Chapter 6

Conclusion

Supersymmetric quantum mechanics has become a powerful tool for the investigations of problems of modern non-relativistic quantum mechanics. It allows one to investigate the spectral properties of a wide class of quantum mechanical systems and to generate new systems with given spectra. SUSY QM gives new insight into the problems of spectral equivalence of the Hamiltonians, which, earlier have been constructed using factorization method in quantum mechanics and Darboux transformations in mathematical physics. Using SUSY QM techniques, the isospectral Hamiltonian approach is utilized to different physical situations.

The information entropy for Pöschl-Teller potential and isospectral Pöschl-Teller potential is calculated in ground state and first excited state. Starting with the Schrödinger equation for $n$-level Pöschl-Teller potential, we calculated the information entropy in position space and momentum space and satisfy the BBM inequality. It is shown that the inequality saturates as the number of bound states of the potential increases. Using isospectral Hamiltonian approach, we calculated the isospectral potential and deformed wave functions and also the information entropy content in the position space. For the reason of algebraic complexity in calculating the isospectral potential, higher excited state wave functions, we restrict to these calculations for 2-level and 3-level potential. The information entropy in position space is calculated for one-parameter and two-parameter deformed potential and it is shown that the information entropy content can be rearranged with the deformation parameter.

The kink solutions for $\phi^4, \phi^6$ model and for the potential model $V(\phi) = \ldots$
$\frac{2}{1}\phi^2\cos^2(\beta \ln \lambda \phi^2)$ have been generalized and studied the field theory admitting the kink solutions. The formalism allows to generate the kink models in two ways. In one case, first the generalized kink solution is obtained and then obtain the corresponding potential. In the second case, we start with the potential, obtain deformed potential and then obtain the kink solution. The change in the value of the deformation parameter in the new potential leads to the change in the associated kink waveform. Thus, the appropriate choice of the parameter enables us to employ a form of the potential that is closer to the situation in a particular physical system.

The FC factors and r-centroids using isospectral Hamiltonian approach have been calculated for a model Hamiltonian. First, we calculated the moments for the different levels of Pöschl-Teller potential. Using isospectral Hamiltonian approach, we deformed the potential and obtain the deformed wave functions. Then the moments for different levels are rearranged using the deformation parameter. After these calculations, this approach is used to calculate Franck-Condon factors for Morse potential. The Morse potential and the corresponding wave function have been deformed and then used for the calculations of Franck-Condon factors which contain a free parameter that can be adjusted considering the experimental data. In case, the calculations for FC factors, r-centroids or transition intensities does not match with experimental data, this approach has benefit to reduce the discrepancy without destroying the energy eigenvalue agreement.

This approach is also used to find the spectrum of charged particle in a class of non-uniform magnetic field. The behavior of a charged particle in an external uniform magnetic field is well known and the spectrum consists of equally spaced levels with level being infinitely degenerate. The problem is not in general solvable for non-uniform magnetic fields, however the ground state is exactly known and the degeneracy of the ground state can be related to the total flux. It has been shown that the spectrum is equispaced but non-degenerate for a wide class of non-uniform magnetic fields. Study of the classical behavior of charged particle in a non-uniform magnetic field is still an open problem.

Many authors have predicted the leptonic widths in various models of quarkonia and after analysis, they reached on the conclusion that in most potential models, it is
difficult to resolve the discrepancy between the values obtained and the experimental values for the spectra and leptonic widths. However, parton fragmentation into $J/\psi$, $\psi'$ at $p\bar{p}$ collision has been examined in experiments at the CDF collaboration at Fermi lab and the discrepancy between the experimental production rates and the production rates calculated from various models has been noted. It is found that the production rates are dependent on the quarkonium wave function. Thus, an accurate calculation of the wave function in quarkonium model is necessary to give correct leptonic widths as well as the production rates. The isospectral Hamiltonian approach provides a simple procedure for generating the partner potential for any one-dimensional potential with the same energy eigenvalues. As the bound state eigenfunctions changes and hence this may be the precise technique required to get agreement of decay widths for quarkonium potentials. The values of the wave functions at origin are calculated using the isospectral potential which consist of a free parameter that can be adjusted in accordance with the experimental data. The work of calculating the leptonic width and the production rates for the isospectral potential is in progress.

In conclusion, we have applied the isospectral Hamiltonian approach to various physical models to understand the role of the deformation parameter.