CHAPTER-7: Numerical modelling of atmospheric nitrogen plasma torch under thermal equilibrium and non-equilibrium condition

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Chapter: 7

Numerical Modelling of Atmospheric Nitrogen Plasma Torch under Thermal Equilibrium and Non-Equilibrium Condition

7.1. Introduction

This chapter presents the CFD simulation of the addressed nitrogen plasma torch. Such simulations are very much necessary to understand the device physics for design optimization, reliability enhancement as well as catering specific application needs. Direct non-invasive experimental investigation fails to probe the distribution of thermal, fluid dynamic and electromagnetic field distribution inside the device due to mechanical obstruction by the torch wall itself. Invasive experimental studies using probes also fail as no probe material can withstand the kind of high temperature, exists inside the torch. Therefore, such simulation studies are extremely important as this may be the only way to understand the thermal, fluid dynamic and electromagnetic field distribution inside the device.

A large number of simulation studies exist on simulation of arc plasma jets [56-69]. However, most of them deal with argon plasma [56-63]. Among very few studies on nitrogen arc, some report on transferred arc nitrogen plasma devices [64-67] and some deals non-transferred arc plasma torches operating with mixture of argon and nitrogen [68,69]. Practically no simulation study is carried out on segmented electrode pure nitrogen plasma torches, operating at moderate power level and delivering long plasma jet.
In the present simulation study it has been assumed that the plasma is an electrically conducting continuum fluid following usual Navier-Stokes equation with source terms appropriately modified by the effects of electromagnetic interactions. It is further assumed that the plasma is steady, axi-symmetric and optically thin. Effect of gravity on the plasma is ignored.

The CFD simulation uses the framework of two temperature modeling. Under this the plasma is assumed to be composed of two sub-gases: one is consisting of electrons and other is composed of all other heavy particles such as atoms, ions and molecules. Under the framework of 2T-model, the electrons are assumed to be in equilibrium among them at temperature $T_e$ and the rest of the heavy species are in equilibrium among themselves at temperature $T_h$ (different from $T_e$ in general). The model is general and equilibrium simulation ($T_e=T_h$) comes as a part of it.

Section 7.2 describes the general form of governing equations and control volume approach. Section 7.3 presents the discretized form of governing equations. The SIMPLE algorithm and pressure-velocity coupling is presented in section 7.4. Section 7.5 presents the final form of governing equations used in the modelling. The mathematical model for the problem is presented in section 7.6. The method of solution is presented in section 7.7, the computational domain is described in section 7.8. Boundary conditions used in the simulation is given in section 7.9. The thermodynamic transport properties used in the calculation is presented in section 7.10 and section 7.11 presents the results and discussion. Conclusions are presented in section 7.12.
7.2. Governing equations:

CFD simulation is the study of transport phenomena occurring in fluid through theoretical modelling. The governing equations of fluid flow represent mathematical statement of the conservation laws of the physics:

1. The mass of the fluid is conserved (conservation of mass)
2. The rate of change of momentum equals the sum of forces on a fluid particle (Newton’s second law or conservation of momentum)
3. The rate of change of energy is equal to the sum of the rate of heat addition to the particle and the rate of work done on the fluid particle. (First law of thermodynamics or conservation of energy)

CFD simulation is nothing but numerical solution of the associated conservation equations for mass, momentum and energy. The fluid is divided into number of infinitesimal volume element which contains large number of fluid particles. The laws of physics are applied to each and every volume element and the solution of the conservation equation is obtained. An infinitesimal volume element is bounded by finite number of surfaces, within which source or sink or both of a quantity can coexist. The behaviour of the fluid is expressed in terms of the macroscopic properties, such as velocity, pressure, density, temperature and their time derivatives. These may be thought of as averages over suitably large numbers of molecules. A fluid particle or a point in a fluid is than the smallest possible element of fluid whose macroscopic properties are not influenced by individual molecules. The control volume is fixed in space and the fluid is moving through the fluid element.
The governing equations and the procedure of solution are described in the following sections.

