's
Teacher explains,"In class VI, you have learnt to solve simple equation in which we get either integer or fraction as a solution." "we need to extend our number system as \( \frac{-3}{2} \) is neither fraction nor integer"

"A rational number is number of the form \( \frac{p}{q} \), where \( \frac{p}{q} \) and q are integers, 
q ≠ 0
"So by definition, if there is 0 if is not a rational no. where as 0 is a rational no."

```
| numerator is smaller than denominator, fraction is proper. | p > q \( \frac{p}{q} \) - improper fraction e.g 10/7 = 1\( \frac{3}{7} \) = mixed number |
| If numerator is greater then fraction is improper. Improper fraction can be expressed in mixed form” | Common factor -1 P/q - simplest form |
| “If \( \frac{p}{q} \) has common factor then \( \frac{p}{q} \) is in its simplest form. |
| If it has common factor say t, then it can be reduced” |
| Need for rational no's Teacher explains,"In class VI, you have learnt to solve simple equation in which we get either integer or fraction as a solution.” "we need to extend our number system as \( \frac{-3}{2} \) is neither fraction nor integer” |
| "A rational number is number of the form \( \frac{p}{q} \), where \( \frac{p}{q} \) and q are integers, q ≠ 0 | p, p and q are integers but q ≠ o |
| "So by definition, if there is 0 if is not a rational no. where as 0 is a rational no.” |
```

2x-4 = 0
2x = 4 => x = \( \frac{4}{2} \) = 2
2x -3 = 0 ,2 is integer
x = \( \frac{3}{2} \) (fraction)
2x + 3 = 0
2x = -3
x = \( \frac{-3}{2} \)

2x-4 = 0
2x = 4 => x = \( \frac{4}{2} \) = 2
2x -3 = 0 ,2 is integer
x = \( \frac{3}{2} \) (fraction)
2x + 3 = 0
2x = -3
x = \( \frac{-3}{2} \)

\( \frac{p}{q} \), p and q are integers but q ≠ 0
\( \frac{p}{a} \) p = numerator
q = denominator
\( \frac{5}{97} \) ≠ rational numbers
\( \frac{0}{0} \) = rational numbers

For e.g \( \frac{-3}{16} \)
\( \frac{4}{100} \)
\( \frac{1}{1} \)

All fraction and integers are
Problem-1 write the numerator and denominator of rational number.

<table>
<thead>
<tr>
<th></th>
<th>Rational numbers.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>( \frac{12}{23} ) Numerator = 12  Denominator = 23</td>
</tr>
</tbody>
</table>

2. \( \frac{-27}{53} \) Numerator = -27  Denominator = 53

3. \( \frac{-99}{-1000} \) Numerator = -99  Denominator = -1000

4. \( \frac{-67}{-167} \) Numerator = -67  Denominator = -167

**Problem 2 : Write the rational number whose numerator and denominator are**

<p>| | |</p>
<table>
<thead>
<tr>
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<th></th>
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</thead>
</table>
| 1. | 5-49 and 55-9  
N = 5-49 = - 44  
D = 55-9 = 46  
Rational no.  = \( \frac{-44}{46} \) |

2. 5 x 3 and 16 ÷ 8  
N = 5 x 3 = 15  
D = 16 ÷ 8 = 2  
Rational no.  = \( \frac{15}{2} \) |

Equivalent from of rational no  
Like wise in fraction,  
\( \frac{mxp}{mxq} \) equivalent to \( \frac{p}{q} \)  
p/q = 3/5
Rational equivalent to $p/q$

For eg. $\frac{3}{19}$

HCF = 1

$\frac{15}{70} = \frac{3 \times 5}{2 \times 5 \times 7} = \frac{3}{2 \times 7} = \frac{3}{14}$

1. Take HCF = $m$
2. $p + m = p'$
3. $p + m = q'$
4. $p'/q'$ = lowest terms

**PROBLEM 3:**

Write rational number in the lowest terms.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$\frac{-36}{180}$</td>
</tr>
<tr>
<td>2.</td>
<td>$\frac{-91}{364}$</td>
</tr>
<tr>
<td>3.</td>
<td>$\frac{24}{64}$</td>
</tr>
</tbody>
</table>

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$\frac{-36}{18} = \frac{-2 \times 3 \times 2 \times 3}{3 \times 6 \times 3 \times 2 \times 5} = \frac{-6 \times 6}{3 \times 6 \times 6 \times 5} = \frac{-1}{15}$</td>
</tr>
<tr>
<td>2.</td>
<td>$\frac{-91}{364} = \frac{13 \times 7}{4 \times 13 \times 7} = \frac{1}{4}$</td>
</tr>
<tr>
<td>3.</td>
<td>$\frac{2 \times 3 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2} = \frac{3}{2 \times 2 \times 2} = \frac{3}{8}$</td>
</tr>
</tbody>
</table>
### Problem 4

Express \( \frac{3}{4} \) as a rational number with

<table>
<thead>
<tr>
<th>a. Denominator = 36</th>
<th>1. ( \frac{36}{4} = 9 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Divide 36 by 4 multiply numerator and dynamotor by 4.</td>
<td>( \frac{1 \times 9}{4 \times 9} = 9 )</td>
</tr>
<tr>
<td>b. denominator = 100 divide - 100 4 Multiply numerator and denominator by - 25</td>
<td>2. ( \frac{-100}{-4} = -25 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Write ( \frac{2}{5} ) a = -750</td>
</tr>
<tr>
<td>&quot;Divide -750 by 2 multiply numerator and dynamotor by -375&quot;</td>
</tr>
<tr>
<td>&quot;what do you get?&quot;</td>
</tr>
<tr>
<td>(b). 500 what will you get on deciding 500 by 2?&quot; &quot;What will you get?&quot;</td>
</tr>
<tr>
<td>Problem -6 fill in the blanks</td>
</tr>
<tr>
<td>(i) ( \frac{2}{3} = \boxed{135} )</td>
</tr>
<tr>
<td>&quot;Divide 135 by 3.&quot;</td>
</tr>
<tr>
<td>&quot;What do you get?&quot;</td>
</tr>
<tr>
<td>&quot;it means both numerator and denominator have to and milt plied by 45&quot;</td>
</tr>
<tr>
<td>(ii) ( \boxed{4} = \frac{90}{120} )</td>
</tr>
<tr>
<td>&quot;You can also do with other method insert x in the box by cross multiplication find the value of x&quot;</td>
</tr>
<tr>
<td>(iii) ( \frac{5}{2} = \frac{20}{216} )</td>
</tr>
<tr>
<td>&quot;Insert x in the box and solve form by cross multiplication&quot;</td>
</tr>
</tbody>
</table>
Standard form of rational number

“A rational number in the lowest form is said to be in the standard form, if the denominator is positive. First make the denominator positive and then reduce it into lowest terms if required.” “This is explained will the help of a examples”

| $x=2\times3\times3\times3$ | Standard form of number  
1. If should be in lowest term  
2. denominator should be positive  
for e.g $\frac{44}{-428}$  
$\frac{44 \times -1}{-428 \times -1} = \frac{44}{428}$  
$\frac{-2 \times 2 \times 11}{2 \times 2 \times 107}$  
$\frac{-11}{107}$  

$\frac{-144}{-144 \times -1} = \frac{144}{144}$  
$\frac{-504}{-504 \times -1}$  

2. $\frac{504}{144} = \frac{2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}{2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7} = \frac{2}{7}$

Practice:
1. Write the rational number in standard form
(1) $\frac{-165}{325}$  
(2) $\frac{168}{-396}$

Recapitulation

(I) A number of the form $\frac{p}{q}$, where $p$ and $q$ are integers and $q \neq 0$, is called a rational number

(II) A rational number is said to be in lowest form, if HCF of $P$ and $q$ is .

(III) A rational number $\frac{p}{q}$ is said to be in the standard form
(I) $\frac{p}{q}$ is in lowest form

(II) $q > 0$

(IV) Every integer and every fraction is a rational number.

(V) If $\frac{p}{q}$ is a rational number and $m$ a non-zero integer

Then $\frac{p}{q} = \frac{p \times m}{q \times m}$

And $\frac{p}{q} = \frac{p \div m}{q \div m}$
LESSON PLAN 2

Topic- Rational Number (contd.)

Entry Behavior
(I) It is assumed now that the students know the concept of rational numbers.
(II) They know how to compare two fractions and integers.
(III) They know how to represent fractions and integers on a number line.
(IV) They know absolute value of an integer.

Instructional objectives:
After the instructional treatment is over, the students will be able to represent the rational number on a number line, compare two rational numbers with unequal denominators, write the absolute value of a rational number.

P. K. Testing
Q1. Which following fraction is the greatest?
\[
\frac{15}{12}, \frac{5}{12}, \frac{1}{12}
\]

Q3. What does p and q represent on a number line?

Presentation of new material:
"We will learn to represent the rational number on a number line, comparing the rational number absolute value of a rational number.

<table>
<thead>
<tr>
<th>Teacher's activity</th>
<th>B.B. work</th>
</tr>
</thead>
<tbody>
<tr>
<td>Representation of a rational number on a number line</td>
<td></td>
</tr>
<tr>
<td>&quot; Any rational number may be represented on a number line, from class VI, recall use method of bisecting a line segment we can divide line segment into equal parts&quot;</td>
<td></td>
</tr>
<tr>
<td>&quot;In the line OA=1, OB=2 OA'=1, OB'=-2</td>
<td></td>
</tr>
<tr>
<td>If we bisect OC</td>
<td></td>
</tr>
<tr>
<td>OD= \frac{1}{4}</td>
<td></td>
</tr>
</tbody>
</table>

-2 -1 -\frac{1}{2} -\frac{1}{4} \frac{1}{4} \frac{1}{2} 1 2

B-1 A-1 C-1 D-1 0 D C A B
In a same way \( OC = \frac{-1}{2} \)
\[ OD = \frac{-1}{4} \]

If we wish to represent \( \frac{12}{7} \) on a number line choose \( P \) corresponding to integer 12. Divide into 7 equal parts
\[ OQ = \frac{12}{7} \text{ i.e. } \frac{1}{7} \text{ of } 12 \]

**Problem 1** Represent the rational number on a number line

<table>
<thead>
<tr>
<th>(I) ( \frac{3}{4} )</th>
<th>( \frac{3}{4} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Again choose ( P ) corresponding to 3. Divide ( OP ) into 4 equal parts ( OQ = \frac{1}{4} \text{ th of } 3 = \frac{3}{4} )</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(II) ( \frac{5}{8} )</th>
<th>( \frac{5}{8} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher asks “what will you choose first”?</td>
<td></td>
</tr>
<tr>
<td>“In have many parts will you divide it”?</td>
<td></td>
</tr>
<tr>
<td>“which part will you mark ( \frac{5}{8} )”?</td>
<td></td>
</tr>
</tbody>
</table>

**Problem 2** Let \( O, P \) and \( Z \) represent the numbers 0.3 and -5 respectively on the number line Mark \( Q, R \) and \( S \) between \( O \) and \( P \) such that \( OQ = QR = RS = SP \) what are the rational numbers represented by the pounds \( Q, R \) and \( S \)? Now choose a pond-\( T \) between \( Z \) and \( O \) so that \( ZT = TO \) which rational number does \( T \) represent?

<table>
<thead>
<tr>
<th>“In how many equal part ( OP ) is divided?” ( OQ = \frac{1}{4} \text{ of } 3 )</th>
<th>( 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( OR = \frac{1}{2} \text{ of } 3 )</td>
<td></td>
</tr>
<tr>
<td>Each part is divided into ( \frac{3}{4} )</td>
<td></td>
</tr>
<tr>
<td>( OZ = -5 )</td>
<td></td>
</tr>
</tbody>
</table>

\[ OQ = \frac{3}{4} \]
\[ OR = \frac{3}{4} + \frac{3}{4} = \frac{6}{4} = \frac{3}{2} \]
Comparison of rational Numbers

To compare any two rational number by writing equivalent form of the given rational numbers having same denominator. If we have same denominator then we can compare two numerators.

"Find LCM of 12,8"
"write the equivalent form of \( \frac{-7}{12} \) and \( \frac{-5}{8} \)
"Out of -14 and -15 which in greater?"

3. \( \frac{4}{3} \) First write \( \frac{4}{3} \) in a standard form
"Find LCM of 3,7"
"What is equivalent form of \( \frac{4}{3} \)?"
"What is equivalent form of \( \frac{-8}{7} \)
"which is greater -28 or -24?"

4. \( \frac{-9}{10} \) and \( \frac{8}{9} \)
"write \( \frac{-9}{-10} \) in standard from "
"What is the Lem of 10&9?"

Which of the two rational numbers is greater?

1. \( \frac{-5}{8} \) and \( \frac{-3}{4} \)
HCF of 8,4=8
\( -5 \times \frac{1}{8} = -5 \)
\( 8 \times \frac{1}{8} = 8 \)
\( -3 \times \frac{2}{4} = -6 \)
\( 4 \times \frac{2}{4} = 8 \)
\( -5 \times \frac{6}{8} = -5 \)
\( 8 \times \frac{8}{8} = 8 \)
\( -7 \times \frac{5}{8} = -7 \)
\( 5 \times \frac{5}{8} = 5 \)

2. \( \frac{12}{8} \)\( \frac{12}{12} \\frac{12}{8} \)
LCM OF 12,8=24
\( -7 \times \frac{2}{12} = -14 \)
\( 12 \times \frac{2}{12} = 24 \)
\( -5 \times \frac{3}{8} = -15 \)
\( 8 \times \frac{3}{24} = 24 \)
\( -14 \times \frac{15}{24} = -24 \)

4. \( \frac{-9}{-10} \) and \( \frac{8}{9} \)
\( -9 \times \frac{9}{9} = -9 \)
\( -10 \times \frac{10}{10} = 10 \)
\( 9 \times \frac{8}{10} = 9 \)
\( 10 \times \frac{9}{9} = 10 \)

LCM of 10 & 9=90
What is the equivalent from of \( \frac{9}{10} \) ?

\[
\frac{9}{10} = \frac{9 \times 9}{10 \times 9} = \frac{81}{90}
\]

What is the equivalent from of \( \frac{8}{9} \)?

\[
\frac{8}{9} \times 10 = \frac{80}{90}
\]

81 > 80

90 > 90

5. \( \frac{-3}{8} \text{ and } \frac{-7}{19} \)

Another way cross multiply and then compare. You can adopt any method.

6. \( \frac{3}{5} \text{ and } \frac{7}{11} \)

Cross multiply. "See which is greater?"

<table>
<thead>
<tr>
<th>Absolute Value of rational number</th>
</tr>
</thead>
<tbody>
<tr>
<td>You know that absolute Value of an integer is an integer</td>
</tr>
<tr>
<td>For integers, the absolute value of a rational no. is also a rational no.</td>
</tr>
<tr>
<td>&quot;It can be shown with the help of examples&quot;</td>
</tr>
</tbody>
</table>

Absolute value of rational number

For integer

\[ | -9 | = 9 \]
\[ | 101 | = 101 \]
\[ | 0 | = 0 \]
\[ | \frac{p}{q} | = \frac{|p|}{q} \]
\[ | \frac{q}{p} | = \frac{|q|}{|p|} \]

Write absolute values of

1. \( \frac{-4}{11} = \frac{|-4|}{11} = \frac{4}{11} \)
2. \( \frac{-3}{7} = \frac{|-3|}{7} = \frac{3}{7} \)
### Problem 3.

Fill in the blanks $>, =,$ or $<$

#### (I)

\[
\begin{array}{c}
\left( \frac{-5}{7} \right) \quad \square \quad \left( \frac{6}{13} \right)
\end{array}
\]

"write the absolute value of both cross multiply and then compare"

We can compare the result when absolute values are not taken

\[
\begin{array}{c}
\left| \frac{-5}{7} \right| \quad \square \quad \left| \frac{6}{13} \right|
\end{array}
\]

\[
\begin{array}{c}
6 \quad \square \quad 6 \\
13 \quad \square \quad 13
\end{array}
\]

\[
\begin{array}{c}
65 > 42
\end{array}
\]

\[
\begin{array}{c}
5 \quad \square \quad 6 \\
7 \quad \square \quad 13
\end{array}
\]

\[
\begin{array}{c}
-65 < 42
\end{array}
\]

\[
\begin{array}{c}
-5 \quad \square \quad 6 \\
7 \quad \square \quad 13
\end{array}
\]

We observe that in this case we obtained same results

\[
168 = -168 \\
7 \quad \square \quad 8
\]

#### (II)

\[
\begin{array}{c}
\left( \frac{-7}{8} \right) \quad \square \quad \left( \frac{21}{-24} \right)
\end{array}
\]

"Write the absolute values of both rational number cross multiply and then compare"

"We can compared the results by not taking absolute values"

\[
\begin{array}{c}
\left| \frac{-7}{8} \right| \quad \square \quad \left| \frac{21}{-24} \right|
\end{array}
\]

\[
\begin{array}{c}
21 \quad \square \quad 21 \\
-24 \quad \square \quad 24
\end{array}
\]

\[
\begin{array}{c}
168 = 168 \\
7 \quad \square \quad 8
\end{array}
\]

\[
\begin{array}{c}
\left( \frac{-7}{8} \right) \quad \square \quad \left( \frac{21}{-24} \right)
\end{array}
\]

\[
\begin{array}{c}
-7 \quad \square \quad 21 \\
8 \quad \square \quad -24
\end{array}
\]

\[
\begin{array}{c}
-168 = -168 \\
-7 \quad \square \quad 21 \\
8 \quad \square \quad -24
\end{array}
\]
(III) \( \left( \frac{-4}{5} \right) \, \square \, \left( \frac{-5}{6} \right) \)

Take absolute values of both rational numbers. Cross multiply and then compare. Compare the result by not taking absolute values.

\[
\left( \frac{-4}{5} \right) \, \square \, \left( \frac{-5}{6} \right)
\]

Cross multiply and then compare we observe in this case different results.

\[
\left| \frac{-4}{5} \right| \times \left| \frac{-5}{6} \right| = \left( \frac{-4}{5} \right) \times \left( \frac{-5}{6} \right)
\]

- \( \frac{-4}{5} \times \frac{-5}{6} = \frac{4}{5} \)
- \( \frac{4}{5} \times \frac{5}{6} = \frac{24}{25} \)
- \( \frac{4}{5} < \frac{5}{6} \)
- \( \left( \frac{-4}{5} \right) < \left( \frac{-5}{6} \right) \)
- \( \frac{-4}{5} \times \frac{5}{6} = \frac{-20}{30} \)
- \( \frac{-4}{5} > \frac{-5}{6} \)

**Problem 4:**

<table>
<thead>
<tr>
<th>Write True or False</th>
<th>Standard form of</th>
</tr>
</thead>
<tbody>
<tr>
<td>(I) The rational number ( \frac{7}{-4} ) lies to the right of zero on the number line</td>
<td>False on the left</td>
</tr>
<tr>
<td>(II) The rational number ( \frac{-8}{-3} ) lies neither to the right nor to the left of the number line</td>
<td>False it will lie towards the right standard form of</td>
</tr>
<tr>
<td>(III) if ( \frac{p}{q} &lt; \frac{r}{s} ), then ( \frac{p}{q} &lt; \frac{r}{s} )</td>
<td>False e.g if</td>
</tr>
</tbody>
</table>

\[
\frac{p}{q} = \frac{-1}{2}, \quad \frac{r}{s} = \frac{-1}{4}
\]

\[
\frac{p}{q} = \frac{-1}{2}, \quad \frac{r}{s} = \frac{-1}{4}
\]

\[
\frac{1}{2} > \frac{1}{4}
\]

\[
\frac{1}{2} > \frac{1}{4}
\]
(IV) if $|x| = |y|$, then $x = y$

| $x = -17$  
$y = 17$ | False $(x) = (-17) = 17$  
$(Y) = (17) = 17$  
$17 \neq 17$ |

(V) if $|x| = 0$, then $x = 0$

| True $x=0$  
$0=0$ |

Practice

1. Represent $\frac{3}{16}$ on a number line

2. Compare $\frac{-7}{21}$ and $\frac{5}{-8}$

3. Absolute value of $\frac{-8}{3}$

Recapitulation:

1. if $\frac{p}{q}$ and $\frac{r}{s}$ are two rational numbers with $q$ and $s$ positive integers then $\frac{p}{q} > \frac{r}{s}$ if $s \times p > q \times r$

2. absolute value of rational number is >0

3. absolute value of rational number is greater than or equal to the number itself.
LESSON PLAN 3

**Topic**: Additional of rational numbers

**Entry Behavior**: it is assumed that the students know
1. About the addition of natural numbers, fractions and integers.
2. About the properties of addition of integers.

**Instructional objectives**: After the instructions the students will able to are over.
1. Add rational numbers
2. prove properties of addition of rational numbers.
3. simplify the complicated expressions.

**P.K. testing**

1. \( \frac{1}{2} + \frac{1}{5} \)
2. \( \frac{2}{5} + 0 + \frac{3}{10} \)

**Presentation of new material**

“You know the addition of natural numbers and fractions we will apply this knowledge to add rational numbers”

<table>
<thead>
<tr>
<th>Teacher activity and Teaching point</th>
<th>B.B.Work</th>
</tr>
</thead>
<tbody>
<tr>
<td>We will solve the sums be adopting the working rule.</td>
<td>Working rule ( \frac{p}{q} + \frac{r}{s} = \frac{p \times s + r \times q}{q \times s} )</td>
</tr>
<tr>
<td>(I) ( \frac{-23}{28} ) and ( \frac{5}{-28} ) Take LCM</td>
<td>( \frac{-23}{28} + \frac{5}{-28} = \frac{-23 + 5}{28} ) ( \frac{-23}{28} + \frac{5}{28} = \frac{-23 - 5}{28} ) ( \frac{-23 + (-5)}{28} = \frac{-28}{28} = -1 )</td>
</tr>
<tr>
<td>(II) ( \frac{-7}{9} + \frac{3}{4} ) Take LCM and divide LCM by denominator &amp; multiply the quotient with numerator.</td>
<td>( \frac{-7}{9} + \frac{3}{4} = \frac{28 + 36}{36} ) ( \frac{8}{36} = \frac{2}{9} )</td>
</tr>
<tr>
<td>(III) ( \frac{-8}{19} + \frac{-2}{57} )</td>
<td>(III) ( \frac{-8}{19} + \frac{-2}{57} )</td>
</tr>
</tbody>
</table>

20
Properties of addition

“property 1” if $x$ and $y$ are two rational numbers then $x + y = y + x$

We will prove by taking examples

$$x + y = y + x$$

**Example:**

$$-5 + \frac{-6}{13} = \frac{-65 + (-66)}{143}$$

**LHS:**

$$\frac{-5}{11} \times \frac{-6}{13} = \frac{-65}{143}$$

**RHS:**

$$\frac{-6}{13} \times \frac{-5}{11} = \frac{-66 + (-65)}{143}$$

$LHS = RHS$

(ii) We take up another example

“Again solve for LHS and RHS”

$$-4 + \frac{-7}{9} = \frac{-4 + (-7)}{9}$$

**LHS:**

$$-4 + \frac{-7}{9} = \frac{-4}{9} - 7$$

$$\frac{-7}{9} \div \frac{-4}{9} = \frac{-43}{9}$$

**RHS:**

$$-4 + \frac{-7}{9} = \frac{-4}{9} - 7$$

$$\frac{-36 - 7}{9} = \frac{-43}{9}$$

$LHS = RHS$
### Property II:

If \( x, y \) and \( z \) are there rational numbers then \( (x+y)+z=x+(y+z) \)

"we will prove with examples"

1. \[ \frac{3}{4} + \left( \frac{-5}{6} + \frac{7}{8} \right) = \left( \frac{3}{4} + \frac{-5}{6} \right) + \frac{7}{8} \]

<table>
<thead>
<tr>
<th>LHS</th>
<th>RHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \frac{3}{4} + \left( \frac{-5}{6} + \frac{7}{8} \right) ]</td>
<td>[ \left( \frac{3}{4} + \frac{-5}{6} \right) + \frac{7}{8} ]</td>
</tr>
<tr>
<td>[ \frac{3}{4} + \left( \frac{-5}{6} + \frac{7}{8} \right) = \frac{3}{4} + \left( \frac{-5 + 21}{24} \right) ]</td>
<td>[ \left( \frac{3}{4} + \frac{-5}{6} \right) + \frac{7}{8} = \frac{3}{4} + \left( \frac{24}{24} \right) ]</td>
</tr>
<tr>
<td>[ = \frac{3}{4} \left( \frac{1}{24} \right) ]</td>
<td>[ = \frac{18 + 1}{24} = \frac{19}{24} ]</td>
</tr>
</tbody>
</table>

**Verify**

\[ \frac{3}{4} + \left( \frac{-5}{6} + \frac{7}{8} \right) = \left( \frac{3}{4} + \frac{-5}{6} \right) + \frac{7}{8} \]

### Property III:

If \( x \) is a rational number then \( o+x=x+o=x \)

\( x+(-x)=o \)
Property IV: Is x is a rational number then \(-x\) is a rational number such that \(-x+(-x)=(-x)+x=0\). \(-x\) is called the negative of x. 

We will simplify the expression by using the above properties.

Simplify

\[
\begin{align*}
\begin{array}{cccccc}
1 & 2 & 8 & -11 & 4 & -2 \\
\frac{5}{3} & \frac{5}{5} & \frac{5}{3} & \frac{5}{3} \\
\end{array}
\end{align*}
\]

(I) First take LCM

(II) Divide LCM by each denominator

(III) Multiply numerator by corresponding quotient

\[
\begin{align*}
\begin{array}{cccccc}
1.2 & 8 & -11 & 4 & 2 \\
\frac{5}{3} & \frac{15}{5} & \frac{5}{3} & \frac{5}{3} \\
\end{array}
\end{align*}
\]

\[
\begin{align*}
6+40+(-11)+12+(-10) &= 15 \\
58-11-10 &= 37 \\
58-21 &= 15 \\
\end{align*}
\]

\[
\begin{align*}
\begin{array}{cccccc}
2 & 4 & 0 & -8 & -13 & 17 \\
\frac{7}{9} & \frac{7}{7} & \frac{7}{21} \\
\end{array}
\end{align*}
\]

Teacher asks, "what will you first?"

Teacher explains, "Divide LCM by denominator and multiply numerator by corresponding quotient."

\[
\begin{align*}
\begin{array}{cccccc}
2 & 4 & 0 & -8 & -13 & 17 \\
\frac{7}{9} & \frac{7}{7} & \frac{7}{21} \\
\end{array}
\end{align*}
\]

\[
\begin{align*}
36+0+(-56)+(-117)+51 &= 63 \\
36-56-117-51 &= 63 \\
87-173 &= -86 \\
\end{align*}
\]

**Practice**

Simplify

\[
\begin{align*}
\frac{3}{7} + 0 + \frac{-16}{21} + \frac{-1}{3} \\
\end{align*}
\]

**Recapitulation**

If x and y are rational numbers, than

I. \(x+y\) is rational number.

II. \(x+0=0+x=x\)

III. \(x+y=y+x\)

IV. For rational number x, y and z

\((x+y)+z=x+(y+z)\)
LESSON PLAN – 4

**Topic:** To find difference between the rational numbers

**Entry Behavior:** It is assumed that students know

(i) subtraction of fractions
(ii) **subtraction of integers:** if x and y are two integers, then subtracting y from x is the same as adding –y to x, i.e. \( x - y = x + (-y) \)

**Instructional Objectives:** After the instructions are over the students will be able to

- subtract two rational numbers
- prove the properties of subtraction
- calculate the other rational number when sum of two rational numbers and one rational number is given.
- calculate the other rational number when difference of two rational numbers and one rational number is given.

**P.K. Testing**

Q1. Add \( \frac{1}{2} + \frac{3}{5} \)

Q2. Fill in the blanks.
   
   (i) \( \frac{-2}{3} + \frac{-5}{7} = \frac{-5}{7} + \)  
   
   (ii) \( \frac{-1}{3} + \left( \frac{4}{9} \right) - \frac{8}{13} = \left( \frac{-4}{9} \right) + \frac{-8}{13} \)
   
   (iii) \( -\left( \frac{-8}{5} \right) = \)  
   
   (iv) \( \frac{3}{7} + \)  

**Presentation of new material:**

"As we have done addition of rational numbers, we will find the difference of rational numbers and solve some more problems to continue learning"

<table>
<thead>
<tr>
<th>Teacher activity and Teaching point</th>
<th>B.B. Work</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem-1. Find the difference:</td>
<td>(1) ( \frac{12}{15} - \frac{12}{25} )</td>
</tr>
<tr>
<td>(1) ( \frac{12}{15} - \frac{12}{25} )</td>
<td>(1) ( \frac{12}{15} - \frac{12}{25} )</td>
</tr>
</tbody>
</table>

24
"Take LCM"  
"Divide LCM by denominator and then multiply the quotient with numerator."

(2) \( \frac{5 \times -8}{63 \times 21} \)

"Find LCM"

3. \( \frac{-6 \times -7}{13 \times 15} \)

\[
\begin{align*}
\text{Problem -2.} & \quad \text{The sum of two rational numbers is -8, one of the numbers is } \frac{-15}{7}, \text{ find the other.} \\
\text{"What is given?"} & \\
\text{"What is to be found?"} & \\
\text{"Solve for x"} & \\
\text{\begin{align*}
\text{Sum} &= -8 \\
\text{One number} &= \frac{15}{7} \\
\text{Let other number} &= x \\
\text{According to the sum} & \\
-\frac{15}{7} + x &= -8 \\
x &= -8 - \left(\frac{-15}{7}\right) \\
&= -8 + \frac{15}{7} \\
&= -\frac{56 + 15}{7} \\
&= -\frac{41}{7} \\
\end{align*}} \\
\text{Sum} &= \frac{5}{9} \\
\text{One number} &= -\frac{7}{8} \\
\text{Other number} &= x \\
\text{According to the sum} &
\end{align*}
\]
"solve for x"

Problem-4: what number subtracted from \( \frac{26}{33} \) so as to get \( \frac{-5}{11} \)

"what is given?"

"what is to be found?"

"solve for x"

"multiply both sides by -1"

Problem -5 find the rational number in the following pair and examine if they are equal.

\[
\left( \begin{array}{c}
-8 \\
9
\end{array} \right) - \left( \begin{array}{c}
-9 \\
4
\end{array} \right) - \left( \begin{array}{c}
-4 \\
9
\end{array} \right) - \left( \begin{array}{c}
11 \\
4
\end{array} \right)
\]

"solve one by one carefully"
"compare the two rational numbers"

No they are not equal

Problem -6 fill in the blanks

1. \[-\frac{4}{13} - \frac{3}{26} = \]

2. \[-\frac{5}{14} + \frac{-1}{\_} = -1\]

Problem -7 Simplify

1. \[-\frac{-2}{3} + \frac{5}{9} - \frac{-7}{6} = \]

"Take LCM, divide LCM by each denominator and multiply the quotient with the corresponding numerator"

2. \[-\frac{3}{8} - \frac{-2}{9} + \frac{-1}{36} = \]

"Do as you have done before"
Practice

1. What number should be subtracted from $-\frac{13}{27}$ so as to get
   \[-\frac{4}{18}?

2. Simplify
   \[-\frac{3}{4} + \frac{9}{24} - \frac{-8}{32}\]

Recapitulation:
If $x$ and $y$ are two rational numbers then
(i) $x-y$ is a rational number
(ii) $x-0=x$
(iii) $-(-x)=x$
(iv) $x-y \neq y-x$
Lesson Plan-5

Topic – Multiplication of rational numbers

Entry Behavior: It is assumed that students know:
(I) multiplication of two natural numbers, two fractions, two integers
(II) Law of multiplication of integers

Instructional objectives. After the instructions are over, the students will able to
♦ compute the product of two rational numbers
♦ prove properties of multiplication of rational numbers.

P. k testing:
1. \( \frac{1}{2} \times \frac{1}{3} = \)
2. \( \frac{1}{3} \times \frac{11}{15} \times 0 = \)
3. \( \frac{2}{7} \times \frac{21}{16} = \)

Presentation of new material.

