Chapter 7

ENHANCING THE ICAL PHYSICS REACH WITH HADRONS

The ICAL experiment would primarily aim to identify the neutrino mass hierarchy from the observations of atmospheric muon neutrinos. It would also improve the precision on the atmospheric neutrino mixing parameters as well as fulfill a number of other goals, which have been mentioned in Chapter 2. The design of ICAL is primarily optimized to measure the muon momentum. Further, the ICAL, being a magnetized detector, would also be able to separate the $\nu_\mu$ and $\bar{\nu}_\mu$ events, through the distinction of the muon and antimuon tracks in the magnetic field. In addition, it is also capable of measuring the hadron energy in each event. Although the hadron energy is measured with relatively poorer resolution, it nevertheless contains crucial information on the event, which may be utilized when taken concomitant with the muon data. The capability of the ICAL, in reconstructing the muon and the hadron information, has been studied using a simulation framework, as described in Chapter 3 and Chapter 4, respectively.

The physics potential of ICAL, in fulfilling its prominent physics goals, can be studied from the event rates expected in the detector. The initial analysis of the

This chapter is based on JHEP 1410, 189 (2014) [20].
7.1. THE EARTH MATTER EFFECT ON NEUTRINO OSCILLATION PROBABILITIES

Physics reach was carried out using the muon momentum \((E_\mu, \cos \theta_\mu)\) only \([18, 19]\). In this section, the ICAL sensitivity, after adding the hadron energy information to the muon energy and muon direction in each event, has been discussed. It is observed that, the hadron energy, when added to the analysis by treating \(E_\mu\) and \(E'_{\text{had}}\) as two separate variables in each event, improve the results significantly. In this approach, the correlation between these two quantities in each event is taken care of. The enhancement of the ICAL physics potential, for determining the neutrino mass hierarchy, the atmospheric mass squared difference and the mixing angle \(\theta_{23}\), and its octant, obtained using the values \(E_\mu, \cos \theta_\mu\) and \(E'_{\text{had}}\) from each event as independent variables has been presented in this chapter.

This chapter is started with a discussion on the Earth matter effects on the neutrino oscillation probabilities, which is the motivation behind the physics study with the ICAL detector. The exploration of the hadronic contribution in the neutrino events of interest in ICAL, the numerical analysis procedure and the enhanced results are then discussed in the subsequent sections.

### 7.1 THE EARTH MATTER EFFECT ON NEUTRINO OSCILLATION PROBABILITIES

The atmospheric muon neutrinos and antineutrinos are produces via the two channels, the survival channel \(\nu_\mu \rightarrow \nu_\mu (\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu)\), and the oscillation channel \(\nu_e \rightarrow \nu_\mu (\bar{\nu}_e \rightarrow \bar{\nu}_\mu)\). For the sake of simplicity, if it is approximated that \(|\Delta m^2_{21}| \ll |\Delta m^2_{31}|\) (or, \(|\Delta m^2_{21}| \ll |\Delta m^2_{32}|\)), then assuming a constant matter density, these two probabilities can be written as

\[
P_{\mu e}^m = \sin^2 \theta_{23} \sin^2 2\theta_{13} \sin^2 \left[1.27(\Delta m^2_{31})^m \frac{L(\text{km})}{E(\text{GeV})}\right],
\]  

(7.1)
and

\[
P_{\mu\mu}^m = 1 - \cos^2 \theta_{13}^m \sin^2 2\theta_{23} \sin^2 \left[ 1.27 \left( \Delta m_{31}^2 + A + (\Delta m_{31}^2)^m \right) \frac{L(\text{km})}{E(\text{GeV})} \right] \\
- \sin^2 \theta_{23}^m \sin^2 2\theta_{23} \sin^2 \left[ 1.27 \left( \Delta m_{31}^2 + A - (\Delta m_{31}^2)^m \right) \frac{L(\text{km})}{E(\text{GeV})} \right] \\
- \sin^4 \theta_{23} \sin^2 2\theta_{13} \sin^2 \left[ 1.27(\Delta m_{31}^2)^m \frac{L(\text{km})}{E(\text{GeV})} \right],
\]