The general form of governing equations assumes the following form,

$$\frac{\partial}{\partial t} (\rho \phi) + \text{div} \left( \rho \vec{v} \phi \right) = \text{div} \left( \Gamma \text{grad} \phi \right) + S \quad (7.1)$$

Where, $\phi$ is the variable, $\rho$ is the mass density, $\vec{v}$ is the velocity, $\Gamma$ is the coefficient of diffusion and $S$ is the source term. The first term in the left hand side of the relation is unsteady term, and the second term is convective term. In the right hand side, first term is the diffusive term and second term is the source term. Thus qualitatively, it may be written as,

**Unsteady term** + **Convective term** = **Diffusive term** + **Source term**

The equation tells that, in any particular control volume, at any particular instant of time, the net production of the variable $\phi$ is exactly balanced by the convection and diffusion of the same variable i.e. the net inward flux of the variable $\phi$ is exactly balanced by the net outward flux. In order to obtain results for velocity distribution, temperature distribution, current density distribution, potential distribution etc., the basic conservation equations including conservation of mass, momentum, energy and charge are solved. Since the computational domain is cylindrical in nature, all the governing equations are written in cylindrical co-ordinate system and solved accordingly.
7.2.1 Continuity Equation:

The continuity equation is based on the law of conservation of mass. So for the mass balance inside the fluid element we need,

\[
\text{Rate of increase of mass in the fluid element} = \text{Rate of mass flowing into the fluid element}
\]

The general form of continuity equation can be written as,

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0
\]

(7.2)

Hence the continuity equation in cylindrical co-ordinate system can be written as,

\[
\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \rho v_r) + \frac{1}{r} \frac{\partial}{\partial \theta} (\rho v_\theta) + \frac{\partial}{\partial z} (\rho v_z) = 0
\]

(7.3)

Since we are dealing with steady state equations, the first term will be zero and the continuity equation or the mass conservation equation becomes,

\[
\nabla \cdot (\rho \mathbf{v}) = 0
\]

(7.4)

Or

\[
\frac{1}{r} \frac{\partial}{\partial r} (r \rho v_r) + \frac{1}{r} \frac{\partial}{\partial \theta} (\rho v_\theta) + \frac{\partial}{\partial z} (\rho v_z) = 0
\]

(7.5)

7.2.2 Momentum conservation equations:

This is based on the Newton’s second law of motion i.e. the net force on a volume element is the product of its mass and its acceleration. In CFD, the forces acting on a fluid element are broadly classified as body forces and surface forces.
**Body forces:**

Body forces are those forces which apply directly on the volumetric mass of the fluid element, such as gravitational force and electromagnetic force.

In a conducting fluid like plasma, there are two types of body forces:

1. Gravitational force per unit volume: \( \rho g \)
2. Electromagnetic body force per unit volume: \( j \times B \)

Hence the net body force per unit volume can be written as:

\[
\vec{F}_B = (F_B, i) + (F_B, j) + (F_B, k) = \rho \vec{g} + j \times B
\]  
(7.6)

**Surface forces:**

Surface forces are those forces which act on the surface of the fluid element e.g. normal stress and tangential stress. As shown in Figure 7.1, there are six tangential stress terms and three normal stress terms which contribute to the surface forces in the three directions.

![Figure 7.1 Stress components in a fluid element in Cartesian co-ordinate and cylindrical coordinate system.](image)
The expression for the three components of the momentum conservation equation in cylindrical coordinate system is written in the following sections.

**Radial component of momentum conservation equation:**

The radial component of momentum equation can be written as:

\[
\nabla \left( \rho \mathbf{v}_r \right) + \frac{\partial (\rho \mathbf{v}_r)}{\partial t} = -\frac{\partial p}{\partial r} + \nabla \cdot \left( \mu \nabla \mathbf{v}_r \right) + \Sigma_r, \tag{7.7}
\]

Where, \( \Sigma_r \) is written as,

\[
\Sigma_r = \frac{1}{r} \frac{\partial}{\partial r} \left( r \mu \frac{\partial v_r}{\partial r} \right) + \frac{1}{r \partial \theta} \left[ \mu \left( \frac{\partial v_r}{\partial r} - \frac{v_r}{r} \right) \right] + \frac{\partial}{\partial z} \left( \mu \frac{\partial v_r}{\partial z} \right) - \frac{2\mu}{r} \left( \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r} \right) + \rho \left( \frac{v_\theta^2}{r} \right) + (F_b)_r, \tag{7.8}
\]

\((F_b)_r\) is the body force given as, \((F_b)_r = (j_\theta B_z - j_z B_\theta)\)

For a steady state solution, the unsteady term can be eliminated. Hence the radial component of momentum conservation equation is written as,

\[
\nabla \left( \rho \mathbf{v}_r \right) = -\frac{\partial p}{\partial r} + \nabla \cdot \left( \mu \nabla \mathbf{v}_r \right) + \Sigma_r, \tag{7.9}
\]