“The product of two rational numbers is just like the product of two fractions”

<table>
<thead>
<tr>
<th>Teacher activity and Teaching point</th>
<th>B.B. Work</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher explains, “If ( \frac{p}{q} ) and ( \frac{r}{s} ) are two rational numbers then you will multiply them as you have done in fractions”</td>
<td>( \frac{p \times r}{q \times s} = \frac{p}{q} \times \frac{r}{s} )</td>
</tr>
</tbody>
</table>

Problem 1 Multiply

1. \( \frac{3}{7} \times \left( -\frac{2}{5} \right) \)

2. \( \frac{-3}{7} \times \frac{7}{5} \)

“you can divide the common factor”
### Teacher activity and teaching point

**Property 1** If \(x\) and \(y\) are rational numbers, then \(x \times y = y \times x\). "As it is true for integers. In a same way it is true for rational number"

"We will prove it with the help of examples"

"First find rational number for LHS and then for RHS compare the two sides"

"We find them equal. Hence the property holds true for rational numbers"

**Property II** if \(x\), \(y\) and \(z\) are three rational numbers, then \(x \times (y \times z) = (x \times y) \times z\)

"We will prove with the help of example"

"Find rational number for LHS and RHS and then equate"

"We find both sides equal"

### B.B. Work

<table>
<thead>
<tr>
<th>Property</th>
<th>Example</th>
</tr>
</thead>
</table>
| Property 1 | \(x \times y = y \times x\)  
1. \(x = \frac{-1}{5}, y = \frac{2}{7}\)  
LHS \(\frac{-1}{5} \times \frac{2}{7} = \frac{-1 \times 2}{5 \times 7} = \frac{-2}{35}\)  
RHS \(\frac{2}{7} \times \left(\frac{-1}{5}\right) = \frac{2 \times -1}{7 \times 5} = \frac{-2}{35}\)  
LHS = RHS |

<table>
<thead>
<tr>
<th>Property II</th>
<th>Example</th>
</tr>
</thead>
</table>
| \(x \times (y \times z) = (x \times y) \times z\)  
1. \(x = \frac{7}{4}, y = \frac{-11}{3}, z = \frac{1}{2}\)  
LHS |
Property III  if $x$ is any rational number, then $x \times 0 = 0 \times x$

Property IV  If $x$ is a rational number then $x \times 1 = 1 \times x = x$

Property V  if $x$, $y$ and $z$ are rational number, then

$(1) x \times (y + z) = x \times y + x \times z$

"Find the rational number for LHS and RHS"

"Solve LHS carefully"

\[
\begin{align*}
\frac{7}{4} \times \left( \frac{-11 \times 1}{3 \times 2} \right) \\
= \frac{7}{4} \times \left( \frac{-11 \times 1}{3 \times 2} \right) \\
= \frac{7}{4} \times \left( \frac{-11}{6} \right) \\
= 7 \times \frac{-11}{4} \times \frac{1}{6} \\
= \frac{-77}{24}
\end{align*}
\]

RHS

\[
\left( \frac{7 \times -11}{3} \right) \times \frac{1}{2} = \left( \frac{7 \times -11}{4 \times 3} \right) \times \frac{1}{2}
\]

\[
= \frac{-77}{24}
\]

LHS = RHS

Property III  $x \times 0 = 0 \times x$

\[
\begin{align*}
73 \times 0 &= \frac{73}{3} \times 0 \\
82 \times 0 &= \frac{82}{3} \times 0 \\
73 \times 0 &= \frac{73}{3} \times 0 \\
82 \times 1 &= \frac{82}{3} \times 1
\end{align*}
\]

Property IV  $x \times 1 = 1 \times x = x$

\[
\begin{align*}
\text{for e.g.} \quad \frac{386}{273} \times 1 &= \frac{386}{273} \times 1 \\
\text{LHS}
\end{align*}
\]

((1) $x \times (y + z) = x \times y + x \times z$

\[
\begin{align*}
x &= -\frac{3}{4}, \quad y = \frac{5}{2}, \quad z = \frac{7}{6}
\end{align*}
\]

LHS
"Solve RHS carefully"

\[
x \times (y + z) = - \frac{3}{4} \times \left( \frac{5}{2} + \frac{7}{6} \right)
\]

\[
x = -\frac{3}{4} \times \frac{15 + 7}{6} = -\frac{3}{4} \times \frac{22}{6} = -\frac{3 \times 22}{4 \times 6} = \frac{-66}{24}
\]

RHS

\[
x \times y + x \times z = - \frac{3}{4} \times \frac{5}{2} + \frac{3}{4} \times \frac{7}{6}
\]

\[
x = \left( -\frac{3 \times 5}{4 \times 2} \right) + \left( \frac{-3 \times 7}{4 \times 6} \right) = \frac{-15}{8} + \frac{-21}{24} = \frac{-45 - 21}{24} = \frac{-66}{24} = \frac{-66}{24}
\]

LHS = RHS

"On comparing we find both sides equal"

(II) \( x \times (y - z) = x \times y - x \times z \)

"First find rational no for LHS and then RHS"

LHS

\[
x \times (y - z) = -\frac{5}{2} \times \left[ \frac{16}{3} \right. - (-1)
\]

\[
= -\frac{5}{2} \times \left[ \frac{16 + 3}{3} \right. 
\]

\[
\]

\[
= -\frac{5}{2} \times \frac{19}{3} = \frac{-95}{6}
\]

RHS
"Compare the both sides and we find them equal. Hence this property holds true for all rational numbers"

$\begin{align*}
xxz &= \frac{f-5}{16^1} \frac{1}{x} \\
&= \frac{12}{3} \\
&= \frac{80}{6} \\
&= \frac{80 - 15}{6} = \frac{-95}{6}
\end{align*}$

Problem 2. Write true or false

(i) $-1 \times \frac{x}{x}$ is positive, if $x$ is negative
(ii) $-1 \times (0-x)$ is negative, if $x$ is positive
(iii) $x \times (y+z)$ is non-zero, if $x$ is non-zero
(iv) if $x \times (y+z)$ is zero, then $y=z$
(v) product of two rational numbers can never be an integer.

LHS=RHS

(i) True
(ii) False
(iii) False
(iv) True
(v) $x \times (z-z) = 0$

False

Practice:

For $x = \frac{-12}{36}, y = \frac{10}{24}, z = -3$

Prove $x \times (y-z) = x 	imes y - x \times z$

Recapitulation

If $x$ and $y$ are two rational numbers, then

(i) $x \times y$ is a rational number
(ii) $x \times y = y \times x$
(iii) $x \times 0 = 0 \times x = 0$
(iv) $x \times 1 = 1 \times x = x$
(v) $(x \times y) \times z = x \times (y \times z)$ for all rational numbers $x$, $y$ and $z$
(vi) $x \times (y+z) = x \times y + x \times z$
(vii) $x \times (y-z) = x \times y - x \times z$
LESSON PLAN-6

**Topic-** Reciprocal of a rational number

**Entry behavior**

(I) The students know how to take the multiplicative inverse of fractional numbers

(II) The students know that if \( \frac{x}{y} \) is a fractional number then \( \frac{x}{y} \cdot \frac{y}{x} = 1 \) and if \( x \) is any integer then \( x \cdot \frac{1}{x} = 1 \)

**Instructional objectives:** After the instructional treatment is over, The students will able to:

♦ write the reciprocal of a rational number

♦ divide the two rational numbers

♦ prove the property of division of rational numbers, for rational number \( x \) and \( y \), \( x \div y \neq y \div x \)

♦ prove that for rational number \( x \) and \( y \)

(a) \( (x + y)^{-1} \neq x^{-1} + y^{-1} \)

(b) \( (x - y)^{-1} \neq x^{-1} - y^{-1} \)

(c) \( (x \times y)^{-1} \neq x^{-1} \times y^{-1} \)

(d) \( (x + y)^{-1} \neq x^{-1} + y^{-1} \)

**P. K. Testing:**

1. Reciprocal of 2 is

   \[ \frac{1}{2} \times \frac{1}{4} = \]

3. \( \frac{1}{2} \times 2 = \)

**Presentation of new material:**

"We have learnt addition, subtraction and multiplication of rational numbers. Now we will take up division of rational numbers by taking reciprocal."
**Teacher activity and Teaching Point**

**Reciprocal of a rational number**

Teacher explains,
For a rational number x,

\[(x^{-1})^{-1} = x\]

"Let us understand by taking examples".

\[x^{-1} = x = 1\]

\[x^{-1} = \frac{1}{x}\] for a non-zero rational number x

"We will see some properties related to reciprocal."

<table>
<thead>
<tr>
<th>Property -1. Verify that</th>
<th>((x + y)^{-1} \neq x^{-1} + y^{-1})</th>
</tr>
</thead>
</table>

"First find rational numbers by substituting the values of x and y on LHS and RHS".

**B.B. Work**

Reciprocal of \[\frac{p}{q} = \left(\frac{p}{q}\right)^{-1} = \frac{q}{p}\]

1. Reciprocal of \(-19 = \frac{-1}{19}\)

2. Reciprocal of \(\frac{8}{13} = \frac{13}{8}\)

\[\left(\frac{3}{7} + \frac{5}{11}\right)^{-1} = \left(\frac{33 + 35}{77}\right)^{-1} = \frac{77}{68}\]

4. \(0\) does not have reciprocal

\[\frac{1}{0}\] is a rational number

\[\left(\frac{3}{5} + \frac{4}{9}\right)^{-1} = \left(\frac{27 + 20}{45}\right)^{-1} = \frac{45}{47}\]

**LHS**

\[\left(\frac{3}{5} + \frac{4}{9}\right)^{-1} = \left(\frac{27 + 20}{45}\right)^{-1} = \frac{47}{45}\]

**RHS**

\[\left(\frac{3}{5} + \frac{4}{9}\right)^{-1} = \left(\frac{3}{5} + \frac{4}{9}\right)^{-1} = \frac{47}{45}\]
**Property -2** Verify that 
\[(x - y)^{-1} \neq x^{-1} - y^{-1}\]

"Find the rational numbers by substituting the values of \(x\) and \(y\) for LHS and RHS"

On comparing both sides, we find them unequal

\[
\text{LHS} = \frac{9}{19}, \text{RHS} = \frac{11}{13}
\]

\[
(x - y)^{-1} = \frac{7}{19} - \frac{11}{13}
\]

\[
(\frac{7}{19} - \frac{11}{13})^{-1} = \frac{11}{13} - \frac{7}{19}
\]

\[
\text{LHS} = \frac{247}{118}, \text{RHS} = \frac{277}{187}
\]

**Property -3** verify that 
\[(x \times y)^{-1} = x^{-1} \times y^{-1}\]

"Find the rational numbers for LHS and RHS"

On comparing, both sides are found to be equal. Thus, this rational no. holds true for reciprocals of rational number"
For Property-4 verify that
\[(x + y)^{-1} = x^{-1} + y^{-1}\]

"Find the rational numbers for LHS and RHS"

\[
\begin{align*}
\text{LHS} &\quad (x + y)^{-1} = \left(\frac{-15}{26} + \frac{5}{13}\right)^{-1} \\
&= \left(\frac{-15 \times 13}{26 \times 5}\right)^{-1} = \left(\frac{-195}{130}\right)^{-1} \\
&= \frac{-130}{195}
\end{align*}
\]

\[
\begin{align*}
\text{RHS} &\quad (x + y)^{-1} = \left((-15)^{-1} + \left(\frac{5}{13}\right)^{-1}\right) \\
&= \frac{26}{15} \div \frac{5}{13} = \frac{26 \times 13}{5 \times 15} = \frac{-26 \times 5}{-130} = \frac{-130}{195}
\end{align*}
\]

LHS = RHS

"On comparing, both sides are found to be equal. Thus this RHS rational no. holds true for reciprocals of rational number"

Practice

For \(x = \frac{-10}{7}\) and \(y = \frac{5}{14}\)

insert two rational numbers between \((x + y)^{-1}\) and \(x^{-1} + y^{-1}\)
LESSON PLAN 7

**Topic** – Division of rational numbers

**Entry Behaviour:** The student know the division of
(i) two rational numbers
(ii) fractional numbers and
(iii) two integers.

**Instructional objectives:**
After the instructional treatment is over, the students will be able to
♦ calculate the rational number when product of two rational numbers and one of the numbers is given.
♦ prove the properties of division of rational numbers.

**P.K. Testing**

Q1. Find reciprocal of

\[
(i) \quad \frac{-13}{29} \quad (ii) \quad \frac{3}{7} \times \frac{5}{11} \quad (iii) \quad \frac{-3}{7}
\]

**Presentation of new material:**
"we will solve more problems related & division of two rational numbers to continue learning"

<table>
<thead>
<tr>
<th>Teacher activity and teaching point</th>
<th>B.B. work</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-4 \div \left(\frac{-3}{5}\right))</td>
<td>(i) (4 \div \left(\frac{-3}{5}\right))</td>
</tr>
<tr>
<td>As you know from previous class, take reciprocal and (\div) changes into (\times)</td>
<td></td>
</tr>
<tr>
<td>(= -4 \times \frac{-5}{3})</td>
<td>(= \frac{-4 \times -5}{3})</td>
</tr>
<tr>
<td></td>
<td>(= \frac{-4 \times -5}{1 \times 3} = \frac{20}{3})</td>
</tr>
</tbody>
</table>

\[
\left(\frac{-7}{12}\right) \div \left(\frac{-2}{3}\right)
\]

"Take reciprocal and \(\div\) changes into \(\times\)

<table>
<thead>
<tr>
<th>(ii) (\left(\frac{-7}{12}\right) \div \left(\frac{-2}{3}\right))</th>
<th>(ii) (\left(\frac{-7}{12}\right) \div \left(\frac{-2}{3}\right))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(= -7 \times -3)</td>
<td>(= -7 \times -3)</td>
</tr>
<tr>
<td>(12 \times 2)</td>
<td>(12 \times 2)</td>
</tr>
<tr>
<td>(= \frac{21}{7})</td>
<td>(= \frac{24}{8})</td>
</tr>
</tbody>
</table>
Problem 2

The product of two rational numbers is $\frac{-8}{9}$ number is $\frac{-4}{15}$, find the other

"what is given?"  
product = $\frac{-8}{9}$  
one number = $\frac{-4}{15}$  
other number = $x$

According to the sum

\[
x \times \frac{-4}{15} = \frac{-8}{9}
\]

\[
x = \frac{-8 \times 15}{9} = \frac{-120}{3} = 40
\]

other number = $\frac{10}{3}$

Problem 3: With what number should be multiply $\frac{-15}{28}$, so that the product be $\frac{-5}{7}$?

"what is given?"  
what is to be found?

product = $\frac{-5}{7}$  
one number = $\frac{-15}{28}$  
other number = $x$

According to sum

\[
x \times \frac{-15}{28} = \frac{-5}{7}
\]

\[
x = \frac{-5 \times 28}{7} = \frac{-140}{7} = 20
\]

\[
\frac{4 \times 7}{15} = 105
\]
Problem 4.
verify \((x + y) \times z \neq x + (y \times z)\)
"substitute the values in both LHS and Rhs and find the rational number for both sides"

"On comparing both sides we find that they are not equal"

\[
\text{Lhs} \quad (x + y) \times z = \left( \frac{8}{15} + \frac{2}{5} \right) \times \frac{4}{10} \\
= \left( \frac{8 \times 2}{15 \times 10} \right) + \left( \frac{2 \times 4}{5 \times 10} \right) \\
= \frac{16}{150} + \frac{8}{50} \\
= \frac{8}{50} + \frac{8}{50} \\
= \frac{16}{50} \\
= \frac{8}{25} = \frac{32}{100}
\]

\[
\text{Rhs} \quad x + (y \times z) = \frac{8}{15} + \left( \frac{2}{5} \times \frac{4}{10} \right) \\
= \frac{8}{15} + \frac{8}{50} \\
= \frac{8}{25} = \frac{32}{100}
\]

LHS \neq RHS

Problems 5
verify \(x \times (y + z) \neq x \times y + x \times z\) for \(x = \frac{3}{7}, y = \frac{5}{11}, z = \frac{1}{2}\)
Is it true for \(x = \frac{3}{7}, y = \frac{5}{11}, z = \frac{5}{11}\)

"Find rational number for LHS and RHS by substituting values of \(x, y\) and \(z\)"

\[
\text{LHS} \quad x \times (y + z) = \frac{3}{7} \times \left( \frac{5}{11} + \frac{1}{2} \right) \\
= \frac{3}{7} \times \left( \frac{10 + 11}{22} \right) \\
= \frac{3}{7} \times \frac{21}{22} \\
= \frac{66}{147}
\]

\[
\text{RHS} \quad x \times y + x \times z = \left( \frac{3}{7} \times \frac{5}{11} \right) + \left( \frac{3}{7} \times \frac{1}{2} \right) \\
= \frac{3}{7} \times \frac{5}{11} + \frac{3}{7} \times \frac{1}{2} \\
= \frac{3 \times 5}{7 \times 11} + \frac{3 \times 1}{7 \times 2} \\
= \frac{15}{77} + \frac{3}{14} \\
= \frac{33}{77} + \frac{33}{77} \\
= \frac{66}{77}
\]

LHS \neq RHS

"Comparing two sides, we find them unequal"
Case II

"Again when \( y = z \), find LHS and RHS."

"On comparing both sides they are not equal, thus, the relation does not hold true even for \( y = z \)

<table>
<thead>
<tr>
<th>LHS</th>
<th>RHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x + (y + z) = \frac{3}{7} \cdot \left( \frac{5}{11} + \frac{5}{11} \right) )</td>
<td>( x \cdot (y + x + z) = \frac{3}{7} \cdot \left( \frac{5}{11} + \frac{5}{11} \right) )</td>
</tr>
<tr>
<td>( = \frac{3}{7} \cdot \left( \frac{10}{11} \right) )</td>
<td>( = \frac{3}{7} \cdot \left( \frac{11}{11} \right) )</td>
</tr>
<tr>
<td>( \frac{3}{7} \cdot \frac{11}{10} = \frac{33}{70} )</td>
<td>( \frac{3}{7} \cdot \frac{10}{11} = \frac{33}{70} )</td>
</tr>
</tbody>
</table>

LHS \( \neq \) RHS

Problem 5: verify that \(- (x + y) \cdot z = x + z + y \cdot z\) for \( x = \frac{3}{5}, y = -\frac{6}{7}\) and \( z = \frac{10}{11}\)
"Find rational number for LHS and RHS by substituting the given of x, y and z"

\[
\text{LHS} = (x + y) + z = \frac{3}{5} - 6 + 5 + 7 = \frac{3}{5} - 6 + \frac{35}{5} = \frac{35}{5} - 9 = \frac{21}{11} \]

\[
\text{Rhs} = x + z + y = \frac{3}{5} - 3 + \frac{11}{5} = \frac{3}{5} + \frac{11}{5} = \frac{36}{11} \]

"On comparing both sides, we find them equal. This relation holds true"

"Consequently it is true for \((x - y) + z = x + z - y + z\)"

<table>
<thead>
<tr>
<th>Practice</th>
</tr>
</thead>
<tbody>
<tr>
<td>With what number should we multiply (-\frac{17}{60}) so that the product be (-\frac{5}{12})?</td>
</tr>
<tr>
<td>for (x = \frac{-3}{7}, y = \frac{2}{14}) and (z = \frac{6}{7}) verify ((x - y) + z = x + z - y + z)</td>
</tr>
</tbody>
</table>

\((x + y) \times z \neq x \times (y + z)\)

\((z + y) \div z = x + z - y + z\)

\((x - y) \div z = x + z - y + z\)
LESSON PLAN -8

**Topic**: To rational numbers between two rational numbers

**Entry Behaviour**

It is assumed that student know

(i) if \( x \) and \( y \) are integers then \( \frac{x+y}{2} \) is an integer between \( x \) and \( y \)

(ii) how to insert integers between two integers.

**Instructional objectives**:

After the instructional treatment is over

The students will be able to

- calculate the rational number between \( x \) and \( y \).
- recall that there are infinitely many rational numbers between \( x \) and \( y \)
- calculate the rational numbers between \( \frac{x+y}{2} \) and \( \frac{|x| + |y|}{2} \)
- calculate rational numbers between \( (x+y)^{-1} \) and \( x^{-1} + y^{-1} \)

**P.K. Testing**

1. The integers between 2 and 8 are
2. The integers between -6 and 3 are

**Presentation of new material**

"We will solve some more problems like finding of rational numbers between the two rational numbers"

**Teachers activity and teaching point**

| "Rational number between two rational numbers" Consider the integer between 4 and 8 " | integer \( \frac{4+8}{2} = \frac{12}{2} = 6 \) |
| "How ever if we take 6 and 7, we know that there is no integer between 6 and 7 but a rational number \( \frac{13}{2} \) lies between 6 and 7 " | no integer between 6 & 7 \( \frac{6+7}{2} = \frac{13}{2} \) |
| if \( x \) and \( y \) are two rational numbers, \( x < y \) | \( \frac{x < x+4}{2} < y \) |
| There are infinitely many rational numbers between \( x \) and \( y \) |

**Problem** 1. Find two rational numbers between

\( \frac{43}{2} \)
We need to add two rational numbers and divide by 2.

Another way is to make denominators equal.

Now simplify. Write any two integer between -110 and -70.

<table>
<thead>
<tr>
<th>Rational Numbers</th>
<th>Operations</th>
<th>Simplification</th>
</tr>
</thead>
<tbody>
<tr>
<td>-10 and -10/11</td>
<td>-10/7 + (-10/11)</td>
<td>-180/77</td>
</tr>
<tr>
<td>28/35 and 39/15</td>
<td>2.28/35 + 39/15</td>
<td>LCM of 35 and 15 = 105</td>
</tr>
<tr>
<td>-5 and -5/9</td>
<td>-5/9 + (-5/9)</td>
<td>-10/9</td>
</tr>
</tbody>
</table>

The denominators are already the same. So write in between rational numbers having numerators between -5 and 5.

Problem 2: For \( x = \frac{3}{5} \) and \( y = \frac{-3}{7} \) is \(|x + y| = |x| + |y| \) if not, find two rational numbers, between \(|x + y|\) and \(|x| + |y|\).
"find LHS and RHS after substituting x and y"

<table>
<thead>
<tr>
<th>LHS</th>
<th>( x+y = \frac{3}{5} - \frac{3}{7} = \frac{3}{5} + \frac{3}{7} = \frac{21-15}{35} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>RHS</td>
<td>(</td>
</tr>
</tbody>
</table>

On comparing both sides are not equal.

"write the rational numbers between the two, having numerator between 6 and 36 and same denominator"

<table>
<thead>
<tr>
<th>LHS</th>
<th>( x-y = \frac{-4}{9} - \frac{5}{11} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>RHS</td>
<td>(</td>
</tr>
</tbody>
</table>

Problem 3. For \( x = \frac{-4}{9} \) and \( y = \frac{5}{11} \), find \( |x-y| = |x|-|y| \) if not, then insert two rational between them.

Problem 4. Insert a rational number between \((x+y)^{-1}\) and \(x^1 + y\) where \( x = \frac{3}{5}, y = \frac{7}{9} \)
### Find $(x + y)^{-1}$

$(x + y)^{-1} = \left( \frac{3}{5} + \frac{7}{9} \right)^{-1} = \left( \frac{27 + 35}{45} \right)^{-1}

\[ \frac{62}{45} \]

### Find $x^{-1} + y^{-1}$

$x^{-1} + y^{-1} = \left( \frac{3}{5} \right)^{-1} + \left( \frac{7}{9} \right)^{-1} = \frac{5}{3} + \frac{9}{7}

### Write equivalent rational numbers and insert a rational number in between

\[
\begin{align*}
35 + 27 &= \frac{62}{21} \\
45 + 21 &= \frac{945}{21} \\
62 + 21 &= \frac{1302}{21} \\
62 + 62 &= \frac{3844}{21} \\
21 + 21 &= \frac{1302}{21} \\
945 + 946 &= \frac{13844}{21} \\
1302 + 1302 &= \frac{13844}{21}
\end{align*}
\]

**Problem 5.** Insert a rational number between $(x - y)^{-1}$ and $x^{-1} - y^{-1}$, where $x = \frac{2}{3}$ and $y = \frac{3}{4}$

### Find $(x - y)^{-1}$ and $x^{-1} - y^{-1}$

"write equivalent rational number and insert a rational number in between"

\[
\begin{align*}
(x - y)^{-1} &= \left( \frac{2}{3} - \frac{3}{4} \right)^{-1} = \left( \frac{8 - 9}{12} \right)^{-1} = \left( \frac{-1}{12} \right)^{-1} = -12 \\
x^{-1} - y^{-1} &= \left( \frac{2}{3} \right)^{-1} - \left( \frac{3}{4} \right)^{-1} = \frac{3}{2} - \frac{4}{3} = \frac{9 - 8}{6} = \frac{1}{6}
\end{align*}
\]

\[
\begin{align*}
-12 \cdot \frac{6}{6} &= \frac{-72}{6} \\
-72 \cdot \frac{1}{6} &= \frac{-9}{6} \\
\frac{-72}{6} - \frac{9}{6} &= \frac{-81}{6}
\end{align*}
\]

**Practice**

For $x = \frac{-15}{28}$ and $y = \frac{-5}{7}$

Verify $(x + y)^{-1}$ and $x^{-1} + y^{-1}$

**Recapitulation**

If $x$ and $y$ are two rational numbers,

- $(x + y)^{-1} \neq x^{-1} + y^{-1}$
- $(x - y)^{-1} \neq x^{-1} - y^{-1}$
- $(x \times y)^{-1} = x^{-1} \times y^{-1}$
- $(x + y)^{-1} = x^{-1} + y^{-1}$
LES S O N  P L A N -9

Topic: Decimal Representation of Rational Numbers

Entry Behaviour: It is assumed that students are able to
(i) Convert fractions into decimals
(ii) convert decimals into fractions.

Instructional Objectives.
The students will able to
♦ represent the rational number in its decimal form (terminating or non-terminating repeating decimal)
♦ state when the given rational number will be terminating decimal or non-terminating repeating decimal without actual division.
♦ recall that if two rational numbers are terminating then their product, sum or difference is also terminating.

P.K. Testing
1. convert into decimals
   (i) \(\frac{1}{2}\)
   (ii) \(\frac{25}{4}\)
   (iii) \(\frac{3125}{1000}\)
   (iv) \(\frac{1455}{10}\)

Presentation of new material:
"We know that a fraction may be represented as a positive decimal number. Likewise every finite decimal number which is greater than zero may be expressed as a fraction. It is natural if we expect a similar type of relation between rational numbers and decimal numbers. In chapter we explore this relationship."

<table>
<thead>
<tr>
<th>Teacher activity</th>
<th>B.B. Work</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;Every rational number may be expressed as a decimal number of terms in the decimal part may be converted in to a rational number&quot; &quot;We will see with the help of examples&quot;</td>
<td>Decimal representation Terminating Non-terminating or finite or infinite</td>
</tr>
</tbody>
</table>
Problem -1 convert the following rational numbers in to decimals

"As you have seen in case of fractions, to find its decimal, we divide numerator by denominator. Here also, we divide numerator by denominator"

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
</table>
| 1 (i) | $\frac{15}{4}$ | $3.75$
|   | 12 |   |
|   | 30 |   |
|   | 28 |   |
|   | 20 |   |
|   | 20 |   |
| = | 3.75 |   |
| 2. | $\frac{481}{200}$ | $0.2405$
|   | 400 |   |
|   | 810 |   |
|   | 800 |   |
|   | 1000 |   |
|   | 1000 |   |
| = | 0.2405 |   |
| 3. | $\frac{-13}{20}$ | $-0.65$
|   | 120 |   |
|   | 100 |   |
|   | 100 |   |
| = | -0.65 |   |
| 4. | $\frac{-21}{16}$ |   |
| "In all the above examples you must have observed that remainder is zero., decimal is terminated. Now let us move on next problem to see the deference between non-terminating decimals."
**Problem -2 “Express as non-terminating repeating decimals.”**

(i) \[
\frac{10}{11}
\]

“Since the remainder 1 is repeating, the sequence of digits 9, 0 would repeat see the representation of non-terminating digits.” “We put bar on the repeating digits”. "As here we put bar on 90 which are repeating"

\[
\begin{array}{c|c}
1.3125 & 21.0000 \\
\hline
-16 & 050 \\
0 & 48 \\
& 20 \\
& 16 \\
& 40 \\
& 32 \\
& 80 \\
& 80 \\
\hline
= -1.31125
\end{array}
\]

(ii) \[
\frac{17}{90}
\]

“Remainder 8 is some as the previous remainder.”

“On which digit will you put bar on it”

\[
\begin{array}{c|c}
10 & 0.909090..... \\
\hline
10 & 99 \\
-99 & 100 \\
100 & -99 \\
-99 & 10 \\
\hline
10 & 0.90909...... \\
\hline
\frac{10}{11} & 0.90909..... \\
\hline
= 0.90
\end{array}
\]

(iii) \[
\frac{-28}{21}
\]

“Remainder 70 is same as the previous remainder. Digit 3 will repeat in the quotient.”

“On which digit will you put bar?”
In the above questions, you must have noticed, which rational numbers will be terminating and which not be. The rational numbers, which have denominators have prime factor 2 or 5 will be terminating and others are non-terminating., which have denominators other than 2 or 5.

\[
\frac{p}{q} = \text{finite i.e terminating}
\]

decimal, if prime factors of q are 2 or 5 or both \[
\frac{p}{q} = \text{non-terminating decimal, if q has a prime factor which is neither 2 or 5}
\]

| 0.188... | 0.188... |
| 90 | 17,000 |
| 90 | 90 |
| 800 | 720 |
| 720 | 0800 |
| 0800 | 720 |
| 720 | 80 |
| \(\frac{17}{90} = 0.18\) | \(\frac{17}{90} = 0.18\) |
| 1.33 | 1.33 |
| 21 | 28.000 |
| 21 | 21 |
| 070 | 070 |
| 63 | 63 |
| 070 | 070 |
| \(\frac{-28}{21} = -1.3\) | \(\frac{-28}{21} = -1.3\) |

50
Problem-3 Which of the following are terminating decimals and why?

1. \(15 + 17\)
   *Take prime factorization of denominator
   \(2^2\).
   \(15 + 17\) non-terminating
   Prime factors of \(17 = 1 \times 17\).
   Prime factors of \(17\) - neither 2 nor 5

2. \(7 + 6\)
   "Though one of the factors of 6, 2 is there but other
   than 2 also there. Therefore decimal is non-terminating"
   \(7 + 6\) non terminating
   \(Prime\ factors\ of\ 6 = 2 \times 3\)

3. \(9 + 8\)

4. \(42 + 9\)

5. \(12.5 + 20\)

Problem-4 Write true and false.

1. \(P\)
   has a terminating decimal
   representation
   If \(q\) is prime.

2. \(\frac{P}{q}\) and \(\frac{r}{s}\) are terminating
   decimals, then
   \(\frac{P}{q} + \frac{r}{s}\) is also terminating
   decimal as is seen in the example.

1. false
   \(q\) should have prime factor
   either 2 or 5 or both.

5. True for e.g
   \(\frac{1}{5} + \frac{1}{20} = \frac{4}{20} + \frac{1}{20} = \frac{5}{20}\)
   (prime factors of 20 = \(2 \times 2 \times 5\))
3. If \( \frac{p}{q} \) and \( \frac{r}{s} \) both have non-terminating repeating decimal representations, then so does \( \frac{p}{q} + \frac{r}{s} \). "You can notice from prime factors of the denominators" does so.

True for e.g.
\[
\frac{1}{6} + \frac{1}{18} = \frac{3+1}{18} = \frac{4}{18} = \frac{2}{9} \quad \text{(prime factors of 18 = 2 x 3 x 3)}
\]

4. If \( \frac{p}{q} \) and \( \frac{r}{s} \) both have terminating decimal representations then so does \( \frac{p}{q} \times \frac{r}{s} \) as is noticed from the example.

True for e.g.
\[
\frac{1}{10} \times \frac{1}{25} = \frac{1}{250} \quad \text{Prime factor 250 = 5 x 5 x 5 x 2}
\]

5. If \( \frac{p}{q} \) has non-terminating decimal and \( \frac{r}{s} \) is terminating decimal, then \( \frac{p}{q} + \frac{r}{s} \) may have terminating decimal representation.

False
\[
\frac{1}{18} + \frac{1}{15} = \frac{5}{9} \quad \text{Prime factor 9 = 3 x 3}
\]

Practice.

Which of following are terminating/non-terminating and why? (tell without actual division)

15 25 16 -15 21
7 64 132 288 3125
Recapitulation

1. Every rational has a decimal representation either terminating or non-terminating.

2. \( \frac{p}{q} \) is terminating decimal, if in its lowest form, the denominator has only 2 and 5 as factors.

3. \( \frac{p}{q} \) in the lowest form, has a non-terminating decimal representation, if a prime number different from 2 and 5 is a factor of q.
LESLIE PLAN 10

**Topic**-Computation with rational numbers  
**Entry Behavior**
It is assumed that students know:
(1) How to convert decimals into fractions.
(2) The representation of rational numbers into corresponding decimals.
(3) The addition subtraction, multiplication and division of decimals.

**Instructional objectives:**
After the instructional treatment is over, the students will able convert the decimals into rational numbers.
solve the expression in decimals and express the resulting number as a rational number

**P.K. Testing**
Q1. Which of the following are terminating decimals?
(1) \(\frac{17}{45}\) (2) \(\frac{23}{40}\) (3) \(\frac{-14}{125}\) (4) \(\frac{-28}{256}\)

**Presentation of new material:**
"Today we will convert decimals into rational numbers in the standard form"

<table>
<thead>
<tr>
<th>Teacher activity</th>
<th>B.B. Work</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Problem 1</strong> Convert the following decimals into rational numbers</td>
<td></td>
</tr>
<tr>
<td>(I) 0.037</td>
<td></td>
</tr>
<tr>
<td>(II) -0.75 Reduce the rational into lowest forms</td>
<td></td>
</tr>
<tr>
<td>(III) -8.625 &quot;write the decimal in expanded form. Reduce the rational number into lowest terms&quot;</td>
<td></td>
</tr>
<tr>
<td>(IV) 9.6</td>
<td></td>
</tr>
<tr>
<td>(I) (0.037 = \frac{3}{100} + \frac{7}{1000} = \frac{37}{1000})</td>
<td></td>
</tr>
<tr>
<td>(II) (-\frac{75}{100} = -\frac{3}{4})</td>
<td></td>
</tr>
<tr>
<td>(III) (\frac{8}{10} + \frac{6}{100} + \frac{2}{100} = \frac{345}{8})</td>
<td></td>
</tr>
<tr>
<td>(IV) (\frac{96}{10} = \frac{48}{5})</td>
<td></td>
</tr>
</tbody>
</table>

**Problem 2.** Add the following numbers and express in the form \(\frac{p}{q}\)

| (1)-5.4 and 9.8 |
| (2) 3.007, 0.587 and 18.341 |
| (1) \(9.8-5.4 = 4.4 = \frac{22}{5}\) |
| (2) \(3.007+0.587+18.341 = 21.935\) |
Problem 3. Simplify and express the result as rational number.