**Axial component of momentum conservation equation:**

In similar manner, the axial component of momentum conservation equation can be written as,

\[
\frac{\partial (\rho \mathbf{v}_z)}{\partial t} + \nabla \left( \rho \mathbf{v}_z \right) = -\frac{\partial p}{\partial z} + \nabla \cdot \left( \mu \nabla \mathbf{v}_z \right) + \Sigma_z, \tag{7.10}
\]

Where, \( \Sigma_z \) is given as,
\[
\Sigma_z = \frac{1}{r} \frac{\partial}{\partial r} \left( r \mu \frac{\partial v_r}{\partial z} \right) + \frac{1}{r} \frac{\partial}{\partial \theta} \left( \mu \frac{\partial v_\theta}{\partial z} \right) + \frac{\partial}{\partial z} \left( \mu \frac{\partial v_z}{\partial z} \right) + (F_b)_z
\]

(7.11)

And \((F_b)_z\) is the \(z\)-component of the body force and it can be written as,

\[(F_b)_z = \left(j_r B_\theta - j_\theta B_r\right)\]

Since we are dealing with steady state solution the unsteady term can be removed and the axial component of the momentum conservation equation can be written as,

\[
\nabla \left( \rho \hat{v}_z \right) = -\frac{\partial p}{\partial z} + \nabla \left( \mu \hat{v}_z \right) + \Sigma_z
\]

(7.12)

**Azimuthal component of momentum conservation equation:**

The azimuthal component of the momentum conservation equation can be written as,

\[
\frac{\partial (\rho v_\theta)}{\partial t} + \nabla \left( \rho \hat{v}_\theta \right) = -\frac{\partial p}{\partial \theta} + \nabla \left( \mu \hat{v}_\theta \right) + \Sigma_\theta
\]

(7.13)

where,

\[
\Sigma_\theta = \frac{1}{r} \frac{\partial}{\partial r} \left( r \mu \frac{\partial v_r}{\partial \theta} \right) - \mu \frac{\partial}{\partial r} \left( \frac{v_\theta}{r} \right) + \mu \frac{\partial v_\theta}{\partial r} + \frac{1}{r} \frac{\partial v_r}{\partial \theta} - 2 \frac{v_\theta}{r}
\]

\[
+ \frac{1}{r} \frac{\partial}{\partial \theta} \left[ \frac{\mu \partial v_\theta}{r} + 2 \frac{v_r}{r} \right] + \frac{\partial}{\partial z} \left[ \frac{\mu \partial v_r}{r} \right] - \rho \frac{v_r v_\theta}{r} + (F_b)_\theta
\]

(7.14)

and the azimuthal component of body force \((F_b)_\theta\) is given as, \((F_b)_\theta = \left(j_r B_z - j_z B_r\right)\)

For steady state solution, the time dependent term is removed and the azimuthal component of momentum conservation equation is written as,

\[
\nabla \left( \rho \hat{v}_\theta \right) = -\frac{\partial p}{\partial \theta} + \nabla \left( \mu \hat{v}_\theta \right) + \Sigma_\theta
\]

(7.15)
7.2.3 The energy conservation equation in LTE plasma:

In case of plasma in thermodynamic equilibrium, energy balance inside a fluid element is governed by the following mechanisms:

a. Work done by the viscous stress inside the fluid. ($W_\mu$)
b. Rate of heat transfer through conduction ($H_X$)
c. Rate of volumetric heat generation due mechanisms such as joule heating and change of energy due to charge particles owing to temperature gradient ($U_P$)
d. Rate of volumetric heat loss such as radiation loss ($U_R$)
e. Change in internal energy ($U_C$)
f. Rate of volumetric heat loss or gain due to collisional exchange ($U_X$)

Hence for an infinitesimal volume element of the plasma with volume $dV$ enclosed by surface area of $dA$ the energy equation can be represented by first law of thermodynamics and it can be written as:

$$U_N = H_X + W_\mu + U_P + U_R + U_C + U_X$$  \hspace{1cm} (7.16)

Where, $U_N$ is the volumetric rate of change of the internal energy. The term $U_P$ can be expressed as the sum of volumetric generation of heat due to Ohmic heating and volumetric change of energy by the charge particles due to temperature gradient. Hence it can be written as, $U_P = U_O + U_J$. Contribution of various mechanisms to the energy equation is explained below.

a. Volumetric rate of change of Internal energy:

If $E$ is the internal energy per unit mass than $U_N$ can be written as,
\[ U_N = \rho \frac{dE}{dt} \]  \hspace{1cm} (7.17)

b. Rate of external heat transfer to the fluid through conduction (\(H_X\))

According to the Fourier’s law, if \(k\) is the thermal conductivity of the fluid, then, the rate at which heat is flowing out of the volume element through the surface area is proportional to the rate of decrease of heat in the volume element. The rate at which heat is flowing out of the volume element through the surface area \(dA\) is given as,

\[
\int \int k\left(\nabla T\right) dA
\]

and the rate at which, heat in the volume element reduces is given as,

\[
\iiint \rho \frac{dQ}{dt} dV
\]

Converting the volume integral into surface integral we obtain,

\[
H_X = \rho \frac{dQ}{dt} = \nabla.(k\nabla T)
\]  \hspace{1cm} (7.18)

where, \(T\) is the temperature and \(k\) is the thermal conductivity of the fluid. In case of plasma in local thermal equilibrium, \(T\) represents both the electron temperature and heavy particle temperature. The thermal conductivity is the total thermal conductivity and can be expressed as the sum of thermal conductivity of electrons and that of heavy particles. So it can be written as, \(k = k_e + k_h\), where, \(k_e\) is the thermal conductivity of the electrons and \(k_h\) is the thermal conductivity of heavy particles.

c. Change of energy due to charge particle content: \((U_P)\)

There are two ways through which energy is associated with charged particle in the plasma and they are generation of energy through the Ohmic heating \((U_O)\) and energy transport through heat conduction by the charge particles because of the
temperature gradient \((U_j)\). Because of the finite resistivity of the plasma, Ohmic heating plays an important role in generating heat. Ions being heavier than electrons, the contribution of ion current to the total current is very small. The volumetric rate of heat generation in the plasma by Ohmic heating is given as,

\[
U_0 = \vec{j} \cdot \vec{E}
\]  

(7.19)

\(\vec{j}\) is the total current density and in cylindrical co-ordinate system it can be written as

\[
\vec{j} = j_r \hat{r} + j_\theta \hat{\theta} + j_z \hat{z}
\]

Since \(\vec{j} = \sigma \vec{E}\) (Generalized Ohm’s law), the volumetric rate of heat generation by Ohmic heating can be written as,

\[
U_0 = \sigma (E_r^2 + E_\theta^2 + E_z^2)
\]  

(7.20)

The other mode through which energy is associated with the charged particle is transport of heat by them through heat conduction by virtue of its temperature gradient and it can be written as,

\[
U_j = \frac{5}{2} k_e \left( \vec{j} \cdot \nabla T \right)
\]  

(7.21)

d. Radiation loss \((U_R)\)

It is very difficult to derive an exact theoretical expression for the volumetric heat loss through radiation because of the associated processes such as emission, absorption and very high temperature gradient. In this study we have used the experimental data by Krey and Morris [106] to include the effect of radiation loss. Appropriate extrapolations have been used where data was not available. The
estimated radiation loss as a function of temperature of nitrogen plasma as obtained by Krey and Morris is presented in Fig. 7.2.

Fig. 7.2 Variation of radiation loss with temperature in nitrogen plasma

e. Rate of change of energy due to chemical reaction: \( (U_C) \)

This quantity depends on particular nature of the reactive gases, and can be assumed to be zero for most of the inert gases in high enthalpy arc plasma devices.

f. Rate of volumetric heat loss or gain due to collisional exchange (\( U_X \))

Plasma contains various species of particles including electron, atom, and ions. In plasma at higher pressure, collisions among the particles play important role in energy transport. For collision among electron and a given species of particles \( S \), the average energy loss by an electron per unit time depends on collision cross-section \( \Sigma_s \), relative speed between electron and heavy particles \( v_{es} \) and average energy loss of an electron
per collision with heavy particle. Hence the rate of volumetric energy loss of electron is given as

\[ U_x = n_e \sum_s n_s v_{es} \Sigma_s (\Delta E_x)_s \]

\[ = \sum_s \frac{2m_e}{m_s} \frac{3}{2} k_B \Sigma_s n_e n_s v_{es} (T_e - T_h) \]  

(7.22)