1. \(32.8-13-10.725+3.517\)  
   (1) \(32.8-13\) 
   \[= \frac{1574}{125}\] 

2. \((2.1)^2 \times (1.5)^2\)  
   (2) \((2.1)^2 \times (1.5)^2\) 
   \[= \frac{99225}{10000} = \frac{3969}{400}\] 

3. \(0.3 \times 0.03 \times 0.003\)  
   (3) \(0.000027\) 

4. \(4.8432 + 0.08\)  
   (4) \(4.8432 + 0.08\) 
   \[= \frac{1522}{25}\] 

5. \((1.2)^2 \times (0.9)^2 + 1000\)  
   (5) \((1.2)^2 \times (0.9)^2 + 1000\) 
   \[= \frac{1.1664}{1000} = \frac{729}{625000}\] 

6. \(6.3 \div (0.3)^2\)  
   (6) \(6.3 \div (0.3)^2\) 
   \[= \frac{63 \times 10}{3 \times 3} = \frac{70}{9}\]

**Practice**

Evaluate and express the resulting number as a rational number:

1. \(257.894 + 0.169\)
2. \((-3.8)^2 \times (0.9)^2 + 3000\)

**RECAPITULATION**

1. Every decimal can be converted back into a rational number.
LESSON PLAN 11

**Topic** Positive Exponents

**Entry Behavior**
It is assumed that students
(1) Know about exponential or power notation of integers.
(2) Are able to solve, or simplify the expressions containing integers with exponents.

**Instructional objectives:**
After the instructional treatment is over, the students will able to
♦ solve the problems using proposition that for a rational number $\frac{p}{q}$, and $m$ is a positive integer, $\left(\frac{p}{q}\right)^m = \frac{p^m}{q^m}$
♦ express the given rational number in power notation
♦ simplify the complex expressions.

**P.K. Testing**
1. $5^4 =$
2. $(-3)^3 =$
3. How do you read $5^4$?
4. What is base in $5^4$. What is 4 called in $5^4$?

**Presentation of new material**
"We will today learn to apply into previous knowledge to rational numbers"

<table>
<thead>
<tr>
<th>Problem 1. Express as rational number $\frac{p}{q}$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\left(\frac{3}{7}\right)^2$</td>
<td>1. $\left(\frac{3}{7}\right)^2 = \frac{3\times3}{7\times7} = \frac{9}{49}$</td>
</tr>
<tr>
<td>2. $\left(-\frac{5}{9}\right)^3$</td>
<td>2. $\left(-\frac{5}{9}\right)^3 = -\frac{5\times5\times5\times9}{9\times9\times9} = -\frac{125}{729}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem 2. Simplify the following</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\left(\frac{3}{5}\right)^4 \times \left(\frac{1}{3}\right)^3$</td>
<td>1. $\left(\frac{3}{5}\right)^4 \times \left(\frac{1}{3}\right)^3 = \frac{3\times3\times3\times3}{5\times5\times5\times5} \times \frac{1\times1\times1}{3\times3\times3}$</td>
</tr>
</tbody>
</table>

56
2. \( \left( \frac{1}{3} \right)^4 \left( \frac{1}{9} \right)^4 \)
\[
\begin{align*}
\left( \frac{1}{3} \right)^4 \cdot \left( \frac{1}{9} \right)^4 &= \left( \frac{1}{3} \right)^4 \cdot \left( \frac{1}{9} \right)^4 \\
&= \left( \frac{1}{3} \right)^4 \cdot \left( \frac{1}{9} \right)^4 \\
&= \frac{1}{81} \cdot \frac{1}{625} \\
&= \frac{1}{50625}
\end{align*}
\]

3. \( \left( \frac{-1}{2} \right)^3 \times 2^2 \times \left( \frac{3}{4} \right)^2 \)
\[
\begin{align*}
\left( \frac{-1}{2} \right)^3 \times 2^2 \times \left( \frac{3}{4} \right)^2 &= -1 \times \frac{1}{4} \times 4 \times \frac{9}{16} \\
&= -1 \times \frac{9}{16} \\
&= -\frac{9}{16}
\end{align*}
\]

4. \( \left[ \left( \frac{1}{2} \right)^2 - \left( \frac{1}{4} \right)^3 \right] \times 2^3 \)
\[
\begin{align*}
\left[ \left( \frac{1}{2} \right)^2 - \left( \frac{1}{4} \right)^3 \right] \times 2^3 &= \left( \frac{1}{4} - \frac{1}{64} \right) \times 8 \\
&= \frac{15}{64} \times 8 \\
&= \frac{15}{8}
\end{align*}
\]

5. \( (3^2 - 2^2) \times \left( \frac{1}{5} \right)^2 \)
\[
\begin{align*}
(3^2 - 2^2) \times \left( \frac{1}{5} \right)^2 &= (9 - 4) \times \frac{1}{25} \\
&= 5 \times \frac{1}{25} \\
&= \frac{5}{25} \\
&= \frac{1}{5}
\end{align*}
\]

**Problem 3. Express in power notation**

(I) \( \frac{1}{243} \)
\[
\begin{align*}
\frac{1}{243} &= \frac{1}{3 \times 3 \times 3 \times 3} = \frac{1}{(3)^3}
\end{align*}
\]

(II) \( -\frac{16}{729} \)
\[
\begin{align*}
-\frac{16}{729} &= -\frac{2 \times 2 \times 2}{3 \times 3 \times 3 \times 3 \times 3} = -\frac{16}{(3)^6}
\end{align*}
\]
Write the prime fractionation of 2401 and 256

\[ 2401 = 7^4 \]
\[ 256 = 4^4 \]

Problem 4. Find the reciprocal of

(I) \( \left( \frac{3}{4} \right)^4 \)

Reciprocal = \( \frac{81}{256} \)

(II) \( \left( \frac{-1}{5} \right)^8 \times \left( \frac{1}{5} \right)^2 \)

Reciprocal = \( \frac{15625}{81} \)

Practice
Simplify

(I) \( \left( \frac{2}{3} \right)^3 - \left( \frac{1}{9} \right)^2 \times 2^3 + \left( \frac{16}{9} \right)^2 \)

Recapitulation
For rational no. \( \frac{p}{q} \) and positive integer \( m \)

\( \left( \frac{p}{q} \right)^m = \frac{p^m}{q^m} \)
LESSON PLAN 12

Topic: Computation of rational numbers with notations

Entry Behaviors: It is assumed that the students are able to:
1. Express the given rational number in power notation.
2. Simplify the expression containing power notations.
3. Compute the absolute value of given rational number.
4. Compare the two rational numbers.
5. Write the reciprocal of the rational number.
6. Insert the rational number between the two rational numbers.

Instructional Objectives: After the instructional treatment is over, the students will able to:
• express the rational number \( \frac{p}{q} \) (not in lowest form) as the power of \( \frac{r}{s} \) (in the lower form)
• distinguish between \( \frac{p^n}{q} \) and \( \left( \frac{p}{q} \right)^n \) where \( m \) is the positive integer

P. K. Testing
1. \( \left( \frac{-1}{2} \right)^3 = \text{__________} \)
2. Reciprocal of \( (-3)^2 \) is _________
3. Express \( \frac{-343}{729} \) as power notation

Presentation of new material:
“Today we will learn to find absolute value of rational number with exponents and solve other problems”

<table>
<thead>
<tr>
<th>Teacher activity</th>
<th>B. B. works</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \frac{81}{625} ) as power of ( \frac{3}{5} ) write prime factorization of 81 and 625</td>
<td>1. ( \frac{81}{625} = \frac{3 \times 3 \times 3 \times 3}{5 \times 5 \times 5 \times 5} = \left( \frac{3}{5} \right)^4 )</td>
</tr>
<tr>
<td>2. ( \frac{-49}{64} ) as a power of ( \frac{7}{8} ) write factorization of 49 in the form</td>
<td>2. ( \frac{-49}{64} = \frac{-7 \times 7}{8 \times 8} = \left( \frac{7}{8} \right)^2 )</td>
</tr>
</tbody>
</table>
3. \(-343\) as power of \(-7\) write \(-343 = -7 \times -7 \times -7 \times -7 = \left(\frac{-7}{9}\right)^3\)

Problem 2. Find the absolute value

\[ \left(\frac{-1}{3}\right)^3 \]
\[ \left(\frac{-1}{3}\right)^3 = \left(\frac{-1}{3}\right)^3 \]
\[ = -\frac{1 \times -1 \times -1}{3 \times 3 \times 3} \]
\[ = -\frac{1}{27} \]
\[ \text{Absolute value} = \frac{1}{27} \]

\[ \left(\frac{2}{7}\right)^3 \]
\[ \left(\frac{2}{7}\right)^3 = \left(\frac{2}{7}\right)^5 = \frac{2 \times 2 \times 2 \times 2 \times 2}{7 \times 7 \times 7 \times 7 \times 7} \]
\[ = \frac{37}{16807} \]
\[ \text{Absolute value} = \frac{37}{16807} \]

\[ \left(\frac{5}{-3}\right)^4 \]
\[ \text{What is the absolute value?} \]
\[ \left(\frac{5}{-3}\right)^4 = \frac{5 \times 5 \times 5 \times 5}{-3 \times -3 \times -3 \times -3} = \frac{625}{81} \]
\[ \text{Absolute value} = \frac{625}{81} \]

\[ \left(\frac{-11}{13}\right)^2 \]
\[ \left(\frac{-11}{13}\right)^2 = \left(\frac{-11}{13}\right)^4 = \frac{121}{169} \]
\[ \text{Absolute value} = \frac{121}{169} \]

Problem 3. Distinguish between the rational numbers \(\frac{3^2}{4}\) and \(\left(\frac{3}{4}\right)^2\). Which is smaller

\[ \frac{3^2}{4} = \frac{3 \times 3}{4} = \frac{9}{4} \]
\[ \left(\frac{3}{4}\right)^2 = \frac{3 \times 3}{4} = \frac{9}{16} \]
of the two. Insert three rational numbers between \( \frac{3}{4} \) and \( \frac{3}{4} \).

"Which is greater?" "Find it by looking at the denominator when numerators are same." "Now pay attention how we find the rational numbers between the two. First add the two rational numbers and divide by 2." "Then again find the rational number between \( \frac{9}{4} \) and \( \frac{45}{32} \).

There is another way of inserting the rational number between the two. Make the denominator same. The three rational numbers can be written simply by keeping the denominator same i.e., 16, write numerators between 9 and 36. "you can adopt any method which you find easy."

**Practice:**

Write the reciprocal and absolute value of 

\[ 1. \left( -\frac{9}{2} \right) + \left( \frac{4}{3} \right)^3 \]
LESSON PLAN-13

**Topic:** laws of Exponents (Positive Powers)

**Entry Behavior:** It is assumed that the students are able to

(i) express the given rational number in powers
(ii) simplify the given expressions with rational numbers in powers.

**Instructional objectives:** After the instructional treatment is over, the students are able to

(i) recall the laws of exponents (Positive Powers)
(ii) solve problems related to the laws

**P.K. Testing**

Find the value

1. \( \left( \frac{3}{7} \right)^2 \)
2. \( \frac{3}{5} ^4 \times \left( \frac{1}{3} \right)^3 \)
3. \( \left( \frac{-1}{5} \right)^3 + \left( \frac{1}{5} \right)^2 \)

**Presentation of new material** “We will solve the problems by using Laws of exponents which we learn today”

<table>
<thead>
<tr>
<th>Teacher activity</th>
<th>B.B. work</th>
</tr>
</thead>
<tbody>
<tr>
<td>“we will arrive at the laws by taking example” “when bases are same powers are added.”</td>
<td></td>
</tr>
<tr>
<td>1. If x is a rational number and in and n are positive integer, then ( x^m \times x^n = x^{m+n} )</td>
<td></td>
</tr>
<tr>
<td>2. if x is a non-zero rational number, m and n are positive integers such that m&gt;n, then ( x^m + x^n = x^m )</td>
<td></td>
</tr>
<tr>
<td>3. If x is a non-rational number, and m such that m&lt;n, then ( x^m + x^n = x^{m-n} )</td>
<td></td>
</tr>
<tr>
<td>1.25x27=</td>
<td></td>
</tr>
<tr>
<td>2x2x2x2x2x2x2x2x2x2x2x2x2= 212 = 25+7</td>
<td></td>
</tr>
<tr>
<td>( x^m \times x^n = x^{m+n} , x &gt; 0 )</td>
<td></td>
</tr>
<tr>
<td>2. 5^4 + 5^3 = \frac{5 \times 5 \times 5 \times 5}{5 \times 5} = 5^2 = 5^{6-3}</td>
<td></td>
</tr>
<tr>
<td>( x^m + x^n = x^{m-n} , x &gt; 0 )</td>
<td></td>
</tr>
<tr>
<td>m &gt; n</td>
<td></td>
</tr>
<tr>
<td>3.5^4 + 5^2 = \frac{5 \times 5 \times 5 \times 5}{5 \times 5 \times 5 \times 5} = \frac{1}{2}</td>
<td></td>
</tr>
<tr>
<td>( x^m + x^n = x^{m-n} , x &gt; 0 )</td>
<td></td>
</tr>
<tr>
<td>m &lt; n</td>
<td></td>
</tr>
<tr>
<td>4. ( (x^n)^m = x^{mn} , x &gt; 0 )</td>
<td></td>
</tr>
</tbody>
</table>
4. If \( x \) is a non-zero rational number and \( n \) are positive integers, then \( (x^n)^m = x^{nm} \).

**Problem -1** fill in the blanks by using various laws.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \frac{2^3 \times 2^4}{2} ) = 2</td>
<td>1. ( 2^3 \times 2^4 = 2^{3+4} = 2^7 )</td>
<td></td>
</tr>
<tr>
<td>2. ( \left( \frac{2}{3} \right)^7 \times \left( \frac{2}{3} \right)^4 = \left( \frac{2}{3} \right)^{7+4} = \left( \frac{2}{3} \right)^{11} )</td>
<td>2. ( \left( \frac{2}{3} \right)^7 \times \left( \frac{2}{3} \right)^4 = \left( \frac{2}{3} \right)^{7+4} = \left( \frac{2}{3} \right)^{11} )</td>
<td></td>
</tr>
<tr>
<td>3. ( \frac{3}{4} \times \frac{3}{4} = \frac{3^5}{4} )</td>
<td>3. ( \left( \frac{3}{4} \right)^8 \times \left( \frac{3}{4} \right)^5 = \left( \frac{3}{4} \right)^{8+5} = \left( \frac{3}{4} \right)^{13} )</td>
<td></td>
</tr>
<tr>
<td>4. ( 8^{13} + 8^{19} = \frac{1}{8} )</td>
<td>4. ( 8^{13} + 8^{19} = \frac{1}{8} )</td>
<td></td>
</tr>
</tbody>
</table>

**Problem -2** Simplify: and express the result in power rotation

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \left( \frac{11}{3} \right)^{11} \times \left( \frac{11}{3} \right)^2 )</td>
<td>1. ( \left( \frac{11}{3} \right)^{11} \times \left( \frac{11}{3} \right)^2 = \left( \frac{11}{3} \right)^{13} )</td>
<td></td>
</tr>
<tr>
<td>2. ( \left( \frac{4}{5} \right)^3 \times \left( \frac{4}{5} \right)^8 )</td>
<td>2. ( \left( \frac{4}{5} \right)^3 \times \left( \frac{4}{5} \right)^8 = \left( \frac{4}{5} \right)^{3+8} = \left( \frac{4}{5} \right)^{11} )</td>
<td></td>
</tr>
<tr>
<td>3. ( \left( \frac{2}{3} \right)^{12} )</td>
<td>3. ( \left( \frac{2}{3} \right)^{12} = \left( \frac{2}{3} \right)^{2+2} = \left( \frac{2}{3} \right)^{8} )</td>
<td></td>
</tr>
</tbody>
</table>

**Problem -3 simplify**

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \frac{2}{3} \times \frac{2}{3} )</td>
<td>1. ( \frac{2^3}{3} \times \frac{2^3}{3} = \frac{2^{3+3}}{3} )</td>
<td></td>
</tr>
<tr>
<td>( = \frac{2^6}{3} = \frac{(2)^3}{3^2} = 32 )</td>
<td>( = \frac{2^6}{3} = \frac{(2)^3}{3^2} = 32 )</td>
<td></td>
</tr>
</tbody>
</table>

63
2. \((-4)^6 + (-4)^9\)

3. \(\left( \frac{1}{2^3} \right)^2\)

2. \((-4)^6 \div (-4)^9\) = \(\frac{1}{(-4)^3}\)

3. \(\left( \frac{1}{2^3} \right)^2 = \frac{1}{(2^3)^2} = \frac{1}{2^6} = \frac{1}{64}\)

### Problem-4 write true or false.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) True</td>
<td>LHS (\left( -\frac{7}{80} \right)^{80} = \left( \frac{7}{80} \right)^{80} ) = RHS</td>
</tr>
<tr>
<td>(2) False</td>
<td>(10^{10} \times 10^{10} \neq 10^{100} \neq RHS)</td>
</tr>
<tr>
<td>(3) True</td>
<td>(\left( \frac{1}{7} \right)^7 = \left( \frac{1}{7} \right)^{49}) Reciprocal = (7^{49})</td>
</tr>
<tr>
<td>(4) False</td>
<td>Actual ((30 \times 30)^3 = 30^{30} \times 30^{30})</td>
</tr>
</tbody>
</table>

### Practice:-

1. \(\left( \frac{4}{8} \right)^{11} + \left( \frac{4}{8} \right)^7 = \) ____________

2. \(\left( \frac{-2}{7} \right)^3 + \left( \frac{-2}{7} \right)^9 = \) ____________

3. \(\left( \frac{3^4}{8} \right)^2 = \) ____________

4. \(\left( \frac{2}{9} \right)^3 \times \left( \frac{2}{9} \right)^4 = \) ____________

### Recapitulation

For rational number \(x\), and \(m, n\) are positive integers then

(i) \(x^m \times x^n = x^{m+n}\)

(ii) \(x^m + x^n = x^{m-n}\), if \(m > n\)

(iii) \(x^m + x^n = \frac{1}{x^{n-m}}\), if \(m < n\)

(iv) \((x^n)^m = x^{mn}\)
LESSON PLAN-14

**Topic:** Law of Exponents (Negative Powers) (Laws I, II, III)

**Entry Behaviors:** It is assumed that the students are able to
(i) Recall 4 laws of exponents (Positive Powers)
(ii) Solve problems related to any of the four laws of exponents (Positive Powers)

**Instructional Objectives:** After the instructional treatment is over, the students are able to:

- use Law I (negative exponents) that for \( x \neq 0 \) and \( m \) is a negative integer, \( x^{-m} = \frac{1}{x^m} \)
- apply Law II that for any integers \( m \) and \( n \) (positive or negative) and for any rational number \( x \neq 0 \), \( x^m \times x^n = x^{m+n} \)
- apply Law III that for any integers \( m \) and \( n \) (positive or negative) and for any rational number \( x \neq 0 \), \( (x^m)^n = x^{mn} \)
- solve complicated problems related to the laws I, II and III.

**P.K. Testing**
Find the value
1. \( \left( \frac{2}{3} \right)^2 \times \left( \frac{2}{3} \right)^3 = \)
2. \( \left( \frac{1}{3^3} \right)^2 = \)
3. \( (-4)^6 + (-4)^9 = \)

**Presentation of new material:**
"Now we will learn about laws of exponents dealing with negative powers. As well. Let us take one by one".

<table>
<thead>
<tr>
<th>Reciprocal of ( \frac{p}{q} ) is denoted by ( \left( \frac{p}{q} \right)^{-1} ), we use exponent -1 to denote reciprocal.</th>
<th>( \left( \frac{p}{q} \right)^{-1} = \text{the reciprocal of } \frac{p}{q} = \frac{1}{\left( \frac{p}{q} \right)} = \frac{q}{p} )</th>
</tr>
</thead>
</table>

65
In similar manner we can say that $3^{-2}$ is a reciprocal of $3^2$, $(-7)^4$ is a reciprocal of $(-7)^4$.

We thus have following law:

**Law I:** If $x$ is a non-zero rational number and $m$ is a positive integer, then $x^{-m}$ is the reciprocal of $x^m$.

**II.** If $x$ is a non-zero rational number and $m$ and $n$ are any integers (positive or negative), then $x^m \times x^n = x^{m+n}$.

**Law III.** If $x$ is a non-zero rational number and $m$ and $n$ are integers, then $(x^n)^m = x^{mn}$.

---

<table>
<thead>
<tr>
<th>$x = \frac{3}{7}$, $x &gt; 0$</th>
<th>$m = 3$, $n = 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^{-3} = \frac{1}{3}$</td>
<td>$x^{-3\times4} = \frac{1}{3^4}$</td>
</tr>
<tr>
<td>$x^{-3} = \frac{1}{x^3}$</td>
<td>LHS $= \left[\left(\frac{3}{7}\right)^{-3}\right]^4 = \left[\frac{7}{3}\right]^4 = \left(\frac{7}{3}\right)^12$</td>
</tr>
<tr>
<td>$x^{-3} = \frac{1}{x^3}$</td>
<td>RHS $= \left(\frac{3}{7}\right)^{-3\times4} = \left(\frac{3}{7}\right)^{-12} = \left(\frac{7}{3}\right)^{12}$</td>
</tr>
</tbody>
</table>

**LAW III** $x^n = x^{mn}$, $n > 0$ where $x > 0$, $m$ and $n$ are integers (+ve or –ve).

"With the help of the law discussed we will solve some problems"
**Problem-1** Find the value of

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>(2^{-3})</td>
</tr>
<tr>
<td></td>
<td>“for finding value you are required to first express as positive exponents by taking reciprocal”</td>
</tr>
<tr>
<td>2.</td>
<td>(\left(\frac{1}{4}\right)^{-4})</td>
</tr>
<tr>
<td>3.</td>
<td>(\left(\frac{4}{-7}\right)^{-3})</td>
</tr>
<tr>
<td>1.</td>
<td>(2^{-3} = \frac{1}{2^3} = \frac{1}{8})</td>
</tr>
<tr>
<td>2.</td>
<td>(\left(\frac{1}{4}\right)^{-4} = (4)^4 = 256)</td>
</tr>
<tr>
<td>3.</td>
<td>(\left(\frac{4}{-7}\right)^{-3} = \left(-\frac{7}{4}\right)^3 = -\frac{343}{64})</td>
</tr>
</tbody>
</table>

**Problem-2** Using the laws of exponents, express each of the following as a rational number with positive exponent.

<p>| | |</p>
<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>(\left(\frac{2}{3}\right)^5)</td>
</tr>
<tr>
<td></td>
<td>“Take reciprocal of the rational number to express it as a rational number with positive exponent”</td>
</tr>
<tr>
<td>2.</td>
<td>(4^3 \times 4^{-5})</td>
</tr>
<tr>
<td></td>
<td>“Sometimes you have to use two laws”</td>
</tr>
<tr>
<td>3.</td>
<td>((2^{-4})^2)</td>
</tr>
<tr>
<td>1.</td>
<td>(\left(\frac{2}{3}\right)^5 = \left(\frac{3}{2}\right)^5) by Law (x^{-n} = \frac{1}{x^n})</td>
</tr>
<tr>
<td>2.</td>
<td>(4^3 \times 4^{-5} = 4^{3-5} = 4^{-2})</td>
</tr>
<tr>
<td></td>
<td>(x^{-m} = \frac{1}{x^m})</td>
</tr>
<tr>
<td></td>
<td>(4^{-2} = \frac{1}{4^2})</td>
</tr>
<tr>
<td>3.</td>
<td>((2^{-4})^2)</td>
</tr>
<tr>
<td></td>
<td>Using law ((x^m)^n = x^{mn})</td>
</tr>
<tr>
<td></td>
<td>(x^{-m} = \frac{1}{x^m})</td>
</tr>
<tr>
<td></td>
<td>(2^{-8} = \frac{1}{2^8})</td>
</tr>
</tbody>
</table>
### Problem-3
Express as a rational number with negative exponent:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>(\left(\frac{1}{2}\right)^3)</td>
</tr>
<tr>
<td>2.</td>
<td>((2^4)^1)</td>
</tr>
<tr>
<td>3.</td>
<td>(4^3 \times 4^4)</td>
</tr>
<tr>
<td>4.</td>
<td>((-3)^4 \times \left(\frac{5}{3}\right)^4)</td>
</tr>
</tbody>
</table>

### Problem-4
By what number should we multiply \(3^7\) so that the product may be equal to 3?“What is given?” “What is to be found?”

- Product = 3
- One number = \(3^7\)
- Other number = \(x\)
- According to sum
  \[X \times 3 = 3\]
  \[X = \frac{3}{3^7} = 3 \times 3^7 = 3^8\]

### Problem-5
by what number should \((-4)^5\) be divided so that the quotient may be equal to \(4^{-2}\)?

- "What is given?"
- "What is that number called which is to be divided?"
- "By using laws of exponent solve for \(x\)"

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Quotient = (4^{-2})</td>
<td></td>
</tr>
<tr>
<td>Dividend = ((-4)^5)</td>
<td></td>
</tr>
<tr>
<td>Divisor = (x)</td>
<td></td>
</tr>
<tr>
<td>According to sum. ((-4)^5 \div x = (4)^{-2})</td>
<td></td>
</tr>
</tbody>
</table>
Practice: By what number should we multiply \((19)^{12}\) so that the product may be equal to \((19)^{-4}\)?

Recapitulation:

(i) \[\left(\frac{p}{q}\right)^{-1} = \frac{q}{p}\]

(ii) For \(x>0\), \(m\) is a positive integer then \(x^{-m}\) is the reciprocal of \(x^m\).

(iii) For \(x>0\), \(m\) and \(n\) are any integers

\[
\begin{align*}
x^m \times x^n &= x^{m+n} \\
x^m + x^n &= x^{m-n} \\
(x^m)^n &= x^{mn}
\end{align*}
\]
LESSON PLAN 15

**Topic:** - Laws of exponents (Negative Powers (Laws IV, V)

**Entry behavior:** It is assumed that are able to the students

(1) Recall Laws I, II, III of exponents (negative powers known).

Laws to various problems.

**Instructional objectives:** After the instructional treatment is over the students will be able to

♦ prove and apply law IV that for any rational number \( x \neq 0 \), integer \( m \) (positive or negative)

\[
(x \times y)^m = x^m \times y^m
\]

♦ apply law V that for any \( x \neq 0 \), \( x^0 = 1 \)

♦ simplify complex expressions related to the laws IV and V.

**P.K Testing**

Fund the value of

1. \((-3)^{-4} = \)
2. \(\left(\frac{5}{7}\right)^{4+2-6} = \)
3. \(2^0 \times 3^0 \times 4^0 = \)
4. \((2^4)^3 = (2)^{\frac{1}{(10)}} = \)

**Presentation of new material**

"we will solve some more problems by apply laws of exponents to continue learning"

<table>
<thead>
<tr>
<th><strong>Teacher activity and Teaching Point</strong></th>
<th><strong>B.B. work</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Law IV:</strong> If ( x ) is a non-zero rational number, then ( x^0 = 1 )</td>
<td>(2^1 \times 2^3 = 2^{1+3} = 2^4)</td>
</tr>
<tr>
<td></td>
<td>(2^1 \times \frac{1}{2^3} = 1) by simplification</td>
</tr>
<tr>
<td></td>
<td>(2^2 = 1)</td>
</tr>
<tr>
<td></td>
<td>(x^0 = 1 ) for ( x &gt; 0 )</td>
</tr>
<tr>
<td></td>
<td>((x \times y)^2 = (x \times y) \times (x \times y))</td>
</tr>
<tr>
<td></td>
<td>(= (x \times x) \times (y \times y))</td>
</tr>
<tr>
<td></td>
<td>(= x^2 \times y^2)</td>
</tr>
<tr>
<td></td>
<td>((x \times y)^-n = \left(\frac{1}{x \times y}\right)^n = \frac{1}{x^n \times y^n})</td>
</tr>
<tr>
<td></td>
<td>(\left(\frac{7}{11} \times \frac{8}{3}\right)^2 = \left(\frac{7}{11}\right)^2 \times \left(\frac{8}{3}\right)^2)</td>
</tr>
<tr>
<td></td>
<td>(\left(\frac{7}{11} \times \frac{8}{3}\right)^3 = \left(\frac{7}{11}\right)^3 \times \left(\frac{8}{3}\right)^3)</td>
</tr>
<tr>
<td></td>
<td>(\left(\frac{7}{11} \times \frac{8}{3}\right)^{\frac{1}{n}} = \left(\frac{7}{11}\right)^{\frac{1}{n}} \times \left(\frac{8}{3}\right)^{\frac{1}{n}})</td>
</tr>
<tr>
<td></td>
<td>((x \times y)^n = x^n \times y^n \times x &gt; 0 ) when ( n ) is any integer</td>
</tr>
</tbody>
</table>

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**Problem 1.** Find the value of

1. \( 4^{x-5} \)
2. \( \left( \frac{5}{7} \right)^{4+2-6} \)
3. \( 2^{a+3} + 4^b \)
4. \( (6^a - 2^b) 	imes (6^a + 2^b) \)

**Problem 2.** Find \( x \) so that

\[
(\frac{5}{3})^{-5} \times \left( \frac{5}{3} \right)^{-11} = \left( \frac{5}{3} \right)^{8x}
\]

"when bases are some powers are added. Equate the powers as bases same and fund the value of \( x \)"

**Problem 3.**
Find \( m \) so that

\[
\left( \frac{2}{9} \right)^{3-6} \times \left( \frac{2}{9} \right)^{2m-1} = \left( \frac{2}{9} \right)^{2n-1}
\]

"Again when bases are same, according to laws of exponents power are added. Equate the powers with RHS to solve \( m \)"

**Problem 4.** Find the reciprocal of rational number

\[
\left( \frac{1}{2} \right)^{-3} \times \left( \frac{2}{1} \right)^{-3}
\]

you know that \( \left( \frac{p}{q} \right)^{-m} = \left( \frac{q}{p} \right)^{m} \)

what is the reciprocal of rational number?"
Problem 5

if \( \frac{p}{q} = \left( \frac{3}{2} \right)^{-2} + \left( \frac{6}{7} \right)^0 \) find \( \left( \frac{p}{q} \right)^{-3} \)

"Express \( \left( \frac{3}{2} \right)^{-2} \) with positive exponent"

what \( \left( \frac{6}{7} \right)^0 \) is ?

"Taking cube on both sides with positive integer to find the value"

\[
\left( \frac{3}{2} \right)^{-2} = \frac{4}{9}
\]

\[
\left( \frac{p}{q} \right)^{-3} = \frac{729}{64}
\]

Problem 6

simplify \( \left( \frac{2}{3} \right)^2 \times \left( \frac{1}{3} \right)^{-2} \times 3^{-1} \times \frac{1}{6} \)

"Apply law \( (x^m)^n = x^{mn} \)

Express terms with negative exponents in positive exponents then solve".

\[
\left( \frac{2}{3} \right)^2 \times \left( \frac{1}{3} \right)^{-2} \times 3^{-1} \times \frac{1}{6}
= \frac{2^2}{3^2} \times 3^2 \times \frac{1}{3} \times \frac{1}{6}
= \frac{2^5}{3^3} \times \frac{1}{2}
= \frac{32}{729}
\]

Problem 7

Show that

\[
\left( \frac{9 \times -11}{13 \times 17} \right)^{-8} = \left( \frac{13}{9} \right)^8 \times \left( \frac{-17}{-11} \right)^8
\]

"Express is the form of positive exponent "By law of exponents we know that

\( (x \times y)^n = x^n \times y^n \)"

LHS

\[
\left( \frac{9 \times -11}{13 \times 17} \right)^{-8} = \left( \frac{13}{9} \right)^8 \times \left( \frac{-17}{-11} \right)^8
= \left( \frac{-99}{221} \right)^8
\]

RHS

\[
\left( \frac{13}{9} \right)^8 \times \left( \frac{-17}{-11} \right)^8
= \left( \frac{13 \times 17}{9 \times -11} \right)^8
= \left( \frac{221}{-99} \right)^8
= \left( \frac{-221}{99} \right)^8
\]

LHS = RHS
### Problem 8

Write the or False i.

i. For all integers \(a\) and \(b\)

\[
(a + b)^2 \geq a^2 + b^2 \quad \text{and} \\
(a - b)^2 \leq a^2 + b^2
\]

you can verify by Taking any values of \(a\) and \(b\).

<table>
<thead>
<tr>
<th>True.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Let (a = 3) (b = 2)</td>
</tr>
<tr>
<td>((a + b)^2 = (3 + 2)^2 = 5^2 = 25)</td>
</tr>
<tr>
<td>(a^2 + b^2 = 9 + 4 = 13)</td>
</tr>
<tr>
<td>((a - b)^2 = (3 - 2)^2 = 1^2 = 1)</td>
</tr>
<tr>
<td>(a^2 + b^2 = 13)</td>
</tr>
<tr>
<td>((a - b)^2 (a^2 + b^2))</td>
</tr>
<tr>
<td>If (a = 2, b = 0)</td>
</tr>
<tr>
<td>((2 + 0)^2 = 2^2 = 4)</td>
</tr>
<tr>
<td>(a^2 + b^2 = 2^2 + 0^2 = 4)</td>
</tr>
<tr>
<td>Here ((a + b)^2 = a^2 + b^2)</td>
</tr>
</tbody>
</table>

II. For all non-zero rational numbers \(x\),

Reciprocal of \(x^m = (\text{reciprocal of } x)^m\)

<table>
<thead>
<tr>
<th>True</th>
</tr>
</thead>
<tbody>
<tr>
<td>Let (x = 2) (m = 4)</td>
</tr>
<tr>
<td>(x^m = (2)^4 = 16)</td>
</tr>
<tr>
<td>Reciprocal of (16 = \frac{1}{16})</td>
</tr>
<tr>
<td>Reciprocal of (x = \frac{1}{2})</td>
</tr>
<tr>
<td>((\frac{1}{2})^4 = \frac{1}{16})</td>
</tr>
<tr>
<td>Lhs = (x^0 \times x^0 = 1 \times 1 = 1)</td>
</tr>
<tr>
<td>Rhs (x^0 = 1 \times 1 = 1)</td>
</tr>
<tr>
<td>Hence, True</td>
</tr>
</tbody>
</table>

(iii) \(x^0 \times x^0 = x^0 / x^0\) is true

For all non-zero values of \(x\)

<table>
<thead>
<tr>
<th>True</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case I</td>
</tr>
<tr>
<td>(a^3 = 3^4 = 81)</td>
</tr>
<tr>
<td>(81 \geq 64)</td>
</tr>
<tr>
<td>Case II</td>
</tr>
<tr>
<td>(b^5 = 4^3 = 64)</td>
</tr>
<tr>
<td>(8 \leq 9)</td>
</tr>
<tr>
<td>(a^b = 3^3 = 9)</td>
</tr>
<tr>
<td>Hence true</td>
</tr>
</tbody>
</table>

#### Practice

73
Find P so that
\[
\left( \frac{-19}{11} \right)^{2p} \times \left( \frac{-19}{11} \right)^{6} = \left( \frac{-19}{11} \right)^{2p-1}
\]

**Recapitulation**

(i) For \( x > 0 \) \( x^0 = 1 \)

(ii) For \( x > 0 \) \( (x \times y)^m = x^m \) is any integer

\( (x \times y)^m = x^m \times y^m \)
LESSON PLAN 16

Topic : Use of exponents in expressing large and small numbers.