Where, \( k_B \) is the Boltzmann constant, \( m_e, n_e \) and \( T_e \) represents the mass, number density and temperature of electrons respectively. \( m_s, n_s \) are the mass and number density of particle of species \( s \) respectively. \( T_h \) is the heavy particle temperature. The average energy loss of an electron per collision is than given by the following expression.

\[ (\Delta E_x)_s = \frac{2m_e}{m_s} \frac{3}{2} k_B (T_e - T_h) \]  

(7.23)

Hence the rate of volumetric heat loss by electrons per unit time is given as the following.

\[ U_x = \sum_s \frac{2m_e}{m_s} \frac{3}{2} k_B \Sigma_s n_e n_s \left( \frac{8k_B T_e}{m_e \pi} \right) (T_e - T_h) \]  

(7.24)

From the above expression it is quite evident that, when the plasma is in thermodynamic equilibrium there is no such energy transfer occurs.

So the final form of the energy equation can be obtained by adding the different contribution to the energy equation. Hence as written earlier, the complete energy equation can be written as,
\[ U_N = H_x + W_x + U_P + U_R + U_C + U_X \]

Substituting the different contribution in the above equation we obtain,

\[
\rho \frac{d(E)}{dt} = \nabla \cdot (k \nabla T) - \rho \left( \nabla \cdot \mathbf{v} \right) + \Phi + U_P - U_R + U_C + U_X
\]  

(7.25)

According to the equation of continuity we have,

\[
\frac{d \rho}{dt} + \rho \left( \nabla \cdot \mathbf{v} \right) = 0
\]

Substituting in equation (7.25), we obtain

\[
\frac{d}{dt} \left[ \frac{E + P}{\rho} \right] = \frac{1}{\rho} \nabla \cdot (k \nabla T) + \frac{1}{\rho} \frac{dp}{dt} + \frac{1}{\rho} \left[ \Phi + U_P - U_R + U_C + U_X \right]
\]

(7.26)

Since, enthalpy of the gas per unit mass is given as \( h = E + P/\rho \), we can write,

\[
\rho \frac{dh}{dt} - \frac{dp}{dt} = \nabla \cdot (k \nabla T) + \Phi + U_P - U_R + U_C + U_X
\]

(7.27)

In plasma, enthalpy \( h \) includes the kinetic energy of electrons, that of heavy particles, and ionization energy of different particles also. For LTE plasma, the exchange term do not appear. When convective losses are included for an nonreactive LTE plasma, the above equation appears under steady state as:

\[
\nabla \cdot (\rho \nabla h) = \nabla \cdot (k \nabla T) + \frac{5}{2} \frac{k_B}{e} \left( j \cdot \nabla T \right) + \sigma (E_r^2 + E_d^2 + E_z^2) - U_R
\]

7.2.4. Final form of governing equations:

This study presents a steady state, two dimensional, axis symmetric model for simulation of nitrogen plasma. Hence the unsteady term is not present in the equation.
Being axis symmetric, the model is two dimensional. The equation (7.1) for general form of the conservation equations in steady state reduces to:

\[
\nabla^2 (\rho f \vec{v} \phi) = \nabla^2 (\Gamma \phi \vec{v} \phi) + S_\phi 
\]

(7.28)

where, \( \phi \) is the quantity conserved, \( \rho_f \) is the mass density of fluid \( f \) for momentum equation and mass density times specific heat for temperature equations, \( \Gamma_\phi \) is the corresponding diffusion coefficient and \( S_\phi \) is the associated source term. \( \vec{v} \) is the velocity vector having axial, radial and azimuthal components, \( v_z, v_r, \) and \( v_\theta \) respectively. The final form of the governing equations used for the simulation is given table-7.1. In the table, \( \phi \) is the electric potential, \( p \) is the pressure. \( j_r \) and \( j_z \) are the radial and axial components of current density. \( \nu, \sigma \) and \( c_{ph} \) are, respectively, viscosity, electrical conductivity and specific heat of heavy particles. Viscous dissipation is neglected in electron energy equation. Since the flow is assumed to be laminar, so no turbulent equation has been included in this study. \( k_e \) and \( k_h \) are, respectively, effective thermal conductivity of electrons and heavy particles. The self induced magnetic field \( (B_\theta) \) at radial location \( r \) is computed by integrating \( j_z \):

\[
B_\phi(r) = \frac{\mu_0}{r} \int_0^r j_z dy
\]

(7.29)

Where, \( \mu_0 \) is permeability of vacuum.
Table 7.1 Terms of the general governing equation (7.28)