Entry behaviour:- It is assumed that the student are able to
(i) express the rational numbers in power notation
(ii) multiply and divide the numbers orally with 10^n(where n is any positive integer)

Instructional objectives : After the instructional treatment is over, the students will able to:
(i) express the rational number in the form of K x 10^n
(ii) write the Kx10^n form in the usual form

P.K testing

(1) \( \left( \frac{1}{2} \right)^3 = \)
(2) 3125=(5)–
(3) 1000000=(10)–
(4) \( \frac{1}{10000} = \frac{1}{(10)^4} = (10)^{-4} \)

Presentation of new material

"Today we will learn to use exponents in expressing Large and small numbers"
"let us see where and how it is used"

In many situations, we come across. Numbers which are very large. For example, the age of universe in years, the mass of Earth in tons, the distance of sun from earth (in Km) are numbers which are very large. Such large numbers are normally approximate, and not exact numbers.

Speed of light in Vacuum = 300000km/s

Or 300000000m/s

=3 x 10^8m/s or 30 x 10^7m/s

k = 3 and n = 8
k = 30 and n = 7
k = 300 and n = 6
1 ≤ K< 10

Speed of light in
Vacuum = 300000 km/s
Or 300000000 m/s

speed of light = 3 \times 10^8 m/s

Age of universes = 8000000000 years
= 8 \times 10^9 years

“For the sake of uniformity we wrote in the form of where K is a terminating decimal such that 1 \leq k < 10 and adjust accordingly.”

“Such large numbers are usually written using exponents with base 1 \leq k < 10
Distance of earth from sun = 1,50,000,000 km

=1.5 \times 10^8 k/m

Exponential notation helps us in writing large numbers, very small number are also written using exponential notation, but this time the exponent n of 10 is negative.

<table>
<thead>
<tr>
<th>Problem 1</th>
<th>Express the numbers in the form of $K \times 10^n$ with the given value of $n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 9800000000, $n = 8$</td>
<td>1. 9800000000, $n = 8$</td>
</tr>
<tr>
<td>&quot;See the value of $n$ II means you have take exponent of $10 = 8$ Adjust the decimal in $K$ accordingly&quot;</td>
<td>$= 9.8 \times 10^8$</td>
</tr>
<tr>
<td>2. 0.0000000000097, $n = -11$</td>
<td>2. 0.0000000000097, $n = -11$</td>
</tr>
<tr>
<td>&quot;Here take exponent of $10 = -11$ Adjust decimal in $K$ accordingly&quot;</td>
<td>$= 9.7 \times 10^{-11}$</td>
</tr>
<tr>
<td>3. 0.000000000000055, $n = -14$</td>
<td>3. 0.000000000000055, $n = -14$</td>
</tr>
<tr>
<td></td>
<td>$= 55 \times 10^{-14}$</td>
</tr>
<tr>
<td>4. 1070000000 ,$n = 9$</td>
<td>4. 1070000000 ,$n = 9$</td>
</tr>
<tr>
<td></td>
<td>$= 1.07 \times 10^9$</td>
</tr>
</tbody>
</table>
Problem 2. Write the numbers in the usual form

<table>
<thead>
<tr>
<th>Problem</th>
<th>Number</th>
<th>Exponent</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$1.001 \times 10^9$</td>
<td>$n = 9$</td>
<td>$1.001 \times 10^9$</td>
</tr>
<tr>
<td>2.</td>
<td>$.65 \times 10^{-6}$</td>
<td>$n = -6$</td>
<td>$.65 \times 10^{-6}$</td>
</tr>
<tr>
<td>3.</td>
<td>$5.9 \times 10^{-9}$</td>
<td></td>
<td>$5.9 \times 10^{-9}$</td>
</tr>
</tbody>
</table>

Problems 3. Express the numbers appearing in the following statements in the form of $K \times 10^n$ where $1 \leq K \leq 10$ and $n$ any integer:

1. Every day, about 1,050,000 kg of pollutants are emitted in Delhi. Pollutants = 1,050,000 kg
   
   "You have to write in form $K \times 10^n$ where $1 \leq K \leq 10$ and $n$ should be less than 10 decimal will come after 1 Adjust n accordingly"

2. In March 2001, the population of India was approximated to be 1,027,000,000, in which 531,200,000 were males and 495,800,000 females.
   
   "What will be the value of $K$?"
   "What is value of $n$?"

1 micron is equal to

<table>
<thead>
<tr>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Females</td>
<td>495,800,000</td>
</tr>
</tbody>
</table>

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\[
\begin{array}{|c|c|}
\hline
\frac{1}{1000000} & = 4.958 \times 10^8 \\
1 \text{ micron} = \frac{1}{1000000} \text{ m} & = \frac{1}{10^6} \text{ m} \\
& = 1 \times 10^{-6} \text{ m} \\
\hline
\end{array}
\]

**Practice:**
Express in the form of \( K \times 10^n \)

1. \( 0.0000000000289, n = -12 \)
2. \( 10800000000000, n = 13 \)

**Recapitulation**
Every number large or small may be expressed in the form of \( K \times 10^n \) where \( K \) is \( 1 \leq k \leq 10 \)
LESSON PLAN 17

Topic : Direct Variation

Entry Behavior : It is assumed that the students are able to
(I) recall the concept of ratio and Proportions
(II) recall the unitary method

Instructional objectives : After the instructional treatment is over, the students will able to
♦ Explain the concept of direct variation
♦ Solve the simple problems related to direct variation.

P.K. Testing
Q1. When speed of car increases, what will happen to time period?
Q2. If 1kg of Potatoes cost Rs10. How much 15kg of potatoes cost?

Presentation of new material
"You know sometimes two related quantities increase or decrease together, but sometimes as quantity increases, the other decreases. This gave rise to the concept of direct variation and increase variation. Today we will take up direct variation.

Teaching Activity and teaching B.B. Work

| Direct variation |
| "suppose that potatoes cost Rs 10kg . To buy 2kg, you have spend double the amount, but to buy half kg you have to spend half the amours . The more you buy, the more you spend and the less you buy the ratio of the quantity of potatoes to its cost remains the same Two quantities varying in this manner are said to vary directly as each other examine the table. And compute the ratio for various values of x and the corresponding values of y" |

<table>
<thead>
<tr>
<th>Wt. In kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>20</td>
</tr>
<tr>
<td>30</td>
</tr>
<tr>
<td>40</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cost In Rs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
</tr>
<tr>
<td>100</td>
</tr>
<tr>
<td>200</td>
</tr>
<tr>
<td>300</td>
</tr>
<tr>
<td>400</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>100</td>
<td>10</td>
</tr>
<tr>
<td>200</td>
<td>10</td>
</tr>
<tr>
<td>300</td>
<td>10</td>
</tr>
<tr>
<td>400</td>
<td>10</td>
</tr>
</tbody>
</table>

| \( \frac{x}{y} = \text{constant (k)} \) |
| \( x = ky \) |

---

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You must have observed but ratio $\frac{x}{y}$ remains constant two quantities $x$ and $y$ vary directly if
i) An increase in $x$ followed by increase in $Y$ in a such a way $\frac{x}{y}$ remains constant
ii) An decrease in $x$ followed by decrease in $x$ $y$ in a such way $\frac{x}{y}$ remains constant

Problem 1. Replace each star in the following tables by a suitable number, if $x$ and $y$ vary directly

<table>
<thead>
<tr>
<th>$X$</th>
<th>7</th>
<th>9</th>
<th>13</th>
<th>*</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y$</td>
<td>21</td>
<td>*</td>
<td>*</td>
<td>63</td>
<td>*</td>
</tr>
</tbody>
</table>

First find the ratio $\frac{x}{y}$

mark each * by a, b, c,

<table>
<thead>
<tr>
<th>$x$</th>
<th>$\frac{7}{21} = \frac{1}{3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>$\frac{9}{a} = \frac{1}{3} \Rightarrow a = 27$</td>
</tr>
<tr>
<td></td>
<td>$\frac{13}{b} = \frac{1}{3} \Rightarrow b = 39$</td>
</tr>
<tr>
<td></td>
<td>$\frac{c}{6} = \frac{1}{3} \Rightarrow c = 21$</td>
</tr>
<tr>
<td></td>
<td>$\frac{25}{d} = \frac{1}{3} \Rightarrow d = 75$</td>
</tr>
</tbody>
</table>

Problem 2: 15 stamps of equal value cost Rs 18.00. How many stamps of the same value can be brought for Rs 36.00?

"Constant a table in which x is no. of stamps $y$= cost of stamps insert the value in the columns The unknown quantity is denoted by x"

<table>
<thead>
<tr>
<th>Stamp</th>
<th>15</th>
<th>X</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost $x$</td>
<td>Rs</td>
<td>Rs</td>
</tr>
<tr>
<td>18</td>
<td>36</td>
<td></td>
</tr>
</tbody>
</table>

$\frac{x}{y} = kz$

15 $\frac{x}{18} = \frac{36}{36}$

$x = \frac{15 \times 36}{18} = 30$

Problems 3: An agent reserves a commission of Rs 73 on the sales of Rs 1000. How much commission will he get on sales on Rs. 100?
Draw a table insert values in the corresponding columns of sales and commission the quantity vary directly

<table>
<thead>
<tr>
<th>Sales (in Rs)</th>
<th>1000</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commission (in Rs)</td>
<td>73</td>
<td>x</td>
</tr>
</tbody>
</table>

\[
\frac{1000}{73} = \frac{100}{x} \\
x = \frac{100 \times 73}{1000} \\
x = Rs.3
\]

Problem 4 If the thickness of 500 sheets of paper is 3.5 cm, then what would be the thickness of 275 sheets of this paper?

<table>
<thead>
<tr>
<th>No of sheets</th>
<th>500</th>
<th>275</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thickness (in cm)</td>
<td>3.5</td>
<td>x</td>
</tr>
</tbody>
</table>

Both quantities vary directly, 
\[
\frac{500}{275} = \frac{3.5}{x} \\
x = \frac{275 \times 3.5}{500} = \frac{275 \times 3.5}{500 \times 10} = \frac{1925}{1000} = 1.925
\]

thickness of 275 sheets = 1.925 cm

Problem 5 If the cost of 13m of a certain kind of plastic sheet is Rs 1395, then what would it cost to buy 105m of such plastic sheet

<table>
<thead>
<tr>
<th>Length of</th>
<th>93</th>
<th>105</th>
</tr>
</thead>
<tbody>
<tr>
<td>plastic sheet (in cm)</td>
<td>1395x105</td>
<td>x = 1525</td>
</tr>
<tr>
<td>------------------------</td>
<td>----------</td>
<td>---------</td>
</tr>
<tr>
<td>Cost (in Rs.)</td>
<td>1395</td>
<td>x = 1525</td>
</tr>
<tr>
<td>93</td>
<td>105</td>
<td></td>
</tr>
<tr>
<td></td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>1395</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>x = 105×1395</td>
<td>93</td>
<td>1525</td>
</tr>
<tr>
<td>cost of 105 m sheet =</td>
<td>Rs 1525</td>
<td></td>
</tr>
</tbody>
</table>

**Problem 6** Salma types 540 words during half an hour. How many words would she type in 6 minutes?

```
"Draw the table make columns. write the information in the table" 
"solve for x"
```

<table>
<thead>
<tr>
<th>No. of words</th>
<th>540</th>
<th>x</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time in mts.</td>
<td>30</td>
<td>6</td>
</tr>
</tbody>
</table>

\[
\frac{1}{2} \text{ hour} = \frac{1}{2} \times 60 = 30 \text{ mts.} \\
\frac{540}{30} = x \\
x = \frac{540 \times 6}{30} = 108
\]

**Practice**

If 30 meters of cloth cost Rs 1455, how many metres of it can be bought for Rs 679?

Recapitulation:

Two quantities x and y are said to vary directly as each other, if they increase or decrease together in such a manner that the ratio of their corresponding values remains constant

\[
\frac{x}{y} = k
\]

**LESSON PLAN 18**

**Topic**: Inverse Variation

**Entry Behaviour**: It is assumed that are students are able to

(1) recall the concept of direct variation
solve problems related to direct variation

**Instructional objectives** :- After the instructional treatment is over the students will able to
- explain the concept of inverse variation
- distinguish between inverse and direct variation.
- solve simple problems of inverse variation.

**P.K Testing.**
Q1. Find a and b in the following table if x and y vary directly

<table>
<thead>
<tr>
<th>X</th>
<th>10</th>
<th>8</th>
<th>a</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>15</td>
<td>b</td>
<td>24</td>
</tr>
</tbody>
</table>

Q2. What is direct variation

**Presentation of new material** : “Today we will learn to solve problems related to inverse variation.”

<table>
<thead>
<tr>
<th>Teacher activity and teaching point</th>
<th>B.B. work</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;In case of direct variation, the two quantities increase or decrease together in the same ratio. Sometimes, corresponding to an increase in the first quantity, these is a decrease in the second, and when there is a decrease in the first quantity there is a corresponding increase in the second. For example, if 4 persons can do a job in 6 days, then if we put only one person on the same job, he would require 4 times as much time. Examine the table In case of direct variation, the ratio between any two entries in row exactly the</td>
<td></td>
</tr>
<tr>
<td>No. of persons</td>
<td>1</td>
</tr>
<tr>
<td>Days</td>
<td>24</td>
</tr>
<tr>
<td>$1 \times 24 = 2 \times 12 = 4 \times 6 = 6 \times 4$</td>
<td></td>
</tr>
<tr>
<td>$8 \times 3 = 12 \times 2 = 24$</td>
<td></td>
</tr>
<tr>
<td>$xy = k$</td>
<td></td>
</tr>
<tr>
<td>$k = \text{fixed number}$</td>
<td></td>
</tr>
</tbody>
</table>
same as that between the corresponding entries. In the other row. Here you observe the product of the corresponding values of two quantities is constant. If $x_1, x_2$ are the values of $x$ corresponding to the values of $y_1, y_2$  

$$x_1y_1 = x_2y_2 (= K)$$

**Problem 1.** $u$ and $v$ vary directly as each other. When $u$ is 10, $v$ is 15, which of following is a possible pair of corresponding values of $u$ and $v$?

(I) 2 and 3  
(II) 8 and 12  
(III) 15 and 20  
(IV) 25 and 37.5

In case of direct variation

$$\frac{u}{v} = \frac{10}{15} = \frac{2}{3} = k$$

Ans: (I) 2 and 3

**Problem 2.** $x$ and $y$ vary inversely as each other when $x$ is 10, $y$ is 6 which of following is not a possible pair of corresponding values of $x$ and $y$?

(I) 12 and 5  
(II) 15 and 4  
(III) 25 and 2.4  
(IV) 45 and 1.3

In case of inverse variation

$$xy = 10 \times 6 = 60 = k$$

Ans: (IV)

**Problem 3:** A train is traveling at a speed of 50km/hr. How much distance would it travel in 12 minutes?

**Teacher activity and Teaching Point**

"If Speed = 50 km/hr. It means that train covers 50km in 1 hour. Convert distance into metres and time into minutes. As we have to find the distance corresponding to 12 minutes."

"What kind of variation is distance and time?. As distance

<table>
<thead>
<tr>
<th>Teacher activity and Teaching Point</th>
<th>B.B. work</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;If Speed = 50 km/hr. It means 50000 x that train covers 50km in 1 hour. Convert distance into metres and Distance in min.</td>
<td>50000 12</td>
</tr>
<tr>
<td>time into minutes. As we have to Time in min.</td>
<td></td>
</tr>
</tbody>
</table>
Problem 4: Shallu cycles to her school at an average speed of 12 km/hr. It takes her 20 minutes to reach the school. If she wants to reach her school in 15 minutes, what should be her average speed?

"It is clear that if you increase your speed you will reach in less time and if you decrease your speed you will reach in more time."

“What kind of variation is it?”

<table>
<thead>
<tr>
<th>Speed(km/hr)</th>
<th>12</th>
<th>x</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time(in min.)</td>
<td>20</td>
<td>15</td>
</tr>
</tbody>
</table>

Inverse Variation

\[
x = \frac{12 \times 20}{15} = \frac{240}{15} = 16
\]

Average speed = 16 km/hr

Problem 5. Twenty pumps can empty reservoir in 12 hours. In how many hours in 45 such pumps do the same work?

“If number of pumps are increased then the work is done is less time.”

<table>
<thead>
<tr>
<th>No of pumps</th>
<th>20</th>
<th>45</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (in hours)</td>
<td>12</td>
<td>x</td>
</tr>
</tbody>
</table>

Inverse Variation

\[
x = \frac{20 \times 12}{45} = \frac{240}{45} = \frac{16}{3} = 5 \frac{1}{3} \text{ hours.}
\]

Practice

A contractor has a workforce of 420 men who can finish a certain piece of work in 9 months. How many extra men must he employ to complete the job in 7 months.
Recapitulation  Two quantities x and y are said to vary inversely as each other if an increase in x causes proportionate decrease in y and vice versa such that their product remains constant $x y = K$
LESSON PLAN 19

Topic: Direct and Inverse variation

Entry Behaviour: It is assumed that the students are able to
(I) recall the concept of direct and inverse variation
(II) solve problems related to direct and inverse variation

Instructional objectives: After the instructional treatment is over, the students are able to:
♦ identify direct or inverse variation in the given statements.
♦ solve various problems related to time and work and time and distance by employing the correct ratio.

P.K. Testing
Q1. What kind of variation is in each case?
   (I) The length $x$ of a journey by bus and the price $y$ of the ticket.
   (II) The number $x$ of the persons hired to construct a wall and time $y$ taken to finish the job.
   (III) The speed $x$ of the car and the time $y$ taken to each the office.
   (IV) Distance covered by the car $x$ and amount of petrol $y$ needed by the car.

Presentation of new material
"you know both direct variation and inverse variation and you have solved some problems related both the variation. Now will solve more problems to continue learning"

Teacher activity and Teaching

B.B. Work

Point

Problem 1.
If 1800 persons can finish the construction of a building in 40 days, how many persons are needed for the construction in 24 days. 

"As you know if no. of person are increased then time required to do the job is less. If no of persons are reduced then time required to do the job is more." What kind of variation is it?" Again you draw the table
Problem 2 A stock of food grain is enough for 500 persons for weeks. How long will the stock last for 400 persons?

As the number of people increased the stock will last for less time. “Again it is inverse variation”

<table>
<thead>
<tr>
<th>No. of person</th>
<th>500</th>
<th>400</th>
</tr>
</thead>
<tbody>
<tr>
<td>No of weeks</td>
<td>2</td>
<td>x</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
500 \times 2 &= 400 \times x \\
\frac{500 \times 2}{400} &= x \\
x &= \frac{5}{2} = 2.5
\end{align*}
\]

no of weeks = \(2\frac{1}{2}\)

Problem 3. A shopkeeper has just enough money to buy 52 cycles worth 525.00 each. If each cycle were to cost Rs. 21.00 more, then how many cycles would he be able to buy with that amount of money?

“what is to be found?” “If the Old Price = Rs. 525 price is raised then what New Price = Rs 525 + 21 = happens to the amount of the Rs 546 commodity in available money?”

“again it is inverse variation”

<table>
<thead>
<tr>
<th>No. of cycle</th>
<th>52</th>
<th>x</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>Rs</td>
<td>Rs 546</td>
</tr>
<tr>
<td></td>
<td>525</td>
<td>546</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
52 \times Rs.525 &= x \times Rs.546 \\
x &= \frac{52 \times 525}{546} \\
No of cycles &= 50
\end{align*}
\]

Problem 4. A contractor who had a work force of 560 persons, undertook complete a portion of a stadium in 9 months. He was asked to complete the job in 7 months. How many extra persons had he to employ?
What kind of variation is it: no. of persons x hired to construct a stadium and time required to finish it "more the people, lesser the time"

Problem 5. The following reading on pressure (P) and volume (V) at a constant temperature of a gas were recorded:

<table>
<thead>
<tr>
<th>Pressure (in cm)</th>
<th>75.00</th>
<th>80.00</th>
<th>b</th>
<th>112.50</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume (in ml)</td>
<td>12.00</td>
<td>a</td>
<td>10.00</td>
<td>c</td>
<td>15.00</td>
</tr>
</tbody>
</table>

Assuming that \( P \) varies inversely as \( V \), find \( a, b, c, \) and \( d \).

"How will you solve for \( a, b, c, \) and \( d \) in inverse variation?".

"what type of relation between x and y will you use?"

\[
560 \times 9 = x \times 7 \\
x = \frac{560 \times 9}{7} \\
x = 720
\]

Problem 6. The following reading on volume (V) and absolute temperature (T) on a constant pressures of gas were recorded.

<table>
<thead>
<tr>
<th>Volume (V)</th>
<th>10</th>
<th>12</th>
<th>15</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absolute Temperature (t)</td>
<td>300</td>
<td>a</td>
<td>b</td>
<td>750</td>
<td>375</td>
</tr>
</tbody>
</table>

\( V \) and \( T \) varies directly as \( T \), find \( a, b, c, \) and \( d \).

"What type of relation between x and y in direct variation you use in direct variation"
"insert the values and solve for a, b , c, d"

\[
\frac{x}{y} = k \text{(constant)}
\]

\[
\frac{10}{300} = \frac{1}{30} \\
\frac{12}{a} = \frac{1}{30} \Rightarrow a = 12 \times 30 \\
a = 360
\]

\[
\frac{1}{30} = \frac{15}{c} \Rightarrow b = 30 \times 45 \\
b = 450
\]

\[
\frac{1}{30} = \frac{c}{750} \Rightarrow c = \frac{750}{30} \\
c = 25
\]

\[
\frac{1}{30} = \frac{d}{375} \Rightarrow d = \frac{375}{306} \\
d = 12.5
\]

**Practice**

1. On a particular day, 200 US dollars are worth Rs 9666. On that day in how many dollars could be bought for Rs 5074.65?

**Recapitulation**

(I) If \( x \) and \( y \) vary directly \( \frac{x}{y} = k \)

(II) If \( x \) and \( y \) vary inversely \( xy = k \)
LESSON PLAN 20

**Topic:** Direct and Inverse Variation (contd.)

**Entry Behaviour:** It is assumed that students are able to:

(I) distinguish between direct and inverse variation

(II) solve problems related to direct and variation.

**Instructional Objectives:** after the instructional treatment is over, the student will able to

♦ solve various problems related to time and work, Distance and time by employing the correct ratio.

**P.K Testing.** Tell the variation

1. number of persons x engaged to complete the hall in y days.
2. Distance x covered in a time y
3. The number of pencils x you can buy with Rs 12 and cost y per pencil

**Presentation of new material:**

"we will solve more problems to continue learning"

Problem 1: working 8 haves a day, Anu can copy a book in 18 days. How many hours a day should she work so as to finish the work in 12 days?

**Teacher activity and B.B.Work**

**teaching point**

"As you work for more hours, how will the time period be affected?" “what kind of variation is it?”

<table>
<thead>
<tr>
<th>no of hours</th>
<th>8</th>
<th>x</th>
</tr>
</thead>
<tbody>
<tr>
<td>no of days</td>
<td>18</td>
<td>12</td>
</tr>
</tbody>
</table>

8\times 18 = x \times 12

"Solve for x"

x = \frac{8 \times 18}{12} = 12

No of hours = 12

**Problem 2.** A 270 m long goods train is running at 40.5 km/h how much time will it take to cross a tree?
“To cross a tree a train needs few seconds. Change the time into second and distance in meters.”

Speed = 40.5 km/hr
Distance = 40.5 km = 40500 m
Time = 1 hr = 3600 seconds

Direct Variation

<table>
<thead>
<tr>
<th>Distance (in m)</th>
<th>40500</th>
<th>270</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (in sec.)</td>
<td>3600</td>
<td>x</td>
</tr>
</tbody>
</table>

\[
\frac{40500}{3600} = \frac{270}{x} \\
x = \frac{270 \times 3600}{40500} = 24 \text{ seconds}
\]

Problem 3: A train 450 m long crosses a pole in 22 ½ seconds. What is the speed of the train in km/hr?

“As speed is required in km/hr so convert the units accordingly”

Distance = 450 m = 0.45 km
Time = 22 ½ = 22.5 sec.

\[
\text{Speed} = \frac{\text{Distance}}{\text{Time}} \\
\text{Speed} = \frac{0.45 \times 3600}{22.5} = \frac{45 \times 10 \times 3600}{100 \times 225} = 72 \text{ km/hr.}
\]

Problem 4: A 250 m long train is running at a speed of 55 km/hr. In how much time will it cross a platform of length 520 m?

“convert the units of the distance into metres as the train will cross the platform in seconds.”
“distance traveled by the train in crossing the platform = length of train + length of platform.”

Speed = 55 km/hr
Distance = 55 km = 55000 m
Time = 1 hr. = 3600 sec.

Distance covered to cross = 250 m + 520 m = 770 m

<table>
<thead>
<tr>
<th>Distance (in m)</th>
<th>55000</th>
<th>770</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (in sec.)</td>
<td>3600</td>
<td>X</td>
</tr>
</tbody>
</table>
Problem 5 A train marring at a speed of so km/hr covers a certain distance in 4.5 hours what should be the speed of train to cover same distance in 3 hours?

“How do speed and time vary?”

“What relation do you use in case of inverse variation between the entries in each column?”

<table>
<thead>
<tr>
<th>Speed (in km/h)</th>
<th>80</th>
<th>X</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time in hours</td>
<td>4.5</td>
<td>3</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\frac{80\times4.5}{x} &= x\times3 \\
x &= \frac{80\times4.5}{3} \\
&= \frac{80\times45}{30} \\
&= 120
\end{align*}
\]

speed should be 120 km/hr

Practice
1. A train is running at 45km/h. If t crosses a pole in 15 seconds, find its length
2. How many days will 1600 persons take to constant a bridge, if 600 person can build the same in 60 days?
LESSON PLAN 21

**Topic**: Percentage and its application

**Entry Behaviour**: It is assumed that the students are able to

(I) recall the concept of percentage

(II) convert fraction and a decimal into a percentage

(III) solve simple problems on percentage

**Instructional objectives.** After the instructional treatment is over, the students are able to

♦ convert the ratio, decimal and rational number into percentage and vice-versa.

♦ solve problems related to percentage.

**P.K Testing:**

1. Write as percentage
   
   (a) \( \frac{1}{2} \)
   
   (b) \( \frac{3}{4} \)
   
   (c) 0.25

2. If you score 480 marks out of 600 what is your percentage?

**Presentation of new material**

"you know the concept of percentage . you know that fraction with denominator 100 we will take up a bit different problems on percentage to continue that learning in previous classes."

**Teaching activity and B.B. work**

**Teaching point**

**Problem 1.**

3.4% of b is Rs 68

To find quantity b, if 3.4% of b is Rs 68 “what is % of b” “what is the meaning of “of” ?”

\[ \frac{3.4}{100} \times b = Rs\ 68 \]

\[ \frac{34}{1000} \times b = Rs\ 68 \]

\[ b = Rs\ \frac{68 \times 1000}{34} \]

\[ Rs\ 2000 \]

**Problem 2.**
a person deposits Rs 600 per month in his savings Bank account . If this is 15% of his monthly income, find his monthly income, “what to be found ?”

Let monthly income = x
According to sum

15% of $x = Rs\ 600$

$15 \times \frac{1}{100} \times x = RS\ 600$

$x = \frac{Rs\ 600 \times 100}{15} = Rs\ 40.00$

Problem 2. Govang went to school for 216 days in a full year. If his attendance is 90% find the number of days on which the school was opened?

“what is to be found?”  
“what is given in the statement?”  
“Now solve for x”

Let the number of days on which the school was opened = $x$

According to sum

$90\%$ of $x = 216$

$90 \times \frac{1}{100} \times x = 216$

$x = 340\ days$

Problem 3: Sudha obtained 504 marks in a certain examination if she obtained 63% of the total marks, then find the total marks,

“what is to be found?”  
“what is given in the statement?”  
“now solve for x”

Let the total marks = $x$

According to sum

$63\%$ of $x = 504$

$63 \times \frac{1}{100} \times x = 504$

$x = \frac{504 \times 100}{63}$

$x = 800$

Total marks = 800

Problem 4. Kishan spends 30% of his salary on food and donates 3% of his salary in a charitable trust in a particular month, he spend Rs 2310 on these two terms. What is his total salary for this month?

“what is to be found?”  
“what is given in the problem?”  
“Solve for x”

Let the salary = $x$

According to sum
30% of + 3% of x = Rs2310
\[
\frac{30}{100} x + \frac{3}{100} x = Rs2310
\]
\[
\frac{33x}{100} = Rs2310
\]
\[
x = Rs\frac{310 \times 100}{33} = Rs7000
\]
salary = Rs7000

Problem 5 The strength of students in a school increased by 8%. If the actual increase in the number of students is 160, find the original strength of the school. Also, find the new strength.

“what is to be found?” Let original strength = x
“what is given in the statement?” 8% of x = 106
“solve for x” \[
\frac{8}{100} x = 160
\]
x = 2000
Original strength = 2000
New strength = 2000 + 160
= 2160

Problem 7. 60% of the students in a school are girls. If the total number of girls in a school is 60%, find the total number of students in the school. Also find the number of boys in the school.

“what is to be found?” Let the total number of students = x
“what is given”
60% of x = 690
\[
\frac{60}{100} x = 690
\]
x = \frac{6900}{6} = 1150
Total number of student = 1150
No. of boys = 1150 - 690 = 460
Problem 8 A football team won 40% of the total number of matches it played during a year. If it lost 12 matches in all and no match was drawn, find the total number of matches played by the team during the year.

“What is to be found?” Let total number of matches played = x

“what is given?” Matches won = 40% of x

“how will you find matches lost?” Match lost = (100-40)% of x

= 60% of x

According to the sum

60% of x = 12

“solve for x”

\[
\frac{60}{100} \times x = 12
\]

\[
x = \frac{120}{6}
\]

Total number of matches played = 20

Practice

The value of car decreases annually by 20% if the present value of the car be Rs 225000. what will be its value after two years?

Recapitulation

(i) By percent means \( \frac{1}{100} \)

(ii) By certain percent means many hundredth
LESSON PLAN 22

Topic: Profit and Loss

Entry Behaviour: It is assumed that the students are able to

1. solve various problems on percentage
2. recall the concept of profit and loss

Instructional objectives: After the instructional treatment is over the students will be able to

♦ solve the problems related to profit and profit %.
♦ solve the problems related to loss and loss %.

P.K Testing

Find $\frac{1}{2}$% of Rs 50

2. If a pen cost Rs 8 to a shopkeeper and he sells at Rs 12. will he gain or lose? How much?

3. Profit = _____________
4. Loss = _____________

Presentation of new material

"you know how to find profit and loss will from your previous classes. Today we will learn to express Profit and loss as a percent of cost price.

Teacher activity and teaching B.B. work

point

Teacher explains "suppose that a dealer boys a cooler for Rs 2200 and spend Rs 300 on its repairs. He sells it for Rs 3000, He will get profit"

"now we will learn the formula of profit% or gain%"

You know that

Profit = Rs 3000 – Rs 2500
= Rs 500

In term of gain% or

Profit = Rs 3000 – Rs 2500
= Rs 500

In terms of gain% or

profit% = \frac{\text{Profit}}{cp} \times 100
= \frac{500}{2500} \times 100
= 20%

It he sells the cooler for Rs 2200, he suffers a
Problem 1. A bookseller bought a book for Rs 15 and spent Rs 5 on its binding. He sold it for Rs 24, find gain percent.

"what is given?" C.P. included overhead charges.

<table>
<thead>
<tr>
<th>Cost price of book</th>
<th>= Rs(15+5) = Rs 20</th>
</tr>
</thead>
<tbody>
<tr>
<td>S.P. = Rs 24</td>
<td>S.P &gt; C.P.</td>
</tr>
<tr>
<td>Profit = S.P - C.P</td>
<td>= Rs 24 - Rs 20</td>
</tr>
<tr>
<td>Rs 4</td>
<td></td>
</tr>
</tbody>
</table>
| Profit % = \( \frac{\text{Profit}}{\text{C.P.}} \times 100 \) | = \( \frac{4}{20} \times 5 = 20\% \)

Problem 2: Joshi bought a second hand car for Rs 70000 and spent Rs 5000 on adrenaline and painting etc. He then sold it for Rs 67500. Find given gain or loss percent.