<table>
<thead>
<tr>
<th>Conservation of</th>
<th>$\phi$</th>
<th>$\Gamma_\phi$</th>
<th>$S_\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Axial momentum</td>
<td>$v_z$</td>
<td>$v_{\text{eff}}$</td>
<td>- $\frac{\partial p}{\partial z} + \frac{1}{r} \frac{\partial}{\partial r} \left[r v_{\text{eff}} \left(\frac{\partial v_{\text{eff}}}{\partial z}\right)\right] + \frac{\partial}{\partial z} \left(v_{\text{eff}} \frac{\partial v_{\text{eff}}}{\partial z}\right)$</td>
</tr>
<tr>
<td>Radial momentum</td>
<td>$v_r$</td>
<td>$v_{\text{eff}}$</td>
<td>- $\frac{\partial p}{\partial r} + \frac{1}{r} \frac{\partial}{\partial r} \left[r \mu \left(\frac{\partial v_r}{\partial r}\right)\right] + \frac{\partial}{\partial z} \left(v_{\text{eff}} \frac{\partial v_z}{\partial z}\right)$</td>
</tr>
<tr>
<td>Azimuthal momentum</td>
<td>$v_\theta$</td>
<td>$v_{\text{eff}}$</td>
<td>$-\rho \frac{v_r v_\theta}{r} - v_{\text{eff}} \frac{v_{\text{eff}}}{r^2}$</td>
</tr>
<tr>
<td>Potential</td>
<td>$\phi$</td>
<td>$\sigma$</td>
<td>0</td>
</tr>
</tbody>
</table>

| Heavy particle temperature | $T_h$ | $k_h + C_{ph} k_{th}$ | $2 v_{\text{eff}} \left[\left(\frac{\partial v_r}{\partial r}\right)^2 + \left(\frac{\partial v_z}{\partial z}\right)^2 + \left(\frac{v_r}{r^2}\right)^2\right]$ | $+ v_{\text{eff}} \left[\left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right)^2 + \left(\frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r}\right)^2 + \left(\frac{\partial v_\theta}{\partial z}\right)^2\right]$ | $- \frac{2}{3} v_{\text{eff}} \left(\nabla \cdot \mathbf{v}\right)^2 - \rho \left(\nabla \cdot \mathbf{v}\right) + E_{\text{ch}}$ |
| Electron temperature | $T_e$ | $k_e$ | $\frac{j_z^2 + j_z^2}{\sigma} + \frac{5}{2} k_B \left(j_z \frac{\partial T_e}{\partial z} + j_r \frac{\partial T_e}{\partial r}\right) - 4\pi U_r - E_{\text{ch}}$ |

$E_{\text{ch}}$ in Table -7.9 is the rate of energy transfer from electrons to heavy particles and can be presented as given in the equation 7.30.

$$E_{\text{ch}} = \sum_h \frac{3}{2} k_h (T_e - T_h) n_e \left(\frac{2 m_e}{m_h}\right) v_{\text{ch}}$$  \hspace{1cm} (7.30)
\( n_e \) and \( n_h \) are, respectively, number density of electrons and heavy particles. \( m_e \) and \( m_h \) are their respective masses. \( \bar{v}_{eh} \) is the average volumetric collision frequency between electrons and heavy particles and it is calculated as given in equation (7.31).

\[
\bar{v}_{eh} = \bar{c}_e n_h Q_{eh}
\]  

(7.31)

Where, \( Q_{eh} \) is the collision cross-section between electrons and heavy particles, and \( \bar{c}_e \) is the average thermal speed of electrons.

**7.3. Method of Solution:**

For solution of the governing equations, two codes are used in unison. The first one is a fluid dynamic solver and the second one is a property routine. The property routine works as explained in the chapter for thermodynamic and transport properties. In the fluid dynamic solver, the solution of the governing equations are performed starting with an initial guess for potential, electron temperature, heavy particle temperature, pressure and velocity inside the computational domain. A SIMPLE like algorithm of Patankar[107] is used to solve the equations under a finite volume discretization technique. The computer code FAST-2D [108], an extension of the program originally developed by Majumdar [109], has been appropriately modified to adopt flexible grid and include additional equations for electron temperature, heavy species temperature, joule heating, radiation loss, electromagnetic body force and collisional exchange term. Updated values of the solved quantities in the CFD solver is passed to the kinetic equations to obtain the updated distribution of number densities in each iteration. The updated number density distributions and associated temperature profiles are then used by the property routine to recalculate the properties for use in
the next iteration. Iteration continues until the solution of the quantities reach a steady state. The finite volume grid used in the study is presented in Fig.7.3. Obtained steady state results are checked for grid invariance.