"what is given?"
"If C.P > S.P, is it loss or gain?"
"How do you find loss?" What is the formula of loss % ?

| C.P. of Car = Rs 70000 + Rs 5000 = Rs 75000 |
| S.P. = Rs 67500 |
| C.P > S.P. |
| Loss = C.P - S.P |
| = Rs 75000 - Rs 67500 |
| = Rs 7500 |
| Loss % = \( \frac{\text{Loss}}{\text{C.P.}} \times 100 \) |
| = \( \frac{7500}{75000} \times 100 \) |
| = 10% |
Problem 3: Karim bought 150 pencils of Rs 20 a dozen. His overhead expenses were Rs 200. He sold the pencils at Rs 2.40 each. What was his profit or loss percent?

```
"what is given?"
No. of dozen = 150
cost per dozen = Rs 20
cost of 1500 dozen = Rs 20 \times 150
= Rs 3000
overhead expenses = Rs 200
Total cost = Rs 3200
SP of 1 pencil = Rs 2.40
SP of 1 dozen = Rs 2.40 \times 12
= 28.80
SP of 150 dozen = 28.80 \times 150
= Rs 4320
SP > CP
Profit = SP - CP
= Rs 4320 - Rs 3200
= Rs 1120
Profit % = \frac{1120}{3200} \times 100
= 35%
```

Problem 4: By selling a TV for Rs 10240, John suffered a loss of 20%. Find the CP of the TV.

```
"what is to be found?"
Let the CP of TV = x
SP = 10240
Loss = 20%
Loss% = \frac{Loss}{CP} \times 100
Loss% = \frac{CP - SP}{CP} \times 100
```

"substitute the values which are known and do the calculations carefully"
Problem 5 : By selling bucket for Rs.240, a blacksmith loses 20% of his cost. If he sells it for Rs 360, what profit or loss would be there for him?

**“what is to be found?”**

In 1st case, SP = Rs 240
And Loss 20% so we need to find CP

Let the CP = x

Case 1
SP = Rs240
Loss = 20%

\[
\text{Loss}\% = \frac{\text{Loss}}{\text{CP}} \times 100
\]

\[
\frac{x - 240}{x} \times 100 = 20
\]

\[
x = \frac{5 \times 1200}{4} \Rightarrow x = \text{Rs} 12800
\]

CP of TV = Rs 12800

Problem 6 : If by selling a bed sheet for Rs 150, a passion loses 4% for what amount should he sell it so as to gain 20%
"have also, we need is find C.P first." "substitute the values in the formula and calculate CP?"

**Case 1**

\[ SP = \text{Rs} 150 \]

Let \( CP = X \)

Loss\% = 4\%

Loss \% = \Rightarrow \left( \frac{CP - SP}{CP} \right) \times 100

\[
4 = \left( \frac{x - 150}{x} \right) \times 100
\]

\[
1 = \frac{x - 150}{x}
\]

\[
x = 25 \times 150
\]

\[
x = 3750
\]

\[
x = \frac{3750}{24}
\]

\[ CP = \text{Rs} 156.15 \]

**Case II**

"In the second case, we need to find S.P. if we want to have gain 20%"

Let S.P = X

Gain = 20\%

Gain = \left( \frac{SP = CP}{CP} \right) \times 100

\[
20 \times 156.15 = X - 156.15
\]

\[
\frac{312.30}{10} = x - 156.15
\]

\[
312.30 + 156.15 = x
\]

\[ SP = \text{Rs} 187.380 \]

**Practice**

A dealer gains 7\% by selling a sofa set for Rs 3852. Find his gain percent if he sells it for Rs 4050.

**Recapitulation**

(I) If S.P > C.P, Profit = S.P - C.P

(II) \[ \text{Profit} \% = \left( \frac{\text{Profit}}{CP} \right) \times 100 \]

(III) If C.P > S.P, Loss = C.P - S.P

(IV) \[ \text{Loss} \% = \left( \frac{\text{Loss}}{CP} \right) \times 100 \]
LESSON PLAN 23

**Topic**: Simple Interest

**Entry Behaviour**: It is assumed that students are able to

(I) recall the concept of simple interest

(II) calculate simple interest by unitary method

**Instructional objectives**: After the instructional treatment is over, the student will be able to

♦ solve problems related to simple interest.

♦ calculate amount in the given problems.

♦ solve some complex problems related to simple interest and amount.

**P.K. Testing**

Q1. Find the interest on 5000 for 3 at the rate of 5% per annum.

How do you calculate amount after 3 years?

**Presentation of new material**

"you have read S I in class VI and you have calculated it by using the unitary method. we now develop a formal for calculating SI."

<table>
<thead>
<tr>
<th>Teacher activity and teaching point</th>
<th>B.B. work</th>
</tr>
</thead>
</table>
| Now let the Principal be P, rate of interest R% pr annum and time T years. "you have calculated SI by using unitary method." | Rate =R% Per annum  
Interest on Rs 100 for 1 year = Rs R  
Interest on Rs 100 for 1 year = Rs R

Interest on Rs P for 1 year = Rs \( \frac{P\times R}{100} \)  
Interest on Rs P for T years = \( \frac{P\times R\times T}{100} \)  
\[ I = \frac{P\times R\times T}{100} \]  
\[ P = \frac{100\times I}{R\times T}; R = \frac{I\times 100}{P\times T}; T = \frac{100\times I}{P\times R} \] |

From the formula of I, we can calculate other quantities when they are unknown. When word interest is used, it means simple interest."
Problem 1. Find Simple interest when Principal =Rs2850 , rate of interest per annum = $3\frac{1}{2}\%$, Time period = 8 months

"First write the given information"

Principal (P) = Rs 2850
Rate of Interest (R) = $3\frac{1}{2}\% = \frac{7}{2}$
Time period = 8 months = $\frac{8}{12}$ years

"Convert months into year as rate is per annum"

"Calculate simple interest"

\[
\text{SI} = \frac{P \times R \times T}{100} = \frac{95 \times \frac{7}{2} \times \frac{8}{12} \times \frac{1}{100}}{10} = 66.50
\]

Problem 2. Find Principal when SI= Rs72, Rate = 3% per annum and time = 3 months

"first write the given information convert months into years if required, write the formula and substitute the values in it"

\[
\text{SI} = \frac{P \times R \times T}{100}
\]

\[
72 = P \times 3 \times \frac{3}{12} \times \frac{1}{100}
\]

\[
P = \frac{72 \times 12 \times 100}{3 \times 3} = \text{Rs 9600}
\]

Problem 3 : Veena deposited Rs 7200 in a finance company which pays 15% interest per year. Find the amount she is expected to get after $4\frac{1}{2}$% year
Problem 4: A sum of money becomes $\frac{7}{4}$ of itself in 6 years at a certain rate of interest. Find the rate of interest.

Let the principal = $P$

Time = 6 years

Amount = $\frac{7}{4}$ of $P$

Rate of interest = $R\%$

\[
A = P + SI
\]

"Find the value of SI in terms of $x$"

\[
-x \times x = SI = \frac{3x}{4}
\]

"Substitute the values"

\[
SI = \frac{P \times R \times T}{100} = \frac{3 \times 100 \times 6}{4 \times 6} = 12.5\%
\]

Problem 5. In how much time will a sum become double of itself at $4\frac{1}{2}\%$ per simple interest.
"In this problem, amount doubled the Principal sum at rate of \( \frac{25}{2} \) % in a certain period?

You have to calculate Time period. "again calculate Interest in term of \( x \) and the find the value of \( T \) by substituting in the formula"
LESSON PLAN 24

**Topic**: Multiplication of monomials

**Entry behaviour**: It is assumed that the students are able to

(I) identify monomials, binomials and trinomials

(II) add and subtract the like monomials and like term in binomicals and trinomials.

(III) recall the laws of exponents

**Instructional objectives**: After the instructional treatment is over, the students will be able to

♦ multiply the simple and complex monomials.

♦ evaluate the product for the given values of literals.

**P.K. Testing**
1. Identify monomials binomials and trinomial
   - $2xy, -4x + 7y, a + b + t, \frac{4}{9}yx^2$ which of following are like terms
   - $2x, 6xy, 8x, \frac{-11}{4}x, -19yx^2$

2. Add $x^2 + yz - 2$ and $-5x^2 + 9yz$
3. Subtract $2xy + z^2$ from $3z^2 - 4xy$
4. $2^2 \times 2^2 = 2^5$
5. $x^6 \times x^7 = x^{13}$

**Presentation of new material**

"Today we will learn about the multiplication of monomials"

<table>
<thead>
<tr>
<th>Teacher activity and teaching point</th>
<th>B.B. work</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiplication of monomial</td>
<td></td>
</tr>
<tr>
<td>1. $2a$ and $3a$</td>
<td>Multiply</td>
</tr>
<tr>
<td>&quot;To multiply the Monomials first separate and multiply the numerical coefficient. Literals can be separated and like ones are solved by applying laws of exponents&quot;</td>
<td>$1.2a \times 3a = 2a \times 3a = (2 \times 3)(a \times a) = 6a^2$</td>
</tr>
<tr>
<td>2. $7x^2 \times 7xyz$</td>
<td>$(7x - 7)(x^2 \times xyz)$</td>
</tr>
</tbody>
</table>
### Problem 2.7

Multiply the like base what will \(x^2 \times x\) becomes?

### Problem 3

\(\frac{2}{5} x^2 y^3\) and \(\frac{10}{17} xy^2\)

"Separate the coeff. and literals. Separate the like bases and multiply them by using laws of exponent"

### Problem 4

\(5a^2b^3\left(3b^2c^4\right)\)

"Separate the numerically coeff. and literals what will \(a^2 \times a, b \times b^2\) become?"

### Problem 5

\(\left(\frac{2}{3}xyz\right)\left(\frac{3}{4}x^2y^3z^2\right)\left(\frac{4}{5}x^3y^2z^3\right)\)

"Separate the numerical coefficients and literals and multiply them. what will \(x \times x^3 \times x^3, y \times y^2 \times y^3\) and \(2 \times 2^2 \times 2^3\) become?"

### Problem 6

\(\left(a^{50}b^{57}\right)\left(b^{49}c^{67}\right)\left(c^{33}d^{100}\right)\left(d^{100}a^{100}\right)\)

"Separate the like literals what will \(a^{50}d^{100}, b^{51}b^{49}, c^{67}c^{33}\) become?"

### Problem 7

\(0.9ab\left(-0.3ab^2c^3\right)\left(-2.0a^2c^3\right)\left(0.5\right)\)

"what is the product of \((0.9 \times 0.3 \times -2.0 \times 0.5)\)?"

"what will \(a \times a^2, c^3 \times c^3\) become?"
8. Multiply 0.3 and -100x²y³ verify the result for x = 0.1 and 0.3xy - 100x²y³
y = -10
"find the product of 0.3xy and -100x²y³."
"To verify the result substitute the values of x = 0.1 and y = -10 the monomials and in the product obtained."
"what do you find "
"the result is sovled"

= (0.3)(-100)(x²y³)
= (-30)(x²y³)
for x = 0.1 and y = -10
lhs
(0.3 x 0.1 x -10) x (-100 x (0.1²))

9. Find the value of (32a⁴b²)(-100a⁶b³)

at a = 1, b = \frac{1}{2} after expressing it as a monomial

"separate the numerical coefficients and literals and multiply them."
"substitute the value of a and b in the product and obtain the numeric value."

(32a⁴b²)(-100a⁶b³)(0.5a³b³)
(32-100 x 0.5)(a⁴b² x a³b³)
(-1600)(a⁵b⁵) = -1600a⁵b⁵
for a = 1, b = \frac{1}{2}

= -1600 x (\frac{1}{2})⁵
= (-1600) x 1 x \frac{1}{2} x \frac{1}{2} x \frac{1}{2} x \frac{1}{2}
= -1600 x \frac{1}{32}
= -50

Practice:
Evaluate the product of 2x³, 9y³ and -0.2 when x=1 and y=2

Recapitulation
Product of two monomials is the product of their coefficients and the literals in the two monomials
LESSON PLAN 25

Topic: Multiplication of monomial (contd.)

Entry behaviour: It is assumed that the students are able to
(I) multiply the given monomials
(II) verify the product at simple values of the literals

Instructional objectives: After the instructional treatment is the students will able to

♦ write the factors of the given monomial.
♦ represent the products of two monomials geometrically

P.K Testing
Q1. Find the product of
(I) \(2ab^2c\) and \(-12a^2bc^2\).
(II) \((0.5x^3yz^2)(0.25 \times x^2y^2z^5)\)
(II) \(\left(\frac{24}{100} \times a^{10}b^2\right) \left(\frac{100}{24}a^2b^{10}\right)\)

Q2. what is the value of \((2xy^2)(4x^2y)(2xy)\) at \(x = 1, y = 1\)

Presentation of New material
Today we will simplify problems with bit complicated values and will try to represent the products geometrically."

Problem 1. Evaluate the product of 6.4
\(6.4x^3, 8.0y^3\) and \(-1.6x^2y^5\) when \(x = 1.0\) and \(y = 0.5\).
First multiply the monomial
"what is the product of two monomials ?"
Find the value at \(x = 1.0,\) and \(y = 0.5.\)

\[
\begin{align*}
(6.4x^3)(8.0y^3)(-1.6x^2y^5) &= (6.4 \times 8.0 \times 1.6) (x^3 \times x^2 \times x \times y^3 \cdot y^5) \\
&= (-81.92) (x^5 \times y^{15}) \\
&= -81.92 (x^5 \times y^{15}) \\
&= 81.92 (x^5 \times y^{15}) \\
&= 81.92 \times (0.5)^5 \\
&= 81.92 \times 0.03125 \\
&= 2.56
\end{align*}
\]
### Problem 2.
Simplify

1. \((-3a) \times (-4a^2x^2) \times (5.5x^3)\)

```
separate the numeric and literals

\[ 2 \left( \frac{3}{4} p^2q^r \right) \times (5pq^2) \times \left( \frac{-8}{150} r^2 \right) \]
```

"separate the coefficients and literals"

- \((-3a) \times (-4a^2x^2) \times (5.5x^3)\)
- \((-3 \times 4 \times 5.5) \times (a^3x^5)\)
- \(63a^3x^5\)

- \(\frac{3}{4} p^2q^r \times (5pq^2) \times \left( \frac{-8}{150} r^2 \right)\)
- \(\frac{3}{4} \times 5 \times \frac{-8}{150} \times (p^2q^r \times pq^2 \times r^2)\)
- \(-\frac{1}{150} \times p^5 \times p \times q \times q^2 \times r \times r^2\)
- \(-p^5q^r r^3\)

### Problem 3:
Verify that the following relation are true.

\[
(5abc)^3 \left( \frac{-1}{500} a^5b^{10}c^{300} \right) - \left( \frac{-1}{500} a^5b^{10}c^{300} \right)(5abc)
\]

**LHS**

\[
\left( \frac{-1}{500} a^5b^{10}c^{300} \right)(5abc)
\]

**RHS**

\[\left( \frac{-1}{500} a^5b^{10}c^{300} \right)(5abc)\]

### Problem 4:
Find the monomials with positive integer coefficients, whose product is the given monomial.
1. $\text{xyz}$
   “Tell the two monomials whose product is $\text{xyz}$”
   “You can write any one of it”

2. $a^2b$
   “tell the monomials whose product is $a^2b$”

<table>
<thead>
<tr>
<th>Problem 5: Represent the following products geometrically.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3x \times 4x$</td>
</tr>
<tr>
<td>“3x can be represented by a breadth, 4x can be represented by a length.”</td>
</tr>
<tr>
<td>“what is the area of the rectangle?”</td>
</tr>
<tr>
<td>“You can check the products also”</td>
</tr>
<tr>
<td>$3x \times 4x$</td>
</tr>
<tr>
<td>$3x$</td>
</tr>
<tr>
<td>$12x^2$</td>
</tr>
<tr>
<td>Length=4x</td>
</tr>
<tr>
<td>Breadth=3x</td>
</tr>
<tr>
<td>Area of rectangle=$L \times B$</td>
</tr>
<tr>
<td>Area=$3x \times 4x=12x^2$</td>
</tr>
<tr>
<td>$3x \times 4x=(3 \times 4)(x \times x)=12x^2$</td>
</tr>
</tbody>
</table>

Practice
Find the monomials whose product is the given monomials
(I) $p^q r^s$  (II) $a^2b^c$
LESSON PLAN 26

**Topic**: Multiplication of monomial and a binomial.

**Entry behaviour**: It is assumed that the students know how to:

(I) multiply the complex monomial

(II) verify the product at the given values of literals

**Instructional objectives**: After the instructional treatment is over, the students are able to

♦ identify the numerical coefficient in the product of monomials.

♦ identify the literal part in the product of monomials.

♦ multiply monomial with a binomial.

♦ calculate and verify the product for the given values of literals.

♦ represent the product of monomial and binomial geometrically.

**P.K. Testing**

Q1. What is the product of

(I) \(0.2x^2y \times 0.5xy^2 \times 2\)

(II) \(\frac{1}{2}a^2bc \times \frac{4}{3}ab^2c \times 9abc^2\)

Q2. Find the value of \((1.5 p^2q)(0.2pq^2) \times (p^2q^3)\) at \(p = 1, q = -1\)

Q3. Tell two monomials whose product is 7 pq

**Presentation Of New Material**

"Today we will learn to find the product of monomial and binomial and simplification of complex expression"

**Problem 1**: Find the numerical coefficient in the product
Problem 2 Find the literal part in the product $3.998a^2, -171.47b^2, 36.01c^2$ and $2d^2$.

$$(-3.998 \times -171.47 \times 36.01 \times 2)$$

"separate the numerical coefficient and literals here you need not multiply the numerical coefficients"

"what is the literal part?"

Literal part = $a^2b^2c^2d^2$

---

Problem 4: Express as numerical and then evaluate at $a=2$ and $b=1$

<table>
<thead>
<tr>
<th>Expression</th>
<th>Numerical Form</th>
<th>Evaluation at $a=2$ and $b=1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3a + 13 - 2a^2$</td>
<td>$(3a + 13) - 2a^2$</td>
<td>$-2a^2$</td>
</tr>
<tr>
<td>$3ab^2$ and $\left(\frac{7}{9}a^2c + \frac{2}{3}a^2b^3\right)$</td>
<td>$3ab^2 \times \left(\frac{7}{9}a^2c + \frac{2}{3}a^2b^3\right)$</td>
<td>$\frac{7}{3}a^2b^3$</td>
</tr>
<tr>
<td>$\frac{7}{3}a^2b^3 - 2a^2b^3$</td>
<td>$\frac{7}{3}a^2b^3 - 2a^2b^3$</td>
<td>$\frac{7}{3}a^2b^3 - 2a^2b^3$</td>
</tr>
</tbody>
</table>
"Multiply first by \(-2.7a^2\) with \(-0.4a^2\)

Find the value of product at \(a = 2\) and \(b = 1\),

Find the value of product at \(a = 2\) and \(b = 1\)

\[
\begin{align*}
-2.7a^2(0.3b^2 - 0.4a^2) &= (-2.7a^2 \times 0.3b^2) + (2.7a^2 \times 0.4a^2) \\
&= (-2.7 \times 0.3 \times a^2 \times b^2) + (-2.7 \times 0.4 \times a^2 \times b^2) \\
&= (-8.1ab^2) + (1.08a^4)
\end{align*}
\]

\[
\begin{align*}
\text{At } a &= 2 \text{ and } b = 1 \\
&= -0.81a^2b^2 + 1.08a^4
\end{align*}
\]

**Problem 5:** Verify the result for \(x = 2, y = 1\) and \(z = -1\)

"First multiply

\[
\frac{1}{2} x^2 y z^3 (x^2 + y^2)
\]

solve and express it as binomial"

"To verify substitute the values of \(x = 2, y = 1\) and \(z = -1\) in LHS and then in the Product at RHS"

"what do you find?"

\[
\begin{align*}
\text{LHS} &= \frac{1}{2} \times (2)^2 \times (1)(-1)(4 + 1) \\
&= \frac{1}{2} \times 8 \times (-1)(5) = -20
\end{align*}
\]

\[
\begin{align*}
\text{RHS} &= \frac{1}{2} \times (2)^2 \times (1)^2 \times (-1)^2 + \frac{1}{2} \times (2)^2 \times (1)^2 \times (-1)^2 \\
&= \left(\frac{1}{2} \times 32 \times 1 \times -1\right) + \left(\frac{1}{2} \times 1 \times -1\right) \\
&= (-16) + (-4) \\
&= -20 = \text{LHS}
\end{align*}
\]
**Problem 6** Simplify

1. \( a^2 - b^2 + a(a + b) \)
   "Solve the \( a(a + b) \) in the same way as you have done before"
   "Add the like terms" tell the like terms"
   \[
   \begin{align*}
   a^2 - b^2 + a(a + b) &= a^2 - b^2 + (a \times a + a \times b) \\
   &= a^2 - b + a^2 + ab \\
   &= a^2 + a^2 - b^2 + ab \\
   &= 2a^2 - b^2 + ab
   \end{align*}
   \]

2. \( 10p^2 - 6p(p + q) + p(3 - 7p) \)
   "Solve the \(-6p(p + q)\) and \(p(3 - 7p)\)"
   "Take care of -ve sign while opening the bracket"
   "Group the like terms and solve then."
   \[
   \begin{align*}
   10p^2 - 6p(p + q) + p(3 - 7p) &= 10p^2 - [6p \times p + 6p \times q] + [p \times 3 - p \times 7p] \\
   &= 10p^2 - [6p^2 + 6pq] + [3p - 7p^2] \\
   &= 10p^2 - 6p^2 - 6pq + 3p - 7p^2 \\
   &= 10p^2 - 6p^2 - 6pq + 3p + 3p \\
   &= -3p^2 - 6pq + 3p
   \end{align*}
   \]

**Practice**
Simplify \( p^2q^2(2a^2 + b^2) + 4q^2(a^2 - b^2) \)

**Recapitulation**
To multiply a monomial by a binomial, we multiply the monomial with each term of the binomial and add the products.
LESSON PLAN -27

**Topic:** Simplification of complex algebraic expressions.

**Entry Behaviour:** It is assumed that

(I) solve a monomial and a binomial

(II) evaluate and verify the products at the given values of literals

**Instructional objectives:** After the instructional treatment is over, the students will be able to

♦ multiply binomial with binomial

♦ multiply binomial with trinomial

♦ simplify and verify complex algebraic expressions for given values of literals.

**P.K. Testing**

1. multiply \((5a+6 \text{ by } 3a)\) and then find the value of product at \(a = 2\)

2. simplify find the value of product at \(a=2\) \(a(a-b)+b(a-b)\)

**Presentation of new material**

"Today we will do multiplication of two binomials and binomial and trinomial" 

**Teacher activity and teaching point**

1. \(2x + 9\) and \(6x + 5\)

   "From one of the binomial multiply the other binomial with first factor and then multiply with second factor"

   "Add the like terms"

   \(\frac{3}{4}a^2 + 7b\) and \((a^2 + \frac{2b^3}{9})\)

2. "From one of the binomial multiply the other binomial with first factor and then multiply with second factor"

**B.B. Work**

<table>
<thead>
<tr>
<th>1. (2x + 9) and (6x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2(2x+9)(6x+5))</td>
</tr>
<tr>
<td>(= (2 \times 6 \times x \times 2 \times 5 \times x))</td>
</tr>
<tr>
<td>(+ (9 \times 6x + 9 \times 5))</td>
</tr>
<tr>
<td>(= (120x^2 + 135) + (54x + 45))</td>
</tr>
<tr>
<td>(= 120x^2 + 10x + 54x + 45)</td>
</tr>
<tr>
<td>(= 120x^2 + 64x + 45)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2. (\frac{3}{4}a^2 + 7b) and ((a^2 + \frac{2b^3}{9}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{3}{4}a^2 + 7b) and ((a^2 + \frac{2b^3}{9}))</td>
</tr>
<tr>
<td>(= \frac{3}{4}a + \frac{2b^3}{9})</td>
</tr>
<tr>
<td>(= \frac{3}{4}a^2 + \frac{2b^3}{9})</td>
</tr>
<tr>
<td>(= \frac{27}{4}a^2 + \frac{2b^3}{9})</td>
</tr>
<tr>
<td>(= \frac{3}{4}a^2 \times a^2 + \frac{3}{24} \times a^2 b^2 + 7b \times a^2 + \frac{2b^3}{9})</td>
</tr>
<tr>
<td>(= \frac{3}{4}a^2 + \frac{14}{9}a^2 b^2 + \frac{14}{9}b^3)</td>
</tr>
</tbody>
</table>
3. \( \left( 2x - \frac{1}{2} y \right) \) and \( \left( \frac{3}{4} x - 10y + 8 \right) \)

"From the binomial and trinomial multiply"
"the factors with the binomial or trinomial"
"Add the like terms"

\[
\begin{align*}
3. & \quad \left( 2x - \frac{1}{2} y \right) \text{ and } \left( \frac{3}{4} x - 10y + 8 \right) \\
& = \left( 2x - \frac{1}{2} y \right) \left( \frac{3}{4} x - 10y + 8 \right) \\
& = 2x \cdot \frac{3}{4} x - 10y \cdot \frac{3}{4} x - 10y \cdot \frac{1}{2} y + 10y \cdot 8 - \frac{1}{2} y \cdot 8 \\
& = \frac{1}{2} x \cdot \frac{3}{4} x - \frac{1}{2} y \cdot 10y + \frac{1}{2} y \cdot 8x + 10y \cdot 2x \\
& = \frac{3}{2} x^2 - 20xy + 16x - \frac{3}{2} xy + 5y^2 + 4y \\
& = \frac{3}{2} x^2 - 20xy + \frac{3}{8} xy + 5y^2 + 16x - 4y \\
& = \frac{3}{2} x^2 - \frac{160xy - 3xy}{8} + 5y^2 + 16x - 4y \\
& = \frac{3}{2} x^2 - \frac{163xy}{8} + 5y^2 + 16x - 4y \\
\end{align*}
\]

Problem 2.

4. Simplify and verify the result for \( x = 2 \) and \( y = 1 \)

\[
\begin{align*}
\frac{1}{4} \left( 2x^2 - 10y^2 \right) \left( 2x^2 + 10y^2 \right) \\
& = \frac{1}{4} \left( 2x^2 - 10y^2 \right) \left( 2x^2 + 10y^2 \right) \\
& = \frac{1}{4} \left( 2x^2 \cdot 2x^2 + 2x^2 \cdot 10y^2 - 10y^2 \cdot 2x^2 - 10y^2 \cdot 10y^2 \right) \\
& = \frac{1}{4} \left( 4x^4 + 20x^2y^2 - 20x^2y^2 - 100y^4 \right) \\
& = \frac{1}{4} \left( 4x^4 - 25y^4 \right) \\
& = x^4 - 25y^4 \\
\text{LHS} \quad & = \frac{1}{4} \left( 2 \times 4 - 10 \times 1 \right) \left( 2 \times 4 + 10 \times 1 \right) \\
& = \frac{1}{4} \left( 8 - 10 \right) \left( 8 + 18 \right) \\
& = \frac{1}{4} \left( -2 \right) \left( 26 \right) = -9 \\
\text{RHS} \quad & = \left( 2 \right)^4 - 2 \left( 5 \right)^2 \\
& = 16 - 25 = -9 \\
\text{LHS} = \text{RHS}.
\end{align*}
\]
5. \( x^2 + (3x - y)(3x + y + y^2) \)
   "solve \((3x - y)(3x + y + y^2)\) in a same way as you have done before" 

\[
\begin{align*}
5. \ x^2 + (3x - y)(3x + y + y^2) \\
\quad = x^2 + 3x(3x + y + y^2) - y(3x + y + y^2) \\
\quad = x^2 + (3x \times 3x + 3x \times y + 3x \times y^2) \\
\quad \quad - (3x \times y + y \times y + y^2 \times y) \\
\quad = x^2 + 9x^2 + 3xy + 3xy^2 - 3xy - y^2 - y^3 \\
\quad = 10x^2 + 3xy^2 - y^2 - y^3
\end{align*}
\]

"To verify the result substitute the values of \(x\) and \(y\) in LHS and RHs" 

"what do you find?"

6. \((1.5x - 4y)(1.5x + 4y + 3)\)
   Multiply \((1.5x - 4y)\) and \((1.5x + 4y + 3)\)

Verify the result by substituting \(x=2\) and \(y=1\) at LHS and RHS

"What do you find?"
7. express as algebraic expression  
(a + b)(1 - d) + (c + d)(a - b) + 2(ac + bd)  
"solve (a + b)(c - d), (c + d)(a - b) and 2(ac + bd) and then add like term"

$$7. (a + b)(c - d) + (c + d)(a - b) + 2(ac + bd)$$
$$= a(c - d) + b(c - d) + c(a - b) + d(a - b) + 2ac + 2bd$$
$$= ac - ad + bc - bd + ac - bc + ad - bd + 2ac + 2bd$$
$$= ac + ac + 2ac - bd - bd + 2bd$$
$$= 2ac + 2ac - 2bd + 2bd = 4ac$$

**Practice**

$$7(a + b)(a^2 + b) + (2a + b^2)(2a + b^2)(a^2 + b^2 + 2)$$

| Practice | 7. express as algebraic expression  
(a + b)(1 - d) + (c + d)(a - b) + 2(ac + bd)  
"solve (a + b)(c - d), (c + d)(a - b) and 2(ac + bd) and then add like term"

$$7. (a + b)(c - d) + (c + d)(a - b) + 2(ac + bd)$$
$$= a(c - d) + b(c - d) + c(a - b) + d(a - b) + 2ac + 2bd$$
$$= ac - ad + bc - bd + ac - bc + ad - bd + 2ac + 2bd$$
$$= ac + ac + 2ac - bd - bd + 2bd$$
$$= 2ac + 2ac - 2bd + 2bd = 4ac$$

<table>
<thead>
<tr>
<th>7. (a + b)(c - d) + (c + d)(a - b) + 2(ac + bd)</th>
<th>7. (a + b)(c - d) + (c + d)(a - b) + 2(ac + bd)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a + b)(1 - d) + (c + d)(a - b) + 2(ac + bd)</td>
<td>(a + b)(c - d) + (c + d)(a - b) + 2(ac + bd)</td>
</tr>
<tr>
<td>&quot;solve (a + b)(c - d), (c + d)(a - b) and 2(ac + bd) and then add like term&quot;</td>
<td>&quot;solve (a + b)(c - d), (c + d)(a - b) and 2(ac + bd) and then add like term&quot;</td>
</tr>
</tbody>
</table>
**Topic** – Standard identities

**Entry Behaviour**: it is assumed that the student are able to
(I) multiply two binomials (identical and different)
(II) solver algebraic expression

**Instruction objectives**: After the instruction treatment is over, the student will able to
♦ write the product of binomials using standard identities.
♦ represent the identities geometrically.

**P.K. Testing**
1. Find the value of \((x + y)(7x - y)\) at \(x = 1, y = 0\)
2. Find the value of \(p^2 - q^2(p - q)\) at \(p = 2, q = 0\)

**Presentation of new material**:
Now we will learn how to multiply the two like binomials with the help of formulas which are called standard identities.

<table>
<thead>
<tr>
<th>Teacher activity and teaching point</th>
<th>B.B.Work</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Standard identities</strong></td>
<td></td>
</tr>
<tr>
<td>1. ((a + b)(a + b) = a^2 + 2ab + b^2)</td>
<td>((a + b)(a + b))</td>
</tr>
<tr>
<td></td>
<td>(= a(a + b) + b(a + b))</td>
</tr>
<tr>
<td></td>
<td>(= a^2 + ab + ab + b^2)</td>
</tr>
<tr>
<td></td>
<td>(= a^2 + 2ab + b^2).</td>
</tr>
<tr>
<td>2. ((a - b)(a - b) = a^2 - 2ab + b^2)</td>
<td>((a - b)(a - b))</td>
</tr>
<tr>
<td></td>
<td>(= a(a - b) - b(a - b))</td>
</tr>
<tr>
<td></td>
<td>(= a^2 - ab - ab + b^2)</td>
</tr>
<tr>
<td></td>
<td>(= a^2 - 2ab + b^2).</td>
</tr>
<tr>
<td>3. ((a + b)(a - b) = a^2 + b^2)</td>
<td>((a + b)(a - b))</td>
</tr>
<tr>
<td></td>
<td>(= a(a - b) + b(a - b))</td>
</tr>
<tr>
<td></td>
<td>(= a^2 - ab + ab - b^2)</td>
</tr>
<tr>
<td></td>
<td>(= a^2 - b^2).</td>
</tr>
</tbody>
</table>

Teacher explains "we can easily obtain products easily by applying standard identities."
Find the product using a suitable identity

1. \((2y + 5)(2y + 5)\)
   
   "which identity will you use?"

   \[2y + 5\] = \(a^2 + 2ab + b^2\)
   
   \((2y + 5)^2 = 4y^2 + 20y + 25\)

2. \((1.1m + 2.1)(1.1m + 2.1)\)
   
   "Which identity will you use?"

   \((a + b)^2 = a^2 + 2ab + b^2\)
   
   \((1.1m + 2.1)^2 = (1.1m)^2 + 2 \times 1.1m \times 2.1 + (2.1)^2\)
   
   \(= 1.21m^2 + 4.62m + 4.41\)

3. \(\left(\frac{5}{2}x - 7\right)\left(\frac{5}{2}x - 7\right)\)
   
   "Which identity will you use?"

   \((a - b)^2 = a^2 - 2ab + b^2\)
   
   \(\left(\frac{5}{2}x - 7\right)^2 = \left(\frac{5}{2}x\right)^2 - 2 \times \frac{5}{2}x \times 7 + (7)^2\)
   
   \(= \frac{25}{4}x^2 - 35x + 49\)

4. \(2(-8x^3 + 7y^2)^2\)
   
   "Which identity will you use?"

   \((-8x^3 + 7y^2)^2 = 64x^6 - 112xy^2 + 49y^4\)

5. \((2x^3 + 9y^3)(2x^3 - 9y^3)\)
   
   "Which identity will you use?"

   \((a + b)(a - b) = a^2 - b^2\)
   
   \((2x^3 + 9y^3)(2x^3 - 9y^3) = (2x^3)^2 - (9y^3)^2\)
   
   \(= 2x^8 - 81y^6\)
6. \((2r^2 - \frac{1}{400}t^2)^2 - (2r^2 + \frac{1}{400}t^2)^2\)

"which identity will you use so that it can be solved = easily?"

Using a suitable identity:

\[ a^2 - b^2 = (a - b)(a + b) \]

\[
\begin{align*}
&= \left[2r^2 - \frac{1}{400}t^2 - 2r^2 - \frac{1}{400}t^2\right]^2 \\
&= \left[-\frac{2}{400}t^2\right]^2 \\
&= \frac{-2}{400}t^2 \\
&= \frac{-1}{50}t^2.
\end{align*}
\]

7. \((3p + 8q)^2 + (3p - 8q)^2\)

"which identity will you use?"

Using a suitable identity:

\[(a + b)^2 = a^2 + 2ab + b^2\]
\[(a - b)^2 = a^2 - 2ab + b^2\]

\[
\begin{align*}
&= 9p^2 + 2 \times 3p \times 8q + 6q^2 \\
&= 9p^2 + 48pq + 64q^2. \\
&= 18p^2 + 12q^2.
\end{align*}
\]

8. \((m^2 - n^2m)^2 + 2m^2n^2\)

"which identity will you use?"

Using identity:

\[(a - b)^2 = a^2 - 2ab + b^2\]

\[
\begin{align*}
&= 2m^2n^2 + \left(m^2 - n^2m\right)^2 = \left(m^2\right)^2 - 2 \times m^2 \times n^2m + \left(n^2m^2\right) \\
&= m^4 - 2m^3n^2 + n^4m^2 + 2m^3n^2. \\
&= \left(89p - 5q\right)^2 + 1780pq.
\end{align*}
\]

9. \((89p - 5q)^2 + 1780pq\)

"which identity will you use?"

Using identity:

\[(a - b)^2 = a^2 - 2ab + b^2\]

\[
\begin{align*}
&= \left(89p - 5q\right)^2 + 1780pq \\
&= 7921p^2 + 25q^2 - 1780pq + 1780pq \\
&= 7921p^2 + 25q^2.
\end{align*}
\]

**Practice**

\[(am - an)^2 + (am + an)^2\]

**Recapitulation**

\[(a + b)^2 = a^2 + 2ab + b^2\]
\[(a - b)^2 = a^2 - 2ab + b^2\]
\[a^2 - b^2 = (a - b)(a + b)\]
LESSON PLAN 29

Topic standard identities contd.

Entry Behaviours: It is assumed that the student are able to

(I) recall the three identities

(II) apply the suitable identities in multiplying two binomials

(III) represent $x\times y$ geometricly

Instructional objectives. After the instructional treatment is over, the students will able to.

♦ solve square of big numbers with the help of suitable identities

♦ solve complicated algebraic expressions using suitable identities

P.K. Testing

Using a suitable identity find

\[
\begin{align*}
(a^2 + b^2)^2 \\
(6x^2 - 5y)^2 \\
(m^2 - n^2)\cdot 2m'n^2 \\
(1.7p^3 + 1.2q^3)(1.7p^3 - 1.2q^3)
\end{align*}
\]

Presentation of new material

Teacher told “now we will solve the squares of number or multiplication of two numbers with the help of suitable identities."
$92^2$
solve it write the help of identity
write 92 as difference of 100 and 8
break the number in such that a
way squaring of taking squares
of a number easy
$(103)^2$
write 103 as a sum of 100 and 3

$105 \times 95$
"writ 105 as sum of 100 and 5 write 95
as a difference of 100 and 5"

$297 \times 303$
write 297 as difference of 300 and 3
and 303 as sum of 300 and 3".

$51^2 - 49^2$
"which identity will you use?"

$92^2$
$(100 - 8)^2$
$(a - b)^2 = a^2 - 2ab + b^2$
$= (100)^2 - 2 \times 100 \times 8 + 64$
$1000 - 1600 + 64$
$= 10064 - 1600 = 8464$
$(103)^2 = (100 + 3)^2$
$(a + b)(a - b) = a^2 - b^2$
$= (100)^2 - (5)^2$
$= 10000 - 25 = 9975$

$105 \times 95$
"writ 105 as sum of 100 and 5 write 95
as a difference of 100 and 5"

$297 \times 303$
write 297 as difference of 300 and 3
and 303 as sum of 300 and 3".

$51^2 - 49^2$
"which identity will you use?"

$297 \times 303$
$(300 - 3)(300 + 3)$
$\sin g(a - b)(a + b) = a^2 - b^2$
$= (300)^2 - (3)^2$
$= 90000 - 9 = 89991$

$51^2 - 49^2$
using identity
$a^2 - b^2 = (a - b)(a + b)$
$51^249^2 = (51 - 49)(51 + 49)$
$= (2)(100)$
$= 200$
<table>
<thead>
<tr>
<th>Expression</th>
<th>Simplification</th>
</tr>
</thead>
<tbody>
<tr>
<td>$233^2 - 227^2$</td>
<td>using identity $a^2 - b^2 = (a - b)(a + b)$</td>
</tr>
<tr>
<td>Find the value of $a$ if (1) $8a = 35^2 - 27^2$</td>
<td>$8a = 35^2 - 27^2$ using identity $a^2 - b^2 = (a - b)(a + b)$ $8a = (35 - 27)(35 + 27)$ $8a = (8)(62)$ $8a = 8 \times 62$ $a = 62$</td>
</tr>
<tr>
<td>$pqa = (3pq)^2 - (3p - q)^2$</td>
<td>using $a^2 - b^2 = (a - b)(a + b)$ $pqa = (3pq + 3p)(3p + q - 3p + q)$ $pqa = (6p)(2q)$ $pqa = 12pq - a \frac{12pq}{pq}$</td>
</tr>
</tbody>
</table>
Arrange the aboard pieces into a square

- A 5 × 5 square piece
- A 6 × 6 square piece
- Two 5 × 6 rectangular pieces

Arrange the pieces in such a way that their lengths or edges of the pieces coincides with the other pieces.

Practice

Find P

(I) \(10p = 9912 - 9812\)

(II) Arrange 2×2 square piece, 3×3 square piece and two 2×3 rectangular pieces to form a square

Recapitulation

\[(a + b)^2 - a^2 + 2ab + b^2\]

\[(a - b)^2 = a^2 - 2ab + b^2\]

\[(a + b)(a - b) = a^2 - b^2\]
LESSON 30

**Topic** – Factorization of algebraic expressions

Entry Behaviour:- It is assumed that the student are able to complete
(I) compute the HCF of the given numbers
(II) identify the factor of the monomial

Instructional objectives. : After the instructional objectives is over, the students will able to
♦ compute the HCF of the given monomials or algebraic expression.
♦ factorize the given algebraic expression by taking out HCF of the terms constituting the expression.
♦ factorize the expression by regrouping the terms

**P.K. Testing**
1. Find the factors of 21, 30
2. what are common factor 42 and 28
3. what is the HCF of 70 and 90

Presentation of new material
"you know how to find the factors & HCF of numbers we will learn to find factors and HCF of the algebraic expressions methods.

**Problem 1.** Find highest common factors of the monomials

<table>
<thead>
<tr>
<th>Teacher Activity</th>
<th>BB Work</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2x^2$ and $10xy$</td>
<td>$2x^2$ and $10xy$</td>
</tr>
<tr>
<td>&quot;can you tell what will be the factors of $2x^2$?&quot;</td>
<td>$2x^2 \times 1 = 2x^2$</td>
</tr>
<tr>
<td>&quot;what will be the factors of $10xy$?&quot;</td>
<td>$2x \times x = 2x^2$</td>
</tr>
<tr>
<td>&quot;write all the possible products&quot;</td>
<td>$2 \times x^2 = 2x^2$</td>
</tr>
<tr>
<td>&quot;what are the common factors of $2x^2$ and $10xy$?&quot;</td>
<td>factors of $2x^2 = 2, x, x^2, 2x, 2x^2$</td>
</tr>
<tr>
<td>&quot;which is the HCF?&quot;</td>
<td>$10xy \times 1 = 10xy$</td>
</tr>
<tr>
<td>$2x^2$ and $10xy$</td>
<td>$2x \times 2 \times 5 = 10xy$</td>
</tr>
<tr>
<td>$2x \times x \times y = 10xy$</td>
<td>$10xy \times 2 \times y = 5 = 10xy$</td>
</tr>
<tr>
<td>$2x \times 2 \times x \times y = 10xy$</td>
<td>$10 \times xy = 10xy$</td>
</tr>
<tr>
<td>$10 \times xy = 10xy$</td>
<td>Factor of $102x^2$ &amp; $10xy$</td>
</tr>
<tr>
<td>$= 2x \times 2x$</td>
<td>$= 2, x, 2x$</td>
</tr>
<tr>
<td>HCF = $2x$</td>
<td></td>
</tr>
</tbody>
</table>
"you can proceed in a simple manner"
"now tell what HCF = 7 pq common between two?"

<table>
<thead>
<tr>
<th>$21p^2q$ and $49pq^2$</th>
<th>$11abc^3, 13a^2b^2c$ and $100a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>11abc$^3, 13a^2b^2c$ and $100a$</td>
<td></td>
</tr>
<tr>
<td>you can break the monomials into simpler terms</td>
<td></td>
</tr>
<tr>
<td>what is the HCF of numerical HCF = 5a</td>
<td></td>
</tr>
<tr>
<td>HCF of numerical coefficients is 5 which is common between the three</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$2a^2 + 10a^2 + 20a^2$</th>
<th>$2a^2 + 10a^2 + 20a^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;Take out highest common factor of coefficients 2 is the HCF as it is present in all the three terms.&quot;</td>
<td></td>
</tr>
<tr>
<td>&quot;Take out HCF as it is present in all the three terms as exponent for a is 2,3,4, the lessor of 2,3,4, is 2&quot;</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$-4a^2 - 16a^2b - 20a^2b^2$</th>
<th>$-4a^2 - 16a^2b - 20a^2b^2 - 4a^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>what will be the HCF of coefficients?&quot;</td>
<td></td>
</tr>
<tr>
<td>The hcf of - 4,-16,-20 is - 4&quot;</td>
<td></td>
</tr>
<tr>
<td>&quot;The hcf of literal as $a^2b$ and $a^2$ is present in all three term&quot;</td>
<td></td>
</tr>
</tbody>
</table>

Q now will factorize the algebraic expressions by taking out HCF

<table>
<thead>
<tr>
<th>$10x + 5x^2$</th>
<th>$10x + 5x^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;what is HCF of coefficients 10 and 5 is 5&quot;</td>
<td></td>
</tr>
<tr>
<td>&quot;The hcf of literals $x$ &amp; $x^2$ x, a the exponent of is 1 &amp; 2, lessor is so HCF is $x$&quot;</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$3x^2y + 6xy^2$</th>
<th>$3x^2y + 6xy^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;The exponent of $x$ in $x^2y$ and $xy^2$ is 2 &amp; 1 respectively, the lessor of 2 1 is 1&quot;</td>
<td></td>
</tr>
<tr>
<td>The exponent of $y$ in $x^2y$ and $xy$ is 1 and 2 The lessor of 1 and 2 is 1 The HCF is $xy$&quot;</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$2x^2 - 6x^4 - 10x^2$</th>
<th>$2x^2 - 6x^4 - 10x^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>the HCF of 2, 5,10 is 2</td>
<td></td>
</tr>
<tr>
<td>&quot;the exponents of $x$ is 3,4,2 lessor is $x$&quot;</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$-10a^2b +$</th>
<th>$-10a^2b +$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3a^2 - 6a^4 - 10a^2$</td>
<td></td>
</tr>
<tr>
<td>$2x^2 - 6x^4 - 10x^2$</td>
<td></td>
</tr>
<tr>
<td>$2x^2\left[x - 3x^2 - 5\right]$</td>
<td></td>
</tr>
<tr>
<td>$2x^2$ and $\left[x - 3x^2 - 5\right]$</td>
<td></td>
</tr>
</tbody>
</table>
\[-10a^2b + 20b^3 + 40a^3b^2\]

"The HCF of a is 3, 1, b; lessor is 1 the exponents of b is 1, 3, 2; lessor is 1
the HCF of a^3b, b^3a and a^b^3 is ab"

<table>
<thead>
<tr>
<th>Factorization by identity ( a^2 - b^2 = (a - b)(a + b) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>49(p^2 - 36)</td>
</tr>
<tr>
<td>&quot;what is (a^2 - b^2) equal?&quot;</td>
</tr>
<tr>
<td>&quot;49(p^2) is the square of which monomial?&quot;</td>
</tr>
<tr>
<td>&quot;36 is the square of which number?&quot;</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>4(a^4b^4 - 9p^2q^2)</td>
</tr>
<tr>
<td>&quot;whose square is (4a^4b^4) ?&quot;</td>
</tr>
<tr>
<td>&quot;whose square is (ap^2q^2) ?&quot;</td>
</tr>
<tr>
<td></td>
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<td></td>
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<table>
<thead>
<tr>
<th>Problem 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>((m + 2n)^2 - 16m^2)</td>
</tr>
<tr>
<td>&quot;whose square is (16m^2) ?&quot;</td>
</tr>
<tr>
<td>&quot;((m + 2n)^2) will remain as it is&quot;</td>
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**Practice:**

**Factorize:**

\[25a^2b^2c^2 - 15a^2b^2c^2 - 105a^2b^2c^2\]
LESSON PLAN 31

**Topic** – Factorization of Algebraic expressions (Contd.)

**Entry Behaviour**: it is assumed the students are able to

(I) recall the identities

(II) compute the HCF of the expression

**Instructional objectives:** After the instructional to

♦ factorize the algebraic expression by using a suitable identity.

♦ factorize more expressions related to identities or by regrouping the terms.

**P.K. Testing**

Q. what is the HCF of $a^2b^2$ and -$b^2$

Q. what is the hcf of $4x^2 + 20x + 40$

Q. Factorize $6pq -12p^2q + 18pq^2$

$4p^2 - 9$

**Presentation of new material**

"Today we will learn to factorize by applying other identities and regrouping of terms"

<table>
<thead>
<tr>
<th>Teacher’s activity</th>
<th>B.B. work</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem</td>
<td>$a^2 + 8a + 16$</td>
</tr>
<tr>
<td>To express as the square of an expression and factorize</td>
<td>$(a+4)^2 = a^2 + 2ab + b^2$</td>
</tr>
<tr>
<td>$a^2 + 8a + 16$</td>
<td></td>
</tr>
<tr>
<td>write $a^2 &amp; 16$ in form of squares and write $8a$ accordingly</td>
<td></td>
</tr>
<tr>
<td>25$x^2 + 30x + 9$</td>
<td>$25x^2 + 30x + 9$</td>
</tr>
<tr>
<td>write 25$x^2 &amp;$9 in the form of squares</td>
<td>$(5x)^2 + (3)^2 + 2 \times 5x \times 3$</td>
</tr>
<tr>
<td></td>
<td>$(a + b)^2 = a^2 + 2ab + b^2$</td>
</tr>
<tr>
<td></td>
<td>$(5x + 3)^2$</td>
</tr>
<tr>
<td>$a^284ab + 36b^2$</td>
<td>$49a^284ab + 36b^2$</td>
</tr>
<tr>
<td>&quot;write the like form of squares and middle term accordingly&quot;</td>
<td>$= (7a)^2 + (6b)^2 - 2 \times 7a \times 6b$</td>
</tr>
</tbody>
</table>

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\[(x^2 + z^2 - 2xz) - y^2\]

First use identity
\[(a - b)^2 = (a^2 + b^2 - 2ab)\]
to factorize
\[x^2 + z^2 - 2xz\]

Then use
\[a^2 - b^2 = (a - b)(a + b)\]
to solve whole expression
\[(x - z + y)(x - z - y)\]

50x^2 - 72y^2
"HCF of 50 and 72 is 2"
"now write in the form of squares."

\[x(x + y) + 9x + 9y\]
take out HCF of \(ax + 9y\)
take out common factor \((x + y)\) out

50x^2 - 72y^2
\[
\begin{align*}
50x^2 & - 72y^2 \\
2(25x^2 - 36y^2) & - 2(5x - 6y)(5x + 6y)
\end{align*}
\]
\[a^2 - b^2 = (a - b)(a + b)\]

\[(x + y)(x + y) + 9(x + y)\]

50x^2 - 70x - 8y + 2xy
Group the terms by considering the eternals
\[x\) is common in \(s n^2 - 20ny\) common in \(8y & 2xy\]

10xy + 4x + 5y + 2
In this case other you group \(10xy & 5y\)
The HCF of \(10xy & 4x\) is 2x
the HCF of \(5y & 2\) is to find factors of least

\[m^4 - 256\]
"write as squares using \(a^2 - b^2 = (a - b)(a + b)\)
facorizes it

Again you can \(m^2 - 16\) by using \(a^2 - b^2 = (a - b)(a + b)\)

\[(m^2 - 16)(m^2 + 16)\]
\[a^2 - b^2 = (a - b)(a + b)\]
\[\frac{(m^2 - 16)(m^2 + 16)}{m^2} - (a - b)(a + b)\]

Practice
Factorize

1. \(9x^2y - 32x^2y^2 - 24x^2y^2 - 12x^2y^2\)
2. \(\left(p^2 + q^2 - 2pq\right) - r^2\)

Recapitulation
\[(a + b)^2 = a^2 + 2ab + b^2\]
\[(a - b) = a^2 - 2ab + b^2\]
\[(a + b)(a - b) = a^2 - b^2\]
TOPIC: Construction of Triangles (SAS and ASA Criterion)

Entry Behaviour: It is assumed that the students are able to
(i) recall angle sum property of a triangle.
(ii) recall the classification of triangles on the basis of angles and sides.
(iii) construct the angles with the help of protactor and compass.

Instructional Objectives: After the instructional treatment is over, the students will be able to:
♦ Construct triangle when two of its sides and included angle are given.
♦ Construct the triangle when two of its angles and the included side are given.

Instructional aids: Blackboard, chalk, teacher’s geometry box.

P.K. Testing:
Q.1. What type of triangle is it?

Q.2. If $\angle B = 110^\circ$ then $\triangle ABC$ is _______ triangle.

Q.3. In acute angled triangles all angles are less than ________

Q.4. What is the measure of $\angle BAC$?

Presentation of new material
“Today we will learn how to construct triangles with the help of compass while drawing any construction, you should use sharpened pencil. Measurement should be correct.”
Construction of a triangle when two of its sides and the included angle are given (SAS Triangle Construction)

Problem 1
Construct a \( \triangle ABC \) in which \( \angle B = 70^\circ \), \( AB = 4.8 \text{ cm} \) and \( BC = 5.2 \text{ cm} \)

Teacher explains, "Whenever you have to construct a triangle always draw a rough sketch first and indicate the given measures. This will help us to understand the steps to be followed.

1. **Let us draw the rough sketch of \( \triangle ABC \).**
   - Teacher asks, "Now we will draw with the help of compass."
   - Teacher asks, "Draw a line segment \( BC = 5.2 \text{ cm} \)."

2. **Draw the angle of \( 70^\circ \) at \( B \) with the help of protractor as it cannot be made with the compass \( \angle CBX = 70^\circ \). "What is the other given condition?"

3. "Open the compass of radius 4.8 cm and with centre \( B \) draw an arc cutting the ray \( BX \) at \( A \).

4. **Join \( AC \). \( \triangle ABC \) will be the required triangle.**

Problem 2: Construct an isosceles triangle

4.3 cm and \( \angle Z = 80^\circ \)

Teacher asks, "What is an isosceles triangle?" Students respond. Teacher asks, "First draw the rough Sketch."

1. **Teacher asks 'What is the first-step'**
   - Students respond
   - Teacher asks "Draw the line

2. **Draw line segment \( BC = 5.2 \text{ cm} \).**

3. **At \( B \), draw an angle \( XBC = 70^\circ \).**

4. **With \( B \) as centre cut the ray \( BX \) with 4.8 cm**

5. **Join \( AC \). \( \triangle ABC \) is the required triangle.**
segment YZ=4.3 cm
2. Now Draw an angle of 80° on Z
   \( \angle AZY=80° \)
3. Now with centre Z cut the ray AZ with line segment XZ of 4.3 cm.
4. Join X Y

Steps of construction:
1. Draw a line segment YZ=4.3 cm
2. At Z, draw an \( \angle AZY=80° \)
3. Cut ray AZ at X with 4.3 cm from Z
4. Join XY
5. \( \Delta XYZ \) is the required triangle.

Construction of triangle in which two angles and side included in it is given (ASA triangle construction).

Problem 3: Construct a \( \Delta ABC \) in which \( \angle B=70°, \angle C=50° \) and BC = 5.1 cm
Teacher "Draw the rough sketch in your notebooks"
Teacher asks "what is the first step"
"Draw an angle XBC =70° at B with the help of protector
At C drawn, angle YC=50° using a protector.
Produce rays XB and YC to interact at A Thus \( \Delta ABC \) is the required triangle.
Now you can write steps of construction also."

Step of construction
1. Draw line segment BC=5.1 cm
2. At B draw $\angle XBC = 70^\circ$
3. At C draw $\angle YCB = 50^\circ$
4. Let rays XB and YC intersect at A
Thus $\triangle ABC$ is a required triangle.

**Problem 4**: Construct a $\triangle XYZ$ in which $YZ = 4$ cm, $\angle Y = 110^\circ$ and $\angle X = 30^\circ$

Teacher explain “This is a special case. Listen carefully. In case of ASA type of construction, the side should be included between the two angles $\angle Y$ and $\angle X$ is given and $YZ$ is given so we need to find the third $\angle Z$, from the given data.”

Teacher asks “what is the sum of three angles of a triangle?” students respond.

If two angles are $110^\circ$ and $30^\circ$, what will be third angle?”

Students respond

“With $\angle Y$ and $\angle Z$ known, $YZ$ given, construct the required triangle.”

Teacher asks “Draw a line segment $YZ=4$cm on Y draw an angle of $110^\circ$ such that $\angle AYZ=110^\circ$

“On Z draw an angle of $40^\circ$ such that $\angle BZY=40^\circ$

Let BZ and ZY intersect at X
Thus $\triangle XYZ$ is a required triangle. Now you can steps of construction.”

**Problem 5**: Construct a $\triangle ABC$ in which $BC = 4$ cm, $\angle B = 110^\circ$ and $\angle C = 70^\circ$

Teacher asks “Before constructing a triangle, let us first draw a rough
sketch.

"While drawing the rough sketch, we find that it is not possible to draw the triangle. Tell why"

“What is the angle sum property of the triangle?”

Teacher explain
Here the sum of two angles is equal to 180°. It means sum of three angles as greater than 180°. Hence it is not possible to construct a triangle. So whenever you find in the given conditions, that sum of three angles may exceed 180°, then the construction of that triangle is not possible.

“Whenever two of angles and the included side are given i.e. under ASA condition, you are advised to check the angle sum property of a triangle.”

Practice:
Construct a ΔRST in which RS=6 cm, ∠R=110° and ∠T=40°

Recapitulation:
(1) In the construction of triangle with ASA criterion, the two angles cannot be right and obtuse angle at the same time. It is advise to check the angle sum property of a triangle.
TOPIC: Construction of Triangles (SSS, RHS Criterion)

Entry Behaviour: It is assumed that the students are able to
(i) recall angle sum property of a triangle (on the basis of sides and angles)
(ii) construct triangles with SAS and ASA criterion.

Instructional Objectives: After the instructional treatment is over, the students will be able to:
(i) Construct the triangle when three of its sides are given.
(ii) Construct the right angle when its hypotenuse and one side are given.
(iii) Explain which triangles can be constructed or not from the given measures of some of the sides and angles of a triangle.

Instructional aids: Blackboard, chalk, teacher's geometry box.

P.K. Testing:
1. Which of following measures of some of its sides and angles of a triangle are possible?
   (a) \( \angle A = 85^\circ, \angle B = 115^\circ, AB = 5 \text{ cm} \)
   (b) \( \angle Q = 30^\circ, \angle R = 60^\circ, QR = 4.7 \text{ cm} \)
   (c) \( \angle A = 70^\circ, \angle B = 50^\circ, AC = 3 \text{ cm} \)
2. Is it possible to construct a triangle when just all three angles are given? Why

Presentation of new material
"Today we will learn how to construct a triangle when three of its sides are given and construction of right angled triangle."

Teacher Activity
Construction of triangle when three of its sides are given (SSS Triangle Construction)

Problem 1: construct a \( \triangle ABC \) in which \( AB = 4.5 \text{ cm}, BC = 5 \text{ cm} \) and \( CA = 6 \text{ cm} \) "Draw a rough sketch of \( \triangle ABC \)"
"There is no necessary condition that you have to take \( CA \) or \( BC \) or \( AB \) as
base. You have a choice to make any side as a base.”
I have taken BC as BC.
First draw the line segment BC=5 cm as you have done before with C as centre and radius 6 cm (=CA) draw an area of the circle on one side of BC with B as centre and radius 4.5 cm (=AB) cut the previously drawn area at A. Join part A to B and A to C ΔABC is the required triangle. Now you can write steps of construction.

Problem 2: Construct a triangle PQR where PQ=2 cm, QR=3 cm and PR=6 cm.
Teacher asks “First draw the rough sketch of the triangle.”
“While drawing the rough sketches, we find that it is not possible to construct this triangle. Can you tell why?”
“What does triangle on equality from class VI says?”
It states that sum of any two sides of a triangle is greater than the third side. Here PQ&QR=2 cm + 3 cm = 5 cm which is less than 5 cm (=PR) Triangle inequality does not satisfy and triangle is not possible.”
Teacher explains “Every time you take
up the construction of triangle when three of its sides are given, i.e. under SSS condition, you are advised to check whether the three sides satisfy triangle inequality.

Construction of triangle when it hypotenuse and one side are given (RHS Triangle Construction)

**Problem 1**: Construct a right triangle with hypotenuse of length 5 cm and one of its side of length 3 cm.

Teacher asks “As you are given two conditions to draw a triangle from where the third condition will come”

Teacher asks “What is right triangle?”

“Triangle on which one of angle is 90°. So it is understand that you were draw an angle of 90°.”

Now draw the rough sketch

Here vertices of triangle are given. So you can given any same to the triangle.”

Draw line segment BC=3 cm

“Draw an angle of 90° on B. Such that \(\angle XBC=90°\)”

“With centre C, and radius 5cm (i.e. hypotense), an are to intersect ray XB at A Join A and C.”

**Problem 4**: Construct a \(\angle Q=90°\), PR=6cm and QR=4 cm

Teacher asks “First draw the rough sketch” of \(\triangle PQR\)

“Always take the greater side as hypotenise

“Draw a line segment QR=4cm” At Q, draw \(\angle XQR=90°\) with centre R and radius 6cm (=PR as hypotenuse draw
an arc that intersects ray XQ at P. Join P and R."

Now you can write steps of construction

**Steps of construction**

1. Draw line segment QR=4cm
2. At Q, draw ∠XQR=90°
3. With R as centre and radius 6cm (=PR) draw an arc to intersect XQ at P. Join PR.
4. ΔPQR is a required triangle

**PRACTICE**

Which of following triangles can be constructed.

(i) ∠A=115°, ∠B=90°, AB=5cm
(ii) ∠P=60°, PQ=4cm, QR=6cm
(iii) RS=4.5cm, ST = 5cm, ∠S=110°
(iv) ∠R = 90°, ∠P=60°, RS=5cm
(v) AB=5cm, BC = 4.5cm, AC = 4cm
(vi) DE = 2cm, EF = 1.5 cm, DF = 4 cm

**Recapitulation**

In SSS criterion, the given sides should satisfy triangle inequality which states that sum of two sides of a triangle is greater than the third side.
LESSON PLAN 34

Topic : Properties of Isosceles triangle

Entry behaviour : It is assumed that the students are able to :
(i) classify the triangles on the basis of their sides.
(ii) recall the properties of triangle.

Instructional Objectives : After the instructional treatment is over, the students will be able to :
(i) recall the propositions related to an isosceles triangle.
(ii) solve simple figures using the general properties of a triangle and the propositions of an isosceles triangle.

Instructional aids : Backboard, chalk, models of triangles.

P.K. Testing

Q. 1. Classify the following triangles on the basis of their sides.

Q2.

Find $\angle ACB$

Q.3.

Find $\angle PQX$

Q.4

Find $\angle ACY$
Presentation of new material

Teacher explains “we know that in an isosceles triangle, two sides are equal. The angles opposite to these equal sides, are also equal. We will proceed with an activity”

“Teacher asks to do an activity Activity 1

Draw the triangle ABC such that AB=AC=4.5cm and BC= 6cm.

“Now measure ∠ABC and ∠ACB”

“Find ∠ABC-∠ACB.”

What do you observe?

∠ABC-∠ACB either zero or so small that ∠ABC-∠ACB = 0

Activity 2

Teacher asks

Construct a △ABC with ∠B=∠C=50° and BC=5 cm.

“Now measure AB and AC”

“Find AB-AC”. What did you get”

Teacher explains

“The difference AB-AC is either zero or so small that the same may be treated as zero.”

AB-AC = 0

AB=AC

“The above activity illustrate the following proposition : In an isosceles triangle, the sides opposite the equal angles are equal.”

“In an isosceles triangle, the sides opposite the equal angles are equal.”

“Now we will apply these two propositions in various problems.”

Problem 1 : △PQR is an isosceles with PQ=QR. If ∠Q=70°, what is the measure of ∠R “First draw rough sketch of the triangle.”

“What is angle sum property of triangle?”

∠PRQ=180°-100°=80°
\[ \angle PRQ + \angle PQR + \angle QPR = 180^\circ \]
\[ \angle PQR = \angle QPR \]
\[ \angle PRQ + \angle PQR + \angle QPR = 180^\circ \]
\[ 80^\circ + 2 \angle PQR = 180^\circ \]
\[ 2 \angle PQR = 180^\circ - 80^\circ = 100^\circ \]
\[ \angle PQR = \frac{100^\circ}{2} = 50^\circ \]
\[ \angle PRQ = 50^\circ \]

**Problem 2**: In \( \triangle XYZ \), \( \angle X = \angle Z = 40^\circ \)

(in given fig.) which of two sides of the triangle are equal?

Teacher asks

"Which is opposite side to \( \angle X \)?
Which is side opposite to \( \angle Z \)?"

\[ \angle X = \angle Z = 40^\circ \]
\[ \therefore \ XY = YZ \]

(Side opposite the equal angle are equal)

**Problem 3**: \( \triangle PQR \) is isosceles with \( PQ = PR \). If \( \angle R = 45^\circ \) find the measures of other two angles.

Teacher asks. "First draw the rough sketch" what is the measure of \( \angle Q \) if \( PQ = PR \)?

"What is the angle sum property of triangle?"

\[ PQ = PR \]
\[ \angle Q = \angle R = 45^\circ \]

(Angles opposite to equal sides are equal)
\[ \angle P + \angle Q + \angle R = 180^\circ \]
\[ \angle P + 45^\circ + 45^\circ = 180^\circ \]
\[ \angle P + 90^\circ = 180^\circ \]
\[ \angle P = 90^\circ \]

What the same may be treated as zero.
Thus we have
\[ \angle ABC = \angle ACB \]

In an isosceles triangle, the angles opposite the equal sides are equal.

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This activity illustrates the following proposition.
In an isosceles triangle, the angles opposite the equal sides are equal

**Problem 4**: In the drawn fig., find the length of side BC.

In this figure, \( \angle A = 50^\circ \), \( \angle B = 80^\circ \) are given.

If two of three angles of triangle are given, then we can easily calculate the third angle.

"What is angle sum property of triangle?"

\[
\angle A + \angle B + \angle C = 180^\circ
\]

\[
50^\circ + 80^\circ + \angle C = 180^\circ
\]

\[
130^\circ + \angle C = 180^\circ
\]

\[
\angle C = 50^\circ
\]

\[
\angle A = \angle C = 50^\circ
\]

\[\therefore BC = AB\]

(sides opposite to equal angles are equal)

AB = 4 cm

\[\therefore BC = 4 \text{ cm}\]

**Problem 5**: In the given fig. equal sides have been shown by markings. find

(i) \( \angle PRQ \)  (ii) \( \angle PQR \)

PR=QR which angle is opposite to side PR? which angle is opposite to side QR?

"If \( \anglePRS=100^\circ \), what is measure of \( \angle PRQ \)?"
Problem 6: In the given equal sides have been shown by similar markings find x, y and z.

Teacher asks

"In \triangle ABD, AB=BD which angle is opposite to AB?"
Which angle is opposite to BD?"

Students respond

"\angle ADB is denoted by x and \angle BAD=30^\circ"

Now

In \triangle ABD, two angles are known, how will you find the third angle?"
"What is angle sum property?"
"What is linear pair?"

"\angle DBC is denoted by y"

\[ \therefore \angle Q = \angle R = 70^\circ \]
(Angles opposite the equal sides are equal.)

In \triangle ABD

Since, AB=BD

\[ \therefore \angle BAD=\angle ADB \text{ (x)} \]
(Angles opposite equal sides are equal)
\[ \angle BAD = 30^\circ \text{ (given)} \]

\[ \therefore x = 30^\circ \]

\[ \angle ABD+\angle BDA+\angle DAB=180^\circ \]
\[ \angle ABD+30^\circ+30^\circ=180^\circ \]
\[ \angle ABD+60^\circ=180^\circ \]
\[ \angle ABD+60^\circ=180^\circ \]
\[ \angle ABD=180^\circ-60^\circ=120^\circ \]
\[ \angle ABD+\angle DBC=180^\circ \text{ (Linear pair)} \]
\[ 120^\circ+\angle DBC=180^\circ \]
\[ 120^\circ+\angle DBC=180^\circ \]
\[ 120^\circ+y = 180^\circ \]
\[ y = 180^\circ - 120^\circ = 60^\circ \]

In \triangle BDC

BC=DC, which angle is opposite to BC and while angle is opposite to DC?"

\[ \therefore y = z \]
(Angles opposite equal side are equal)

\[ \therefore z = 60^\circ \]
Practice

△ABC is isosceles with AB=BC, if \( \angle C=60^\circ \), find \( \angle BAD \).