7.4. The Boundary Conditions

The computational domain used for the study is shown in figure 7.3. It includes the cathode, anode. ABCDEA is the cathode and it is made up of tungsten. It has a conical tip with cone angle of $24^0$. AB is the base of the cathode and DE is the tip of the cathode. The dimensions of AB and DE are 5.0 and 3.5 mm respectively. Cubic profile is used for the current density. The diameter of current emitting region is 1/3 of the original diameter of the cathode. The torch used for the study is a non-transferred arc DC segmented plasma torch consisting of seven segments including cathode. The anode and the segments between cathode and anode form the nozzle. RF is the torch nozzle exit and its diameter 10 mm. MNHRKM forms the anode, and it is made up of copper. The wall temperature for the anode is 600 K. The current connection is considered to be diffused. BN is the gas inlet. The plasma gas injection point is located at some upstream location. The pressure at the upstream point is 2 atm. The orifice diameter is 2 mm. The gas injection process is swirl injection and the gas enters the torch through an orifice which makes an angle of $20^0$ with the axis. The
axial, radial and azimuthal velocity computed from gas flow rate and orifice diameter are 20.6, 0.0, 6.77 m/s respectively. The details of the employed boundary conditions are given in Table 7.2. The potential along the line DE is updated in every iteration to \( \Phi = \Phi_j \), a profile which is self consistent with the assumed current density profile over the cathode surface. Since the nozzle is conducting, for the solution of the nozzle has been included in the computational domain and a boundary condition is applied in on MLK and RK boundaries. Since the torch is operating in non-transferred arc mode, the boundary condition \( \partial \phi / \partial z = 0 \) is appropriate which means a constant potential at these boundaries.

<table>
<thead>
<tr>
<th>Boundary</th>
<th>p</th>
<th>( V_z )</th>
<th>( V_r )</th>
<th>( V_\theta )</th>
<th>( T_e(K) )</th>
<th>( T_h(K) )</th>
<th>( \phi(v) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>BN</td>
<td>4.74 atm</td>
<td>u</td>
<td>0</td>
<td>- ( \tan(20) )</td>
<td>300</td>
<td>300</td>
<td>( \partial \phi / \partial z = 0 )</td>
</tr>
<tr>
<td>RF</td>
<td>-</td>
<td>( \partial v_z / \partial z = 0 )</td>
<td>0</td>
<td>( \partial v_\theta / \partial z = 0 )</td>
<td>( \partial T_e / \partial z = 0 )</td>
<td>( \partial T_h / \partial z = 0 )</td>
<td>( \phi = 0 )</td>
</tr>
<tr>
<td>EF</td>
<td>-</td>
<td>( \partial v_z / \partial r = 0 )</td>
<td>0</td>
<td>( \partial v_\theta / \partial r = 0 )</td>
<td>( \partial T_e / \partial r = 0 )</td>
<td>( \partial T_h / \partial r = 0 )</td>
<td>( \partial \phi / \partial r = 0 )</td>
</tr>
<tr>
<td>NOHR</td>
<td>-</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( \partial T_e / \partial r = 0 )</td>
<td>600</td>
<td>-</td>
</tr>
<tr>
<td>BCDG</td>
<td>-</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1000</td>
<td>1000</td>
<td>-</td>
</tr>
<tr>
<td>GE</td>
<td>-</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3000</td>
<td>3000</td>
<td>( \phi = \phi_j )</td>
</tr>
<tr>
<td>MLK</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>( \partial \phi / \partial n = 0 )</td>
</tr>
<tr>
<td>RK</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>( \partial \phi / \partial z = 0 )</td>
</tr>
</tbody>
</table>
7.5. Results and discussion:

Temperature distribution:

The distribution of plasma temperature within the plasma torch for four values of electrical current including 170A, 180A, 200A, 210A is presented in figure 7.4. It can be seen that, the zone of high temperature zone gradually extends from cathode to anode as current increases. The axial temperature between the cathode and anode for electrical currents below 200 A is found to drop because the arc is more constricted near the cathode. This causes thermal expansion of the heated gas in the vicinity of the cathode which in turn reduces the temperature in the downstream location. Axial temperature near the anode further increases as the arc is constricted again in this region in the downstream. Observed two lobes in the temperature distribution is a result of this. Significant change in the temperature distribution pattern is observed as one goes from 170A to 180A.