Recapitulation
(i) In an isosceles triangle, the angles opposite the equal sides are equal.
(ii) In a triangle, if two angles are equal, then the sides opposite these angles are also equal.
(iii) Sum of three angles of a triangle is 180°.
(iv) Sum of linear pair is 180°.
(v) Sum of two interior angles is equal to opposite exterior angle.
LESSON-35

Topic: Properties of isosceles triangle (Contd.)

Entry Behaviour: It is assumed that the students are able to
(i) recall the properties of an isosceles triangle
(ii) recall the general properties related to the triangle.

Instructional objectives: After the instructional treatment is over, the students will able to.
♦ Solve complex figures using the propositions of isosceles triangle and other properties of triangle.

Instructional aids: Blackboard, chalk, models of right triangles.

P.K. Testing

Q1. In \( \triangle DEF \), \( \angle D = \angle F \). Which of two sides are equal?

Q2. In \( \triangle ABC \), \( BC = CA \). Which of its two angles are equal?

Q3. In \( \triangle PQR \), \( QR = QP \). If \( \angle P = 36^0 \), what is measure of \( \angle Q \)?

Presentation of new material
"Today we will deal with more complicated figures"

Problem 1. In the given figure, \( \triangle ABC \) is isosceles with \( BC = AC \) if \( \angle A = 70^0 \), FIND (i) \( \angle ABC \) and \( \angle ACB \) (ii) the values of \( x \) and \( y \)
Teacher activity
Teacher asks “It BC = AC, which two angles will be equal?”

“What is the measure of ∠ ABC?”

“What is the measure of ∠ ACB?”
“you know ∠ ABC and ∠ ACB
Now find x° and y° by using the property of Linear pair”

In ΔABC AC = BC
∴ ∠ABC = ∠CAB (angle opposite equal sides are equal)
∴ ∠ABC = ∠CAB = 70°
∠ABC + ∠BAC + ∠ACB = 180°
70° + 70° + ∠ACB = 180°
∠ACB = 180° - 140° = 40°
x° + ∠ABC = 180° (linear pair)
x° + 70° = 180°
x° = 180° - 70° = 100°
y° + ∠ACB = 180° (linear pair)
y° + 40° = 180°
y° = 180° - 40° = 140°

Problem 2 In the given fig. ΔPQR is isosceles with PQ = PR. LM is parallel to QR with L on PQ Give reasons for
(i) ∠Q = ∠R
(ii) ∠PLM = ∠Q
(iii) ∠PML = ∠R
(iv) ∠PLM = ∠PML
(v) ∠PLM is isosceles

Teacher activity
“In ΔPQR, PQ = PR which two angles will be equal?”
“LM || QR and PQ is a transversal which angles are equal?”

Since, PQ = PR
∴ ∠Q = ∠R (Angles opposite the equal sides are equal)--------1.
LM || QR and PQ is a transversal

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"LM || QR and PR is a transversal which angles are equal?"
"You must have observed from (i), (ii) and (iii), that \( \angle PLM = \angle PML \)."
Teacher asks In a triangle it two angles are equal, then what kind is it? Students respond
\[ \text{ZPLM} = \text{ZQR (corresponding angles)} - 2 \]
\[ \text{LM} || \text{QR and PR is a transversal} \]
\[ \angle PML = \angle R (corresponding angles) - 3. \]
From 1, 2 & 3.
\[ \angle PLM = \angle PML \]
\[ \Delta PLM \text{ is isosceles} \]

**Problem 3** In a given fig., \( \Delta ABC \) and \( \Delta DBC \) are both isosceles with a common base BC. The equal sides have been shown with similar markings, If \( \angle A = 60^\circ \), \( \angle D = 40^\circ \), find

(i) \( \angle ABC \) and \( \angle ACB \)
(ii) \( \angle DBC \) and \( \angle DCB \)
(iii) \( \angle ABD \) and \( \angle ACD \)

**Teacher activity**
Teacher asks "It in \( \Delta ABC \ AB=BC \), which of the angles are equal?"

"What is angle sum property of triangle?"
Teacher asks "In \( \Delta BCD \ BD=DC \), which of the two angles are equal?"

**B.B WORK**

\[ AB=AC \]
\[ \therefore \angle ABC = \angle ACB \text{ (angle opposite equal sides are equal)} \]
\[ \therefore \angle ABC + \angle ACB + \angle A = 180^\circ \]
\[ 2 \angle ABC + 60^\circ = 180^\circ \]
\[ 2 \angle ABC = 180^\circ - 60^\circ = 120^\circ \]
\[ \angle ABC = 120^\circ / 2 = 60^\circ \]
\[ \angle ABC = \angle ACB = 60^\circ \]
In \( \Delta BCD \)
\[ BD=DC \]
\[ \therefore \angle DBC = \angle DCB \text{ (Angles opposite the equal sides are equal)} \]
\[ \angle BCD + \angle DBC + \angle BDC = 180^\circ \]
What is angle sum property

\[2 \angle BCD + 40^\circ = 180^\circ\]
\[2 \angle BCD = 180^\circ - 40^\circ\]
\[2 \angle BCD = 140^\circ\]
\[\angle BCD = 70^\circ\]
\[\angle BCD = \angle DBC = 70^\circ\]
\[\angle ACD = \angle ABC + \angle DBC\]
\[= 60^\circ + 70^\circ = 130^\circ\]
\[\angle ACD = \angle ACB + \angle DBC\]
\[= 60^\circ + 70^\circ = 130^\circ\]
\[\angle ABD = \angle ACD\]

What is the measure of \(\angle ABD\)?

What is the measure of \(\angle ACD\)?

Problem 4: In the given with PQ=PR. If \(\angle Q\), find the measures of the triangle.

Teacher activity

B.B WORK

"What is the other condition given"
Teacher asks “If PQ=PR, which of the two angles are equal?”

\[\angle P=2 \angle Q\]
\[\angle P + \angle Q = \angle R = 180^\circ\]
\[2 \angle Q = \angle Q = \angle Q = 180^\circ\]
\[4 \angle Q = 180^\circ\]
\[\angle Q = 45^\circ\]
\[\angle R = 45^\circ\]
\[\angle P = 90^\circ\]

Problem 5: In the given fig, \(\triangle PQR\) and \(\triangle SQR\) are isosceles.

(i) find \(\angle PQR\) and \(\angle PRQ\)

(ii) \(\angle SQR\) and \(x\).
Teacher asks “It is \(\triangle PQR\), PQ=QR, which of two angles will be equal?”
Teacher activity

Teacher asks “what is angle sum property?”

Teacher asks “It in $\triangle SQR$, $SQ=SR$, which of two angles will be equal?”

$\triangle PQR$,

$PQ=PR$

$\therefore \angle PQR = \angle PRQ$ (angles opposite two equal sides are equal)

$\angle PQR + \angle PRQ + \angle P = 180^\circ$

$2 \angle PQR + 30^\circ = 180^\circ$

$2 \angle PQR = 150^\circ$

$\angle PQR = 75^\circ$

In $\triangle SQR$

$SQ=SR$

$\therefore \angle SQR = \angle SRQ = 55^\circ$ (angles opposite two equal sides are equal)

$\angle PQR = x^\circ + \angle SQR$

$x = \angle PQR - \angle SQR$

$x = 75^\circ - 55^\circ = 20^\circ$

**Problem 6:** In the given fig, which angle is greater?

“You can draw fig. properly with compass measure all angles you will find $\angle PQR$ will be greatest”

$\angle PQR$ is greater (angle opposite to longer side is greatest)

Find $\angle BAD$

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**Practice:**

Find $\angle BAD$

**Recapitulation:**

(i) In an isosceles triangle, equal angles have equal and opposite sides.

(ii) In an isosceles triangle, equal sides have equal and opposite sides have equal and opposite angles.
LESSON PLAN 36

Topic : Pythagoras Theorem

Entry behaviour : It is assumed that the students are able to :
(i) identify the kinds of triangle
(ii) identify hypotenuse and other sides of a triangle.

Instructional objectives : After the instructional treatment is over, the students will able to
♦ recall pythagoras theorem.
♦ identify Pythagorean triplet.
♦ solve problems using pythagoras theorem.

Instructional aids : Blackboard, chalk and models of right triangle.

P.K. Testing
Q.1. In ΔPQR, what is PR called as? Name QR and PQ

Presentation of new material
"Today we will learn about the properties of right triangle."

Teacher activity
Teacher asks "Construct a right triangle ABC with BC=5 cm, ∠B=90°, CA = 7.5 cm"
"Now measure AB"
"Find the square of all three sides"
"What do you find?"
This proposition is also known as Pythagoras theorem.
Teacher asks, "which side is the greatest in right triangle?"
Now we will apply Pythagoras theorem to solve various problems.

B.B. Work

AB² + BC² = AC²

Pythagoras Theorem
In a right triangle square of hypotenuse equals the squares of its sides.
AC² > BC² AC > BC
AC² > AB² AC > AB
Greatest side = Hypotenuse
Problem 1: \( \triangle ABC \) is right angled at C. If \( AC=9 \text{cm} \) and \( BC=12 \text{ cm} \), find the length \( AB \).

"Always drawn a rough sketch then proceed"

Teacher asks
"Which side is hypotenuse?"
"What does Pythagoras theorem say?"

"which number when squared represents 225?"

By Pythagoras theorem
\[
AB^2 = AC^2 + BC^2 \\
= 9^2 + 12^2 \\
= 81 + 144 \\
(AB)^2 = 225 \\
(AB)^2 = (15)^2 = 225 \\
AB = 15 \text{cm}
\]

Problem 2: The hypotenuse of a right triangle is 25 cm. If one of its sides is of length 24 cm, find the length of other side.

First draw a rough sketch.
"Which side represents hypotenuse?"
"What does Pythagoras theorem say?"

Teacher asks
"Find the squares of AC and BC"
"Substitute the values and solve it"
"What squared number equals 49?"

By Pythagoras theorem
\[
AC^2 = AB^2 + BC^2 \\
(25)^2 = AB^2 + (24)^2 \\
625 = AB^2 + 576 \\
AB^2 = 625 - 576 \\
(AB)^2 = 49 \\
(AB)^2 = (7)^2 \\
AB = 7 \text{cm}
\]

Problem 3: A ladder 17 m long when set against the wall of a house just reaches a window at a height of 15 m from the ground.
how far is the lower end of the ladder from the base of the wall?
“Draw a rough sketch”
“Which side is hypotenuse?”
“What is Pythagoras theorem?”
Students respond
“Substitute the values let us solve the problem”.

\[
\begin{align*}
AC^2 &= AB^2 + BC^2 \\
(17)^2 &= (15)^2 + BC^2 \\
289 &= 225 + BC^2 \\
289 - 225 &= 64 \\
(BC)^2 &= (8)^2 \\
BC &= 8\text{cm}
\end{align*}
\]

**Problem 4**: If the square of the hypotenuse of an isosceles right triangle is 200 cm², find the length of each side.

“What is given in the problem?”

“Which side is hypotenuse?”

“What is Pythagoras theorem?”

\[
\begin{align*}
AC^2 &= 200\text{cm}^2 \text{(given)} \\
AB &= BC \text{ (given)} \\
AC^2 &= AB^2 + BC^2 \\
200\text{cm}^2 &= AB^2 + AB^2 \\
200 &= 2AB \\
AB^2 &= 100 \text{ cm}^2 \\
(AB)^2 &= (10\text{cm})^2 \\
AB &= 10\text{cm} + BC
\end{align*}
\]

**Problem 5**: Is 15, 10, 25 is Pythagorean triplet or is it represents the sides of a right triangle?

“Which is largest side?”

“Take 25 as hypotenuse”

“Apply Pythagoras theorem”

\[
(25)^2 = (15)^2 + (10)^2 \\
625 &= 225 + 100
\]
“Find the square of each side”. 625×325
“If any three given sides satisfy No Pythagoras theorem then the given sides are the sides of a right triangle or a Pythagorean triplet”

**Problem 6**: A tree broke at a point but did not separate. Its top touched the ground at a distance of 5m from this base. If the point where it broke is at height of 12m from the ground, what was total height of the tree before it broke?

“First draw the rough sketch”

“What is height of tree as represented by figure?”

“Apply Pythagoras theorem”

“Now top D touched the ground at C after it broke it means AD=AC”

“Now find height of tree”.

\[ \text{AC}^2 = \text{AB}^2 + \text{BC}^2 \]
\[ = (12)^2 + (5)^2 = 144 + 25 = 169 \]
\[ \text{(AC)}^2 = (13)^2 = \text{AC} = 13 \text{cm} \]

\[ \text{AC} = \text{AD} = 13 \text{m} \]

Ht. of tree = BD = AB + AD = 12m + 13m = 25 m

**Practice**

Which of the following are sides of a right angle or a Pythagorean triplet.

(i) 96, 28, 100
(ii) 34, 30, 16

**Recapitulation**

(i) In right triangle longest side is hypotenuse.
(ii) In right triangle, if a, b are the lengths of sides and c that of hypotenuse then \( c^2 = a^2 + b^2 \)
(iii) If the sides of a triangle are of lengths a, b and c such that \( c^2 = a^2 + b^2 \), then the triangle is right angled and c is hypotenuse.
LESSON 37

Topic – Construction of a median of a triangle

Entry Behaviour: It is assumed that the students are able to:
(i) construct different types of triangles.
(ii) identify the type of triangle.

Instructional Objectives: After the instructional treatment is over, the students will able to:
♦ recall the meaning of median.
♦ construct median from one vertex to other side in a triangle.
♦ locate the point of concurrence of medians called centroid in the given triangle.

Instructional aids: Blackboard, chalk, models of triangles, teacher’s geometry kit.

P.K. Testing
1. Construct a triangle having $\angle B = 50^\circ$, BC=5cm and AB=4cm
2. What is equilateral triangle?
3. What is an isosceles triangle?

Presentation of new material
Teacher explains “Today we will learn the meaning and construction of median of the triangle.”

Teacher activity
Teacher explains, “A median of a triangle is a line segment joining a vertex to the midpoint of the side opposite to that vertex. Every triangle has three medians, one from each vertex.”

Now we will learn how to draw a median of a triangle.

Problem 1: Construct a $\triangle ABC$ in which $AB=5cm$, $BC=4.5$ cm and $AC=6cm$. Draw its medians.

1. Teacher asks “construct the triangle with the given dimensions”
2. With A as centre and radius with more than half of AC, draw arcs one on each side of AC, with C as the centre and the
same radius draw two arcs, cutting the previously drawn arcs at P and Q respectively.

3. Join PQ, meeting AC in D. Then D is midpoint of AC.

Measure AD and DC.

Teacher asks “Are AD and DC equal?” Then how much is the measure of AD & DC? Join BD-median from B to AC.

4. In a same way with A as centre and open the compass of radius more than half of AB. “What is half of AB?” As \( \frac{1}{2} AB \) is 2.5 cm. Open the compass > 2.5cm. Draw arcs on each side of AB with B as the centre and the same radius draw two arcs, cutting the previously drawn arcs at R and S respectively.

5. Join R and S; meeting AB in E. “measure AE and BE” Are these equal? Then E is the midpoint of AB. Join CE which is the median from C to AB.

6. Mark the point C where BD and CE intersects. C – known as centroid.

Teacher explains “It is sufficient to draw only two medians to locate centroid. The third median can be simply drawn from A to F passing through C.

**Steps of construction**

1. Draw \( \Delta ABC \) with AC=6cm, AB=5cm and BC=4.5 cm

2. With A as centre of radius more than \( \frac{1}{2} \) of AC draw the arc at P and Q with C as centre, cut the arcs at P and join PQ meeting at D. Join BD AD=DC = 3 cm.

3. With A as centre and radius more than \( \frac{1}{2} \) of AB draw an arc at R and S. \( \frac{1}{2} AB = 2.5 \) cm with B as centre, cut the arcs at R and S with same radius.

4. Join RS meeting AB at E. Join CE. AE = BE = 2.5 cm

5. Mark the point C (for centroid) where BD and CE intersects

6. Draw the third median by joining A and F passing through C.
**Problem 2:** ΔPQR is isosceles with PQ=PR=5cm and QR=6cm (i) 
Locate its centroid (ii) which of the medians will be equal?

Teacher asks “Construct the isosceles triangle with the given dimensions.”

What will be next step?”

Student responds.

2. “Open the compass of radius more than \(\frac{1}{2}\) of QR. With centre Q draw arcs on both the sides of Q with R’s centre of same radius, cut the previously drawn arcs at P&Q.

3. Join PQ meeting QR at D. (you can name with any alphabet).

4. Join PD. What is PD?”

“In the same way draw another median Taking Q as centre with radius more than \(\frac{1}{2}\) of QP draw arcs on both sides of PQ at R and S. Cut the drawn arcs from point P with the same radius. Join RS meeting at PQ at E.

Join RE
PD and RE intersect at point C which is the required centroid.

“How many medians are sufficient to locate a centroid?

“How many medians are sufficient to locate a centroid?”

The third median can be simply drawn from D to F passing through C meeting PR

Teacher asks

“Measure all the medians”

---

**Step of construction**

1. Draw ΔPQR with QR=6cm and PQ=PR=5cm

2. With Q’s centre of radius more than \(\frac{1}{2}\) of QR draw arcs on both sides of QR with R as centre of same radius at the previously drawn arcs at A and B respectively.

3. Join A and B meeting at D Join PD.

4. Similarly other median RE is drawn

5. PD and RE intersects at C which is centroid.

6. The third median QF is simply drawn passing through C.
"Which of the two are equal?"
"Which of the two sides were equal in this triangle?"
Teacher concludes
"So medians to the equal sides are equal in isosceles triangle."
What do you think about medians in equilateral triangle?
Teacher explains "In isosceles triangle?"
Teacher explains in isosceles triangle, medians to the two equal sides are equal. In equilateral triangle all sides are equal, so all medians will be equal.

**Practice**: Draw a triangle $\triangle ABC$ such that $\angle B=110^\circ$, $BC = 5$ cm and $AB=4$ cm Locate its centroid. Where does it lie.
1. $\triangle ARST$, if $SU$ is the median, which side will be bisected?
2. If $\triangle RST$ is isosceles with $RS=ST$ $SU$, $RV$ and $TW$ are medians, which medians will be equal.

![Diagram](image)

**Recapitulation**
(1) Median is a line segment that joins a vertex of a triangle to the mid-point of the opposite side.
(2) Centroid of a triangle is a point of concurrence of its medians.
LESSON – 38

Topic: Altitudes of a triangle

Entry Behaviour:- It is assumed that the students are able to

(i) Construct different types of triangles
(ii) Construct median of a triangle.

Instructional Objectives:- After the instructional treatment is over, the students will able to

♦ Recall the meaning of altitudes.
♦ Construct altitude from one vertex to other triangle.
♦ locate that point of concurrence of altitudes called orthocentre lies inside the acute angled triangle, on the vertex of right angle in the right angled triangle, outside the obtuse angled triangle.

Instructional aids:- Blackboard, chalk, models of triangles, teachers Geometry kit.

P.K Testing

Q1. ∆ABC, AD is the median which is the midpoint of BC?

Q2. ∆PQR is isosceles with PQ and QR. Which of the two medians will be equal?

Q3. If the triangle is right angled or obtuse angled where the centered lie?
Presentation of new material.
Teacher explains, “Today we will learn how to construct altitudes of a triangle. As the name suggests, an attitude of a triangle is the line segment from a vertex of the triangle, perpendicular to the line containing the opposite side.

“Every triangle has three altitudes one from each vertex. The point of concurrence of three altitudes is known as orthocenter. To locate an orthocenter, it is sufficient to draw any two altitudes,”

Problem 1: Draw a triangle ABC in which BC=5.4cm $\angle B=65^0$ and $\angle C=53^0$. Locate its orthocenter

Teacher activity and teaching point
Teacher asks “draw a triangle with the given diversions”.
with A as centre and a suitable radius, draw an arc cutting BC at two points P and Q with P as centre of save radius draw and are ..Now, with Q as centre cut the previously drawn are are at R.3. Join AR meeting BC at D. . Then AD is required altitudes.
3. Draw other altitude with B as centre and suitable radius, draw an arc cutting AC at two points and T.
4. Now with S as centre draw an are at P of same radius with T as centre cut the arc at P. Join BP cutting AC at E
Thus BE is the altitude
“BE and AD intersects at point O which is the orthocenter. As I have told earlier, it is sufficient to draw two altitudes. Third
Problem2:- Draw a \( \triangle DEF \) where \( \angle E = 90^\circ \)

1. Draw \( \triangle DEF \) with the given dimensions.
2. With D as centre and a suitable radius draw an arc cutting EF at points P and Q.
3. With P as centre raw and are at R with same radius cut the are with Q as centre.
4. Join DR. As you must have observed that DR meet EF at E
   Thus DE is a attitude to EF
5. In a same way, with E as centre and a suitable radius draw and are cutting DF at points M and N.
6. With M as centre draw an arc at L with same radius cut the arc with N as centre.
7. Join EL cutting DG at V thus EV is altitude to DF.
8. Though it is sufficient to draw two altitudes but you can draw third one for practice.
9. With F as centre and a suitable radius draw an arc cutting DE at S and T. with S as centre draw an arc at U and cut the are at U of same radius with T as centre.
   Join FU meeting DE at E Thus EF is altitude to DE “Now it is quite visible that all three altitudes DE, EF and EV

Steps of construction
1. Draw \( \triangle DEF \) cuts \( \angle E = 90^\circ \), EF=5cm, DE=6cm.
2. With D as centre draw an arc cutting EF at PQ with P as centre draw an arc at R with same radius. Cut this arc with Q as centre of same radius
3. Join DR cutting EF at E
4. In a same way, draw other altitudes also EF is altitudes to DE and a EV is the altitudes to DF.
All the three altitudes coincides at E orthocenter lies on the vertex of right angle.
coincides at E. Thus E is the orthocenter. We can conclude that in any right angled triangle, orthocenter lies on the vertex of right angle.

**Problem 3:** Draw a Triangle PQR such that \( \angle Q = 110^\circ \), QR = 7 cm, PQ = 6 cm.

Teacher asks “Draw a triangle PQR with a given dimensions”. “What kind of triangle do you get?” “Now we will see where orthocenter lies in case of obtuse angled triangle”

**Teacher activity and Teaching Point**

- Produce QR to X with P as centre and a suitable radius draw an arc cutting RX at A and B with A as centre draw an arc at R cut the arc at R with same radius form B.”
- Produce QR to X with P as centre and a suitable radius draw an arc cutting RX at A and B with A as centre draw an arc at R cut the arc at R with same radius from B.”
- Join PR meeting QR produced to

**B.B. Work**

1. Construct \( \triangle PQR \) with the given dimensions.
2. With P as centre of suitable radius, draw arc at AB. With A as centre of suitable radius draw arc at R cut the arc with B. Join PR meeting QR at L.
3. Similarly with R as centre of suitable radius, draw arc at S and T with S as centre of suitable radius draw a arc at U. Cut the arc at U.
X at L. PL is attitude to QR from P
"In a same way. Produce PQ to Y with a suitable radius with R as a centre draw an are cutting PY at ST with s as centre of same radius draw are at O. Cut the are at with T as centre of same radius Join RU meeting PQ produced to Y at M"
"M is attitude to PQ from R"
"Let RM and PL intersect at O"
"As you are already told, that it is sufficient to draw two attitudes. To draw the third attitude join Points O and Q to meet PR at N. QN is required attitude to PR from Q."
What do you observe?
The centre lies outside in care of obtuse angled triangle.

Practice: Drawn a $\triangle DEF$ where $\angle E=75^\circ$ DE=6.5cm and EF=7cm. Locate its orthocenter

Recapitulation
(i) A line segment drawn from a vertex of a triangle, perpendicular to the containing the opposite side
(ii) In a triangle, point of concurrence of altitudes is called as orthocenter.
(iii) In acute triangle orthocenter lies inside the triangle.
(iv) In right triangle orthocenter lies on the vertex of a triangle
(v) In obtuse triangle, orthocenter lies outside the triangle.
LESSON PLAN 39

**Topic:** Perpendicular Bisectors of a triangle

**Entry Behaviour:** It is assumed that the students are able to
(i) Construct triangles.
(ii) Define and construct median and altitudes.

**Instructional objectives:** After the treatment is over, the students will able to
♦ recall the meaning of perpendicular bisectors
♦ construct the perpendicular bisector of a triangle
♦ determine that point of concurrence of perpendicular bisectors called circumcentre lies inside the acute angled triangle, on the hypotenuse in right angled triangle and outside the obtuse angled triangle.

**Instructional aids:** Blackboard, chalk, models of triangles, teacher's Geometry Kit.

**P.K. Testing:**
1. In $\triangle ABC$, where does the orthocenter lie?

![Diagram of triangle ABC]

2. If $\angle Q=90^\circ$, where does the orthocenter lie in $\triangle PQR$? Where does the centroid lie?

![Diagram of triangle PQR]
Presentation of New material:-

"Today we will learn the meaning of perpendicular bisector and how to draw it. Perpendicular bisector is the line segment that is perpendicular to the side and bisects it. Each triangle has three perpendicular bisectors drawn for each side. The point of concurrence of perpendicular bisectors is called circumcentre".

Problem 1:- Draw a triangle ABC with sides AB=5cm, \( \angle B=70^\circ \) and BC=6cm. Find its circumcentre.

1. Teacher asks. "Draw a \( \Delta ABC \) with the given dimensions. Teacher explains.
2. With B as centre the radius \( \frac{1}{2} \) of BC draw arcs on the both sides of BC with C as the centre cut the previously drawn arcs of same radius. Join P and Q cutting BC at M measure BM and CM.
3. With B as centre open the compass of radius greater than \( \frac{1}{2} \) of AB, draw arcs on both sides of AB at D and E with A as centre, out the previously drawn areas of same radius.
4. "Join DE cutting AB at Q measure AQ and BQ."
5. With A as the centre of radius greater than \( \frac{1}{2} \) of AC, at R and S. With C as centre, of same radius cut the previously drawn arcs Join R and S cutting AC at N measure AN and CN, DE, RS and PQ are produced to meet at C is the circumcentre.
Problem 2: Draw a $\triangle ABC$ right angled at C. BC=6cm and AC=5cm.

Teacher asks

"Draw $\triangle ABC$ with given dimensions"

"with C as centre of radius $>\frac{1}{2}$ BC draw an arcs on the both sides of BC at P and Q, with B as centre of same radius cut the previously drawn arcs. Join P and Q cutting BC at M. M is the midpoint of BC"

3. Similarly with B as a centre or radius $>\frac{1}{2}$ AB draw arcs on the both sides of AB at R and S. With A as a centre of same radius cut the previously drawn arcs. Join r and s cutting AB at N. N is the midpoint of AB.

4. With A as centre of radius $>\frac{1}{2}$ draw arcs on the both sides of AC with C as centre cut the previously of same radius drawn arcs. at D and E. Join D and E meeting AC at O . O is the midpoint of AV.

5. DE (when produced) PQ and RS coincides at N (midpoint of hypotenuse)

Hence circumcentre lies on the midpoint of hypotenuse.

Problem 3: Draw a $\triangle PQR$ WITH QR=4.5cm, $\angle R=110^0$, and PR=7cm. Find its circumcentre.

Teacher asks “Draw a $\triangle PQR$ with the given dimensions.
Teacher explain
2. "With R as centre of radius >\(\frac{1}{2}RP\) draw arcs on the both sides of PR at A and B cutting PR at L. Measure RL and PL. L is the midpoint of PR.
3. With P as centre of radius >\(\frac{1}{2}PQ\) draw arcs on the both sides of PQ at R and S. With Q as the centre of same radius cut the previously drawn arcs at r and S. Join R& S. QM= MP meeting PQ at M. Measure QM and MP.
4. With R as centre of radius >\(\frac{1}{2}QR\) draw arcs on the both sides of QR at D and E. With Q as centre of same radius cut the previously drawn arcs at D and C meeting QR at N. Measure QN and NR. Produce DE, RS and AB to meet at C. This is the point of circumcentre.

Practice:- Drawn an isosceles triangle DEF \(\angle D=\angle E=70^\circ\) and DE=5cm. Find its circumcentre.
Q1. \(\triangle ABC\) IS not \(\angle D\) at C. Where does the circumcentre lie.

Q2. In \(\triangle PQR\), \(\angle P=100^\circ\), where does the circumcentre lie?
Recapitulation

(i) Perpendicular bisector is a line segment is the line that is perpendicular to the side and bisects it.

(ii) Point of concurrence of perpendicular bisectors is circumcentre.

(iii) In acute angle, circumcentre lies inside.

(iv) In right triangle, circumcentre lies on the midpoint of hypotenuse.

(v) In obtuse triangle, circumcentre lies outside the triangle.
LESSON – 40

Topic: Angle bisector of a triangle

Entry behaviour: It is assumed that the students are able to
(i) Construct different triangles
(ii) Construct median, altitude and perpendicular bisector.

Instructional Objectives: When treatment is over, the students are able to
♦ recall the meaning of angle-bisectors
♦ construct the angle bisector of the triangle
♦ locate the point of concurrence of angle bisectors called incentre of the triangle

Instructional aids: Blackboard, chalk, Models of triangles, Teacher's Geometry kit, Recapitulatory chart.

P.K. Testing:–
Q1. Which of the following triangles in scalene, equilateral and isosceles?

Q2. In rt. \( \angle d \Delta ABC \)

(a) Where does orthocenter lie?
(b) Where does the circumcentra lie?

Presentation of new material.
"Today we will learn to draw angle bisectors of the triangle. Teacher explains as the name suggest angle bisector means bisecting an angle into two equal parts:
As you have been told earlier, like in the previous concepts, there is necessity of drawing only two angle bisectors. The point of concurrence of angle bisectors is called Incentre. We will learn more from the given problem.

**Problem 1:**
Draw a Triangle ABC with sides \(AB = 5\text{cm}, \measuredangle B = 70^\circ\) and \(BC = 6\text{cm}\). Find its incentre.

Teacher asks “First draw a triangle ABC. with the given dimensions.”

“Now open the compass and draw a arcs at the vertex B of \(\triangle ABC\). Draw the arc from one side. Cut they are form the other side. Join and to end draw a live meeting AC in D.”

BD bisects \(\measuredangle ABC\) into 2 equal angles. Measure the two angles.

In a same way, mark an arc on the vertex C of \(\triangle ACB\). Draw the arc from one end. Cut the arc from the other end.

Join end to end and draw a line segment meeting in E CE divides \(\measuredangle ACB\) into 2 equal angles.

Tell which the equal angles are formed?”

Similarly draw the third angle bisectors.

“The point where two angle bisector meets is called incentre devoted by I”

**Problem 2**

Draw a triangle PQR with QR=4.5cm, \(\measuredangle R = 110^\circ\) and PR=7cm. Find its incentre teacher asks “Draw a triangle with the help of compass with the given dimensions.”
The pencil should be sharpened.

"Open the compass of small radius say 1cm. Draw the arc meeting in QR and RP with that radius, draw the arc from one end, then cut the arc from the other end. Join the points, draw the line segment from R meeting in S. RS will be angle bisector of \( \angle PRQ \) "Which of the two angles will be equal?"

If \( \angle PRQ=110^0 \), what will be the measure of 2 angles?"

In a same way open the compass of small radius. Draw arc meeting in PR and PQ. With that radius, draw the arc from one end, then cut the arc from other end. Join the points, draw the line segment from P meeting in L. PL will be the angle bisector of \( \angle QPR \). which of two angles will be equal?

What have you noticed about the incentre where does it lie? Even in obtuse angled triangle, incentre lie inside.

\[ \angle QRS= \angle PRS \]
\[ \angle QRS= \angle PRS=55^0 \]
\[ \angle QPL= \angle RPL \]

Incentre always lie inside the triangle.
**Practice:**
Draw an equilateral find its incentre.
Triangle with side = 6 cm

**Recapitulation: CHART**

1. **Median**
   - (i) bisects the opposite side drawn
   - (ii) from the vertex part of concurrence is called centroid always lie inside

2. **Attitudes**
   - (i) A line segment from a vertex of a triangle, perpendicular to the Point of concurrence is called orthocentre. In acute angled
   - obtuse angled
   - rt. angled triangle.

   ![Triangle Diagram](image)

   - lies inside
   - lies outside
   - lies on the vertex

3. **Perpendicular bisectors.**
   - Line segment that is perpendicular to the side and bisects it.
   - Point of concurrence of perpendicular bisectors is called circumcentre
   - In acute angled triangle, circumcentre lies inside the triangle.
   - In right angled triangle, it lies on the midpoint of hypotenuse.

4. **Angle bisectors**
   - Bisectors an angle of the triangle.

5. **Point of concurrence**
   - Incentre It always lie inside the triangle.
LESSON PLAN – 41

Topics:– Cuboid and Cube (Introduction)

Entry Behaviour :– It is assumed that the students are able to
(i) identify plane figures like rectangles, squares and triangles
(ii) name rectangles, squares and triangles

Instructional objectives: After the instructional treatment is over the students will able to
♦ distinguish between the planar objects and solid objects
♦ give examples of solid figures( cuboid and cube) from their environment.
♦ represent cuboid and write its 6 faces, 12 edges and 8 vertices
♦ identify all the 4 diagonals of a cuboid or a cube.