Fig. 7.4 Nitrogen plasma temperature distribution inside the plasma torch for different currents, (a) 170A, (b) 180A, (c) 200A and (d) 210A. Flow 17.5 slpm N\textsubscript{2}
However, the axial temperature distribution pattern does not change much as the current goes 200 A and above. The plot of axial temperature profiles and the radial temperature profiles at the nozzle exit are given in figure 7.5 (a) and (b) respectively. Significant jump in the temperature distribution as current switches from 180A to 200A are observed in Figure 7.5. Spectroscopic measurements of plasma temperature at the nozzle exit under the said operating conditions also exhibit similar features as presented in the next section.

![Fig. 7.5 Axial and radial plasma temperature profile of nitrogen plasma. (a) axial temperature profile, (b) radial temperature profile.](image)

![Fig. 7.6 Potential distribution inside the torch at different currents, (a) 170A, (b) 180A, (c) 200A and (d) 210A.](image)
Potential distribution:

The distribution of potential inside the plasma torch for the four current values is presented in figure 7.6. A potential drop exceeding 150V is observed. While nearly similar nature of potential distribution is observed at the considered arc currents, the equipotential zone near the cathode shrinks as current increases. Obtained variation in the potential along the torch axis is shown in figure 7.7. Except near the cathode, almost a linear variation in the potential is observed for all currents. Significant variation in the potential distribution as current changes from 180A to 200A may be noted. Since voltage drop across the arc is an easily measurable quantity, obtained results are verified experimentally in the next section.

![Fig. 7.7 Variation of axial potential for different current values in the plasma column](image)

Velocity distribution:

The distribution of velocity inside the plasma torch is shown in figure 7.8. As the plasma channel is long enough, nearly a fully developed flow is observed. Maximum velocity observed at the exit is around 380 m/s. Overall radial distribution of axial velocity is nearly parabolic. Step like features originate from the sharp
transitions between current carrying and non-current carrying zones in the plasma jet. The current carrying region includes ohmic source terms while non-current regions do not have any.

![Fig.7.8 Velocity distribution inside the plasma torch](image)

**Validation of the simulation result:**

The total arc voltage drop and plasma temperature at the nozzle exit are measured through experiments and compared with the above simulation results under similar operating conditions. Results are presented in Figure 7.9. An excellent agreement between the measured and the simulated voltage drop is observed. It can be seen that, as the current varies from 170A to 180A, the arc voltage maintains almost a constant value both theoretically and experimentally. Similar behaviour is observed as arc current changes from 200A to 220A. However, experimentally observed distinct change in the potential for transition from 180A to 200A is slightly smeared out in the simulation results.

Comparison of experimental and simulated plasma temperature at the nozzle exit is presented in Fig.7.10. Apart from the results at 180A, the observed agreements
are good. Since theoretically it is found that around 180A a transition zone exits, the experimental system may exhibit significant change in behaviour for slight change in experimental condition. Observed mismatch between experimental and theoretical results at 180A might have originated from this.

Nice agreement between the theoretical and the experimental results suggests that the proposed model of the nitrogen plasma torch is reasonably accurate.

Fig. 7.9 Comparison of experimental and simulated arc voltages

Fig. 7.10 Comparison of experimental and simulated plasma temperature
7.6. Conclusions

This chapter of the thesis presents a steady state two-dimensional laminar model of DC non-transferred arc nitrogen plasma torch. The torch is modelled in terms of usual Navier Stokes equations modified by appropriate source terms originating from the current currying nature of the plasma fluid. Associated conservation equations for mass, momentum and energy are solved using SIMPLER like algorithm under finite volume formalism. A modified version of the code FAST-2D is used as solver. Results of the simulation study are presented in terms of temperature, velocity and potential distribution. It has been observed that under the chosen operating conditions there exists a distinct change in the temperature distribution inside the plasma torch for currents less than 200A and that above 200A. Similar feature is observed for the axial potential distribution also. The potential distribution inside the plasma torch remains almost similar for all values of current. However, the voltage drop in the plasma for current more than 200 A is more than that for values of current below 200A. Nearly parabolic velocity distributions with tiny step like features are observed at the torch exit. Reasons for observed behaviour are explained. Obtained temperatures at the nozzle exit and total arc voltage drops are compared with experimental data. Overall very good agreements between experimental and simulation results support correctness of the used model.