Instructional aids: blackboard, chalk, models of cuboid and cube.

P.K Testing
Q1. How will you name these figures?

Q2. How many diagonals are there in the fig.?

Q3. What kind of figures are these? Can you name anyone from your environment.

Q4. What kind of figure is ?

Is this planar?

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Presentation of new material
Teacher explains, “You all know about some simple plane figures like rectangles, squares, and triangles. Today, we will deal with those figures that are not planar. The simplest of these figures are cuboids and cubes. These figures do not lie wholly in a plane. Such figures are called solid or three dimensional figures. In a daily life you have seen a brick, a dice, a room. All these objects have length, breadth and height

**Cuboid** - It is made up of six rectangular regions called face of cuboid.

How many faces are there in a cuboid?
Top bottom from one pair of opposite faces. Tell the other opposite faces?
“Opposite faces are congruent to each other.
Faces other them top and bottom are lateral faces name those faces?
Any two faces other then opposite faces, are called adjacent face and these meet in a line segment called and edge of the cuboid”.

Can you name them?
“How many edges are there?”
“Look at this model. Each face has four adjacent faces. In cuboid we found eight corners. Each of these is called a vertex of the cuboid.”
Teacher asks “Name those vertices?”
“How many edges meet in each vertex?
Since opposite faces of a cuboid are congruent.”
“Which of edges are equal?”

Cuboid (rectangular parallelepiped)
Faces—6 opposite faces.
1. top ABCD bottom EFGH.
2. Front ABEF Back CDGH
3. Side BCEG faces ADFH

**Lateral faces**
ABEF
CDHG
BCGF
ADHE

**Edges of Cuboid**
AB EF AE
BC FG BF
CD GH CG
AD HE DH
Total—12 edges

**Vertices of cuboid**
A E
B F
C G
D H
<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>“Name four side face if EFGH is the base?”</td>
<td>Side face</td>
</tr>
<tr>
<td></td>
<td>A  B  E  F</td>
</tr>
<tr>
<td></td>
<td>B  C  G  F</td>
</tr>
<tr>
<td></td>
<td>A  D  H  E</td>
</tr>
<tr>
<td></td>
<td>C  D  H  G</td>
</tr>
<tr>
<td>“If EFGH is the base name line segment representing height of the cuboid?</td>
<td>BF or CG or DH or AE</td>
</tr>
<tr>
<td>“Name the face opposite to AEHD?”</td>
<td>B  C  G  F</td>
</tr>
<tr>
<td>“Name the face adjacent to BFGC?”</td>
<td>A  B  F  E</td>
</tr>
<tr>
<td></td>
<td>C  D  H  G</td>
</tr>
<tr>
<td></td>
<td>E  F  G  F</td>
</tr>
<tr>
<td>“Name the face which meet in the edge AB?”</td>
<td>A  B  C  D</td>
</tr>
<tr>
<td></td>
<td>A  B  F  E</td>
</tr>
<tr>
<td>“Name three edges which meet in vertex H?”</td>
<td>E  H</td>
</tr>
<tr>
<td></td>
<td>H  G</td>
</tr>
<tr>
<td></td>
<td>D  H</td>
</tr>
<tr>
<td>“Name three faces which have vertex A in common?”</td>
<td>A  D  H  E</td>
</tr>
<tr>
<td></td>
<td>A  B  F  E</td>
</tr>
<tr>
<td></td>
<td>A  B  C  D</td>
</tr>
<tr>
<td>“Name the vertices which are opposite to each other?”</td>
<td>A &amp; G</td>
</tr>
<tr>
<td></td>
<td>B &amp; H</td>
</tr>
<tr>
<td></td>
<td>C &amp; E</td>
</tr>
<tr>
<td>“Now, name the diagonals of the given cuboid?”</td>
<td>A  G</td>
</tr>
<tr>
<td></td>
<td>B  H</td>
</tr>
<tr>
<td></td>
<td>D  F</td>
</tr>
</tbody>
</table>

Teacher explains –

Total-8 vertices
3 edges meet in each vertex.
AB = DC = HG = EF
BC = AD = FG = DH
AE = BF = CG = DH
“We observe that the twelve edges of a cuboid can have only three distinct lengths. Usually longest is called length and out of remaining two, one is called the breadth (or width) and the other the height (or depth or thickness).” A cuboid whose length, breadth height are equal is called cube.

The part of space enclosed by the cuboid is called its interior or cuboidal region.

Teacher asks,
Q1. Name any four objects from your environment having the shape of (1) cuboid students respond.

Q2. The figure of cuboid drawn on board.
The lengths of some of the edge are marked by x, y and z. Tell the lengths of al the other edges

<table>
<thead>
<tr>
<th>EF</th>
<th>FG</th>
<th>CG</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>y</td>
<td>z</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>AB</th>
<th>EH</th>
<th>DH</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>y</td>
<td>z</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CD</th>
<th>AD</th>
<th>AE</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>y</td>
<td>z</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>GH</th>
<th>BC</th>
<th>BF</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>y</td>
<td>z</td>
</tr>
</tbody>
</table>

Cuboid

1. Room
2. a matchbox
3. A brick
4. A book D
If you are given 6 rectangular pieces as drawn on the board. Arrange these rectangular pieces such that edge of one piece with other pieces "What is the shape of the solid you obtain?" Students respond.

**Practice**
Draw a diagram to represent a cuboid, label its vertices as P, Q, R, S, T, U, V and W
Name (i) faces
(ii) edges
(iii) diagonals

**Recapitulation**
A cuboid and cube has
(i) 6 faces (2 top and bottom, 4 side faces)
(ii) 12 edges
(i) 8 vertices
(ii) 4 diagonals.
LESSON 42

**Topic:** Surface area of cuboid and cube

**Entry Behaviour:** It is assumed that the students are able to
(i) recall the concept of perimeter and area
(ii) compute perimeter and area of planar figures like rectangle and square.

**Instructional objectives:** After the instructional treatment is over, the students will able to:
♦ explain the concept of surface area
♦ solve problems related to the surface area of cuboid and cube

**Instructional aids:** black board, chalk, models of cuboid and cube.

**P.K. Testing**

Q1. Identify planar and solid object from.

![Planar and solid objects](image)

Q2. What is the perimeter and area of?

![Rectangle](image)

**Presentation of the material**

Teacher explains “Most of the articles sold in the market are packed in tins, cartons, cardboard boxes are mostly in cuboiddal shape. In order to find out in advance how much tin sheet, cardboard or steel sheet is needed to make the packing boxes, tins, cartons, etc., there is need to calculate surface area of cuboid.”

**Teacher Activity and Teaching Point**

Teacher asks “How many rectangular faces aloes the cuboid has?”

![Cuboid](image)
Teacher explains “Sum of 6 areas of rectangular faces is called total surface area of the cuboid.”

Teacher asks “What is area of rectangle?”

“Name the top and bottom faces?”
“What is the area of bottom and top faces?”
“Name the two single faces?”
“what is length and breadth in these two faces?”
“What is the total surface area?”

Student respond.

What will be the total surface area of cube?

Teacher asks “Give three examples from our environment where funding of surface are may be required”.

Teacher asks “In surface area required in playground?”

Teacher explains “Playground blackboard are not solid objects but planar objects. Area is required”

“Now we apply the formula of surface area to solve various problems”

**Problem 1:** Find the total surface area of lunchbox with dimensions 15cm, 9cm and 8cm

Teacher asks

“What is to be found out?”

“What is the formula of surface area of

<table>
<thead>
<tr>
<th>Area of faces EFGH and ABCD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (ℓ \times b \times h) \text{cm}^2 )</td>
</tr>
<tr>
<td>= 2lb cm(^2)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Area of ADHE and BCGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (b \times h + b \times h) \text{cm}^2 = 2bh \text{ cm}^2 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Area of ABFE and CDHG</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (h \times ℓ + h \times ℓ) \text{cm}^2 )</td>
</tr>
<tr>
<td>= 21hcm(^2)</td>
</tr>
</tbody>
</table>

Total surface area of cuboid

\[ = (2b\ell + 2bh + 2\ell h) \text{cm}^2 \]

\[ = 2(\ell b + bh + ℓ h) \text{cm}^2 \]

In cube \( \ell = \ell = h \)

\[ = 2(\ell \times \ell + \ell \times \ell + \ell \times \ell) \text{Sq units} \]

\[ = 2(3\ell^2) \text{ Sq. units} \]

\[ = 6\ell^2 \text{ Sq. units} \]

Box, room, building,
Packet of biscuits, lunch box.

Length= 15cm
Breadth = 9cm
Height = 8cm

Surface area = 2

\[ (\ell \times b + bh + ℓ h) \]

\[ = 2 (15 \times 9 + 9 \times 8 + 15 \times 8) \text{cm}^2 \]
Problem 2: Find the surface area of a chalkbox whose length, breadth and height are 0.5 cm, 25 cm, and 15 cm respectively.

"What information is given?"

"Are the units same in all the dimensions?"

"Units are needed to make same. How many cms are therein 1 m?"

"Now, what is the surface area of cuboid?"

Problem 3:
Find the surface area of a cube with edge 1.2 m

"What is given?"

"What is the formula of surface area of cube?"

Problem 4: The dimensions of an oil tank are 26 cm x 26 cm x 45 cm. Find the

(i) area of tin sheet required to make 20 such tins

(ii) cost of the tin sheet at rate of Rs 20/m²

"What is the surface area of 20 such tins?"

"What is the area of tin sheet required to make 20 such tins?"
<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>What is the cost of 1m²?</td>
<td>Area of tins sheet = Surface area of 20 tins</td>
</tr>
<tr>
<td>&quot;How many cm² are there in 1cm²?&quot;</td>
<td>= 120640cm²</td>
</tr>
<tr>
<td>&quot;What is cost of 12.064m²?&quot;</td>
<td>1m² = (100×100)cm²</td>
</tr>
<tr>
<td></td>
<td>1m² = 10000cm²</td>
</tr>
<tr>
<td></td>
<td>1cm² = ( \frac{1}{10000} ) m²</td>
</tr>
<tr>
<td></td>
<td>120640cm² = ( \frac{120640}{10000} ) m²</td>
</tr>
<tr>
<td></td>
<td>= 12.064m²</td>
</tr>
<tr>
<td></td>
<td>cost of tin sheet for 1 m² = Rs20</td>
</tr>
<tr>
<td></td>
<td>cost of tin sheet for 12.064 m² = Rs20×12.064</td>
</tr>
<tr>
<td></td>
<td>= Rs 241.28</td>
</tr>
<tr>
<td>Problem 5:</td>
<td></td>
</tr>
<tr>
<td>The length, breadth and height of a cuboidal box are respectively 2m 1cm, 1m and 80cm. find</td>
<td></td>
</tr>
<tr>
<td>(i) are of the canvas required to cover this box</td>
<td></td>
</tr>
<tr>
<td>(ii) cost of the canvas for covering the box at the rate of Rs. 50 per m².</td>
<td></td>
</tr>
<tr>
<td>What information is given?</td>
<td></td>
</tr>
<tr>
<td>&quot;Are the units same?&quot;</td>
<td></td>
</tr>
<tr>
<td>&quot;How many meters are there in 1cm?&quot;</td>
<td></td>
</tr>
<tr>
<td>&quot;What is surface area of cuboid?&quot;</td>
<td></td>
</tr>
<tr>
<td>&quot;What is the area of canvas required to cover the box?&quot;</td>
<td></td>
</tr>
<tr>
<td>&quot;What is the cost of covering per m²?&quot;</td>
<td></td>
</tr>
<tr>
<td>&quot;What is the cost of covering 9.16 m²?&quot;</td>
<td></td>
</tr>
<tr>
<td>Length = 2m 10cm = 2m + ( \frac{10}{100} ) m</td>
<td></td>
</tr>
<tr>
<td>= 2m + 0.1m = 2.1m</td>
<td></td>
</tr>
<tr>
<td>Breadth = 1m</td>
<td></td>
</tr>
<tr>
<td>Height = 80cm</td>
<td></td>
</tr>
<tr>
<td>( \frac{80}{100} ) m = 0.8m</td>
<td></td>
</tr>
<tr>
<td>1cm = ( \frac{1}{100} ) m.</td>
<td></td>
</tr>
<tr>
<td>surface area of cuboid</td>
<td>= ( 2(l \times b + bh + l \times h) )</td>
</tr>
<tr>
<td>= ( 2(2.1 \times 1 + 2.1 \times 0.8 + 0.8 \times 1) ) m²</td>
<td>= ( 2(2.1 + 1.68 + 0.8) ) m²</td>
</tr>
<tr>
<td>= 2×4.58 m² = 9.16 m²</td>
<td></td>
</tr>
<tr>
<td>Area of canvas required to cover box = Surface area of box = 9.16 m²</td>
<td></td>
</tr>
<tr>
<td>Cost of covering 1m² = Rs 50</td>
<td></td>
</tr>
<tr>
<td>Cost of covering 9.16 m² = Rs 50×9.16 = Rs 458</td>
<td></td>
</tr>
</tbody>
</table>
Practice:
A box with a lid is in the form of a cuboid. Its length, breadth and height are respectively 1m 30m, 75cm and 20cm. Find the cost of painting its outside at the rate of Rs. 4Per cm².

Recapitulation
(i) Surface area of cuboid =2(l b+bh+l h)sq. units
(ii) Surface area of cube 6 l²sq. units.
(iii) 1m = 100cm
(iv) 1m²= 10000cm².
LESSON – 43

**Topic:** Lateral surface area

**Entry Behaviour.** It is assumed that the students are able to
1. recall and apply the formulas of surface area of cuboid and cube
2. convert one unit to another

**Instructional objectives:**
After the instructional treatment is over,
♦ solve problems related to the area of four walls or lateral surface area
♦ solve complicated problems related to surface area

**Instructional aids:**
Blackboard, chalk, Model of cuboid.

**P.K. Testing**
1. What is the formula of total surface area of cuboid?
2. What is the formula of total surface area of cube?
3. Find the surface area of cuboid whose length is 10m, breadth 4m and height 5m
4. Find the surface area of cube having edge 1.5cm.
5. What do you meant by total surface area?

**Presentation of new material**
Teacher explains. “You have decided the formula of total surface area by summing up the areas of six rectangular faces of the cuboid, viz. bottom and top faces, side faces and front, and back faces. Now we will calculate lateral surface area”

Teacher asks “Name the top and bottom faces.”
Students respond
“Name the front and back faces?”
“Name the two sides faces?”

ABFE and CDHG
BCGF and ADHE
Teacher explains “Lateral surface includes front, back, and two side faces. So to find Lateral surface area we need to find the area of these 4 faces.”

“What area of ABFE?”
“What is area of CDHG?”
“What is the area of BCGF?”
“What is the area of ADHE?”
“What will be the lateral surface area?”

Area of ABFE = 1 \times h \text{ sq. units}
Area of CDHG = 1 \times h \text{ sq. units}
Area of BCGF = b \times h \text{ sq. units}
Area of ADHE = b \times h \text{ sq. units}

Lateral surface area =
Ar. of ABFE + Ar. of CDHG
+ Ar. BCGF + Ar. of ADHE
= (l \times h + l \times h + b \times h + b \times h) \text{ sq units}
= (2l \times h + 2b \times h) \text{ sq units}
= 2lh + bh \text{ sq. units}

Perimeter of rectangle = 2(l + b)

Problem 1: A barn with a flat roof is rectangular in shape of breadth 10m, length 15m and height 5m. It is to be pointed inside on the walls and on the ceiling but not on the floor. Find the total area to be pointed.

“What is the given in the problem?”

In cube l = b = h
= 2l(l + l) \text{ sq. units}
= 2l(2l) \text{ sq units}
= 4P \text{ sq.units}
"What is to be found?"

"What is the formula calculating area of 4 walls?"

"How will you calculate area of ceiling?"

Look at the fig. ceiling is represented by top face of the cuboid.
What shape does it represent?
It represents the shape of rectangle. How do you calculate area of a rectangle?

"Now you can calculate total area to be pointed"

Length = 15m
Breadth = 10m
Height = 5m
Area of ceiling + Area of four walls.

Area of 4 walls = 2 (l+b) x h
= 2 (15+10) x 5m²
= 2(25) x 5m²
= 250 m²

Area of ceiling = l x b
= (15 x 10) m²
= 150 m²

Total area to be pointed
= 250 m² + 150 m²
= 400 m²

Problem 2: A classroom is 11m long, 8m wide and 5m high. Find the sum of areas of its floor and the four walls.

"What is given?"

"What is to be found?"

"How will you calculate area of four walls?"
Problem 3: A swimming pool is 20m in length, 15m in breadth and 4m in depth. Find the cost of cementing its floors and walls at the rate of Rs.36 per m².

“Which face of the cuboid represents the floor?”
“What is the shape of this face?”
“How do you find the area of rectangular shape?”
“Now you can sum up the areas of floor and four walls what is asked.”

Bottom face

Area of floor = l + b
= (11 x 8) m²
= 88m²

Total area = 190m² + 88m²
= 278 m²

Length = 20m
Breadth = 15m
Depth = 4m

Area of four walls + Area of floor
Area of four walls = 2(l+b)xh
=2(20+15)x4 m²
=2(35) x 4 m² = 280m²

Rectangle

Area of floor = l x b
= (20 x 15)m²
= 300 m²

Total area to be cemented
=Area of four walls + Area of floor
= 280m²+300m²
=580m²

Cost of cementing for 1m²=Rs.36
Cost of cementing for 580m² = 580 x Rs.36 = Rs.20880.

**Problem 4**: The floor of a rectangular hall has a perimeter 250m. If its height is 6m, find the cost of painting its four walls at the rate of Rs.20 per m².

- **What is given?**
- **What is to be found?**
- **What is the area of four walls?**
- What is the perimeter of rectangle?
- **What is cost of painting for 1m²?**
- **What is the cost of painting for 1500 m²?**

```
Perimeter of floor = 250m
Height = 6m
Area of four walls = 2(l+b)xh
Perimeter = 2(l+b)
Area of four walls (Perimeter of base) x h
= (250x6)m² = 1500 m²

Cost of painting for 1m²= Rs.20
Cost of painting for 1500m² = Rs.20 x 1500
= Rs. 30000
```

**Practice**: The length breadth and height of a room are 5m, 4m and 3m respectively. Find the cost of white washing the walls of the room and the ceiling at the rate of Rs.750 per m².

**Recapitulation**
Area of each rectangular regions in cuboid = l x b sq. units per meter of each rectangular region in cuboid = 2 (l+b) unit
Total surface area of cuboid = 2 (l+b+h) sq units.
Total surface area of cube = 6 l² sq. units
Lateral surface area or area of four walls = 2 (l+b) x h sq. units
lateral surface area of = 4 l² sq. units cube
Lesson - 44

Topic: Volumes

Entry Behaviour. It is assumed that the students are able to
(i) recall the concept of area, perimeter and surface area
(ii) Convert one unit to another.

Instructional Objectives: After the instructional treatment is over, the student will able to
♦ explain the concept of volume
♦ distinguish between the concept of area, surface area and volume
♦ solve simple problems related to the volume of cuboid and cube.

Instructional aids: Blackboard, chalk models of cuboid and cube, recapitulatory chart.

P.K. Testing: Before starting the new topic the teacher asked certain Questions related to previous knowledge.
Q.1. What is the surface area of cube with edge 11cm?
Q2. What is the lateral surface of cuboid whose length, breadth and height of a room are 5m, 4m and 3m respectively.
1m = ____ cm. 1 m² = ____ Cm²
Q3. Which of the following figure is bigger.

Presentation of new material concept of volume.
The teacher explains, just as in the case of plane regions, we can talk of one being bigger than the other on the basis of their sizes. “In the same way, of solids too have a magnitude or size or measure. The measure of a solid region is called its volume. In other words, measure of space occupied by the solid.”
Utility of Volume - The teacher explains by giving examples from the real life problems
Teacher asks which tin can store more oil?

(a)

(b) can store more oil than a as b has larger capacity.

(a)

(b)

Teacher asks which drum can store more water?

Teacher explains Student respond.
(b) can store more water than a because b has larger capacity.

Teacher Explains: By observing the sizes can compare two objects
Look classroom is much less than the volume of the cupboard in the classroom is much less than the volume of air in the room. Volume of cricket ball is Teacher as can you give any other example?

Student Respond.
Teacher asks. “Which figure has larger volume?”

Student Respond
Teacher explains. “Sometimes observation may fail to guide is so we need a better method of measuring volume and need a standard unit of area of surface area?”
Students respond.

Teacher explains "In a similar manner we take a cube of side 1cm or 1mm or 1m as a standard unit for measuring volumes and expressed the volume as Cm³, mm³ or m³. Volume of cuboid and cube

```
2
8
4
```

Teacher explains "In these figures drawn on the blackboard, we can say about the volume of each of these cuboids. Since each cuboid is made by putting 64 units cubes together, volume is 64Cm³.

Teacher asks “what is their product of their length (a) breadth (b) and height?”

Student responds

Teacher explains “The product of L.b and h is the same as by cresting the unit Cubes. This activity suggest that volume of cuboid =lxbxh

From the above formal we observe that

\[
\text{length (l)} = \frac{\text{Volume(v)}}{\text{Breath (b)} \times \text{Heigh (h)}}
\]

\[
\text{Breadth (b)} = \frac{\text{Volume(v)}}{\text{length (l)} \times \text{Heigh (h)}}
\]

\[
\text{Height (h)} = \frac{\text{Volume(v)}}{\text{Breath (b)} \times \text{Length (l)}}
\]

In case of cube l=b=h

\[\therefore \text{Volume of a cube} = \text{l} \times \text{l} \times \text{l} = \text{l}^3 \text{ cubic units}\]

In case \(l = \text{1cm} \quad \ell = \text{10mm}\)

\[1\text{cm}^3 = 10\text{mm} \times 10\text{mm} \times 10\text{mm} = 1000\text{mm}^3\]

In case \(l = \text{1m} \quad \ell = \text{100cm}\)

\[1\text{m}^3 = 10\text{cm} \times 10\text{cm} \times 10\text{cm} = 100000\text{cm}^3\]

"Now we will solve questions with the above formulas."
1. Find the volume of the cuboid whose length = 12cm, breadth = 10cm, Height = 8cm. Teacher asks what is the volume of cuboid.

Teacher explains

\[ V = \ell \times b \times h \]
\[ = 12\text{cm} \times 10\text{cm} \times 8\text{cm} \]
\[ = (12 \times 10 \times 8)\text{cm}^3 \]
\[ = 960\text{cm}^3 \]

Q2. Find the volume of cube who edge is 15mm Teacher asks what is the length of cube?

Student respond. \[ \ell = 15\text{mm} \]

Teacher asks “what is the volume of cube?”

\[ V = \ell^3 \]

Students respond. \[ V = 15\text{mm} \times 15\text{mm} \times 15\text{mm} \]

Teacher explains side by side
\[ = (15 \times 15 \times 15)\text{mm}^3 \]
\[ = 3375\text{mm}^3 \]

Q3. A cuboidal solid wooden block contains 36cm³ word. If its length and breadth are respectively 4cm and 3cm, find its height.

Teacher asks “what information is given?”

Student responds.

Teacher explains.

\[ V = 36\text{cm}^3 \]
\[ \ell = 4\text{cm} \]
\[ b = 3\text{cm} \]
\[ h = ? \]

\[ V = \ell \times b \times h \]
\[ h = \frac{V}{\ell \times b} = \frac{36\text{cm}^3}{(4 \times 3)\text{cm}^2} = \frac{36}{12}\text{cm} \]
\[ h = 3\text{cm} \]

Q4. A match box measures 4cm by 2.5cm by 1.5cm. What will be the volume of a pocket containing a maximum of 12 such match boxes?

Teacher asks “What is the length, breadth and height of match box?”

Students respond. length of match box = 4cm
Breadth = 2.5cm  
Height = 1.5cm

“What is volume of cuboid?” Volume of 1 match box = \( \ell \times b \times h \)
\[ = 4\text{cm} \times 2.5\text{cm} \times 1.5\text{cm} \]
\[ = (4 \times 2.5 \times 1.5)\text{cm}^3 \]
\[ = 15\text{cm}^3 \]

“How many match boxes are there in 1 packet?”
Students respond.
Teacher explains “The size of one packet will be sizes of 12 match boxes.”
\[ = 12 \times 15\text{cm}^3 \]
\[ = 180\text{cm}^3 \]

Q5. A cuboidal water tank is 6m long, 5m wide and 4.5m deep. How many liters of water can it hold?
Teacher asks “What is the length, breadth and height of cuboidal tank?”
Student responds.

Length of tank = 6m  
Breadth = 5m  
Height = 4.5m

“What is the volume of cuboid?” Volume of tank = \( \ell \times b \times h \)
Student responds
\[ = 6\text{m} \times 5\text{m} \times 4.5\text{m} \]
\[ = (6 \times 5 \times 4.5)\text{m}^3 \]
\[ = 135\text{m}^3 \]

Teacher asks “How many cu cm are there in litre?”
Students respond.
Teacher asks “How many cu cm are there in 1m^3? 
\[ 1\text{m}^3 = 1000000\text{cm}^3 \]
Students responds.
\[ 1000\text{cm}^3 = 1\text{litre} \]
\[ 1\text{m}^3 = 1000\text{litres} \]
\[ \therefore \text{Volume of tank} = 135000\text{litres} \]

Recapitulation

<table>
<thead>
<tr>
<th>CHART</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume of cuboid = length \times breadth \times height</td>
</tr>
<tr>
<td>Volume of cube = (length)^3</td>
</tr>
</tbody>
</table>

Standard units of volume.
\[ 1\text{m}^3 = (100 \times 100 \times 100)\text{cm}^3 = 1000000\text{cm}^3 \]
\[ 1\text{cm}^3 = (100 \times 100 \times 100)\text{mm}^3 = 1000\text{mm}^3 \]
$1 \text{cm}^3 = 1 \text{ml}, \quad 1000\text{cm}^3 = 1\text{litre}$

$1 \text{m}^3 = 1000\text{l}^3 = 1\text{kl}$

The students revise the formulae and units with the help of chart.
LESSON – 45

Topic: Volumes (Contd.)

Entry Behaviour. It is assumed that the students are able to
(i) are able to compute volume of cuboid and cube
(ii) identify the word problems related to surface area and volume.

Instructional objectives:
After the instructional treatment is over the students will able to
♦ convert one kind of unit of volume to another
♦ solve complex problems related to volume of cuboid and cube.

Instructional aids: Blackboard, chalk, Model of cuboid and cube.

P.K. Testing
Q1. What is the volume of cuboid whose length, breadth and height are 10cm, 20cm and 15cm respectively?
Q2. Volume of cube =
Q3. Volume of cube when edge is 1.5m.
Q4. Find the depth of a tank having capacity. 60m³, length and breadth are respectively 5m and 4m.

Presentation of new material
“Today we will take problem having some complex calculations related to volume”

<table>
<thead>
<tr>
<th>Q.1. Find the cost of digging a cuboidal pit 8m long, 6m broad and 3m deep at the rate of Rs.30 per m³.</th>
<th>BB Work</th>
</tr>
</thead>
<tbody>
<tr>
<td>What is to be found out?</td>
<td>Cost of digging cuboidal pit</td>
</tr>
<tr>
<td>What is given in the problem?</td>
<td>Length, breadth and height of pit and cost per m³.</td>
</tr>
<tr>
<td>What is the volume of cuboid?</td>
<td></td>
</tr>
<tr>
<td>Students respond</td>
<td>Length = 8m</td>
</tr>
<tr>
<td></td>
<td>Breadth = 6m</td>
</tr>
<tr>
<td></td>
<td>Height = 3m</td>
</tr>
<tr>
<td></td>
<td>Volume of pit = l x b x h</td>
</tr>
<tr>
<td></td>
<td>= 8m x 6m x 3m</td>
</tr>
<tr>
<td></td>
<td>= (8x6x3) m³</td>
</tr>
<tr>
<td></td>
<td>= 144 m³</td>
</tr>
<tr>
<td>Question</td>
<td>Answer</td>
</tr>
<tr>
<td>----------</td>
<td>--------</td>
</tr>
</tbody>
</table>
| What is the cost per m³? | Cost of digging of m³ = Rs.30  
Cost of digging of 144m³  
= Rs.30 x 144  
= Rs. 3420 |

**Problem 2**: The capacity of a cuboidal tank is 50000 l of water. Find the breadth of the tank, if its length and depth are respectively 2.5m and 10m.

What is given in the problem?  
Teacher explains  
The units of volume are in litres whereas that of length and depth are in litres whereas that of length and depth are in meters. There is need to make the units same otherwise we can’t solve unlike terms.”

Teacher asks “How many litres are in there 1m³?  

1m³ = 1000 l  
V = 50 m³  

What is the volume of cuboid?  
Students respond  
How can we find breadth?  

V = l x b x h  
b = \frac{V}{l h}  
= \frac{50}{(2.5x10)m^2} = \frac{50}{25} m  
Breadth = 2m

**Problem 3**: A village, having a population of 4000, requires 150 l water per head per day. It has a tank measuring 20m by 15m by 6m. For how many days the water of this tank will last?

**Content**  
Teacher asks  
What is given in the problem?  
What is to be found out?  
Students respond.  
Teacher explains “Two things are given. First the capacity of the tank and the second Requirement of water per head per day of 4000 villagers. First
let us find out volume of the tank."

<table>
<thead>
<tr>
<th>Requirement of water by 1 person per day = 150 l</th>
</tr>
</thead>
</table>

Teacher asks "How will you find the no. of days the water of this tank will last?"

Teacher explained “The need of 4000 persons will be met from capacity of tank.”

How many litres are there in 1m³?

<table>
<thead>
<tr>
<th>Volume of tank = 1800m³</th>
</tr>
</thead>
<tbody>
<tr>
<td>1m³ = 1000 l</td>
</tr>
<tr>
<td>= 1800 x 1000 l</td>
</tr>
<tr>
<td>= 1800000 l</td>
</tr>
<tr>
<td>No. of days = 3</td>
</tr>
</tbody>
</table>

**Problem 4**: A godown measures 40m x 25m x 15m. Find the maximum number of wooden creates each measuring 1.5m x 1.25m x 0.5m that can be stored in the godown?

Teacher asks

What is to be found?

Length of godown = 40m
Breadth of godown = 25m

What is given information in the problem?

Height of godown = 15m
Volume of godown = l x b x h
= (40x25x15) m³
= 15000 m³
Length of wooden crate = 1.5m
Breadth of wooden crate = 1.25 m
Height of wooden crate = 0.5 m
Volume of wooden crate = l x b x h
= (1.5 x 1.25 x 0.5) m³
= 0.9375 m³

How will find the no. of crates that can be stored in a godown?

<table>
<thead>
<tr>
<th>No. of crates = \frac{Volume of godown}{Volume of wooden crate}</th>
</tr>
</thead>
<tbody>
<tr>
<td>= \frac{15000 m³}{0.9375 m³} = 16000</td>
</tr>
</tbody>
</table>
Problem 5: Two cubes each of side 6cm, are joined end to end. Find the volume of resulting cuboid.

Teacher asks
"What is the given information?"
Teacher explains. "If we join two cubes end to end a cuboid is formed."
Teacher asks "Look at the picture. What is the length, breadth and height of the cuboid?"
Now find the volume of cuboid.

<table>
<thead>
<tr>
<th>Side of each cube = 6cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of cuboid = 12cm</td>
</tr>
<tr>
<td>Breadth = 6 cm</td>
</tr>
<tr>
<td>Height = 6 cm</td>
</tr>
</tbody>
</table>

Volume of cuboid = \( L \times b \times h \)  
= \((12 \times 6 \times 6)\) cm\(^3\)  
= 432 cm\(^3\)

Problem 6:
How many wooden cubical blocks of edge 12cm can be cut from another cubical block of edge 3m 60 cm?

Teacher asks
"What is the information given in problem?"
How many cms are there in 1m?
Students respond

<table>
<thead>
<tr>
<th>Edge of bigger cubical block</th>
</tr>
</thead>
<tbody>
<tr>
<td>= 3m 60 cm</td>
</tr>
<tr>
<td>= 3m + 60 cm</td>
</tr>
<tr>
<td>= 3 \times 100 \text{ cm} + 60 \text{ cm}</td>
</tr>
<tr>
<td>= 300\text{ cm} + 60 \text{ cm}</td>
</tr>
<tr>
<td>= 360 \text{ cm}</td>
</tr>
</tbody>
</table>

What is the volume of the cube.

<table>
<thead>
<tr>
<th>Volume of cube = ( l^3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>= (360 \times 360 \times 360) \text{ cm}^3</td>
</tr>
<tr>
<td>= 46656000 \text{ cm}^3</td>
</tr>
</tbody>
</table>

Edge of small cube = 12cm

<table>
<thead>
<tr>
<th>Volume of cube = ( l^3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>= (12 \times 12 \times 12) \text{ cm}^3</td>
</tr>
<tr>
<td>= 1728 \text{ cm}^3</td>
</tr>
</tbody>
</table>

No. of cubes  
= \( \frac{\text{Volume of bigger cube}}{\text{Volume of small cube}} \)

200
Problem 7:
What will happen to the volume of a cube, if its each edges is
(i) doubled (ii) halved?

Let the edge of original cube = 1 cm
Volume of cube = l³

Case I
Edge of cube = 2 l
Volume of cube = (2 l)³ = 8 l³
It becomes 8 times

Case II
Edge of cube = l/2
Volume of cube = (l/2)³ = \frac{l³}{8}
It becomes one-eighth.

Practice:
How many wooden cubical blocks of edge 0.2m can be cut from a log
of wood of size 9m by 80cm by 90 cm, assuming there is no wastage.

Recapitulation
(1) Surface area of cuboid = 2 (lb+lh+bh)
(2) Surface area of cube = 6 l²
(3) Volume of cuboid = lbh
(4) Volume of cube = l³