(7.2)

where the mass–squared difference \((\Delta m_{31}^2)^m\) and the mixing angle \(\theta_{13}^m\) can be expressed in terms of their values in vacuum, as

\[
(\Delta m_{31}^2)^m = \sqrt{(\Delta m_{31}^2 \cos 2\theta_{13} - A)^2 + (\Delta m_{31}^2 \cos 2\theta_{13})^2},
\]

(7.3)

and

\[
\sin 2\theta_{13}^m = \frac{\Delta m_{31}^2 \sin 2\theta_{13}}{\sqrt{(\Delta m_{31}^2 \cos 2\theta_{13} - A)^2 + (\Delta m_{31}^2 \cos 2\theta_{13})^2}}.
\]

(7.4)

Here \(A = 2\sqrt{2}G_F n_e E\) is the MSW matter potential which depends on the Fermi coupling constant \((G_F)\), the number density of the electrons \((n_e)\) and the energy of the neutrinos \((E)\). The corresponding probabilities for antineutrinos can be written by replacing \(A\) by \(-A\) in the above probability expressions.

From the approximate probability expressions above, it can be seen that, for a non–zero \(\theta_{13}\), the values of \(P_{e\mu}^m\) and \(P_{\mu\mu}^m\) would be different for the normal hierarchy \((\Delta m_{31}^2 > 0)\) and the inverted hierarchy \((\Delta m_{31}^2 < 0)\). Also, the probability is different for neutrinos and antineutrinos. Fig. 7.1 illustrates this difference in the two probabilities, for a fixed \(L = 7000\) km, and in the energy range \((0.5 - 20)\) GeV. Note that, the probability values plotted in this figure have been generated numerically, using the 24–layer PREM [130] profile of the Earth matter density and without neglecting the value of \(\Delta m_{21}^2\). The set of oscillation parameters used to generate this figure has been mentioned in the figure caption. This distinction, through the detected event rates in ICAL, can be used to determine the true hierarchy. Since the ratio of atmospheric \(\nu_\mu\) to \(\nu_e\) is nearly 2 in the sub–GeV energy range and the ratio increases at
7.1. THE EARTH MATTER EFFECT ON NEUTRINO OSCILLATION PROBABILITIES

Figure 7.1: (a) The probabilities $P_{\mu e}$ and (b) $\bar{P}_{\mu e}$, (c) $P_{\mu \mu}$ and (d) $\bar{P}_{\mu \mu}$, as a function of the neutrino (antineutrino) energy in the range $0.5 - 20$ GeV, at fixed $L = 7000$ km. Note that, the 24 layer PREM [130] profile of the Earth matter density has been used in the calculation of these probability values. The oscillation parameters used are, $\sin^2 2\theta_{13} = 0.1$, $\sin^2 2\theta_{23} = 0.5$, $\sin^2 2\theta_{12} = 0.84$, $\Delta m^2_{21} = 7.5 \times 10^{-5}$ eV$^2$, $|\Delta m^2_{32}| = 2.4 \times 10^{-3}$ eV$^2$, $\delta_{CP} = 0$ deg.

higher energies, it is clear from Fig. 7.1 that the events in the detector would dominantly come from the survival channel of $\nu_\mu$s.

The recent discovery of a non–zero $\theta_{13}$ at the reactor $\bar{\nu}_e$ disappearance experiments [99, 100, 101, 102, 103, 104] and at the accelerator $\nu_e$/\bar{$\nu}_e$ appearance experiments [108, 105, 106, 107], has done been a very good news for the experiments looking for the detection of the true mass hierarchy. The moderately large value of $\theta_{13}$ opens the gateway to probe the sub–leading three–flavor effects in current and future neutrino oscillation experiments. The mass hierarchy (MH), can be probed through the measurement of the matter effects on neutrinos as they pass through the
Earth over long distances \([109, 110, 111, 112, 113]\). The differences in the oscillations probabilities of the neutrino and antineutrino, arising due to the matter effects, are crucial to determine the MH \([114, 115]\) and have been illustrated in the following paragraph.

The baseline \(L\) in the case of the atmospheric neutrinos traveling through earth (i.e., the neutrinos reaching the ICAL detector from below) may vary from a few km to approximately 13,000 km depending on its incident direction on the detector. The probabilities, being functions of \(L\) and \(E_\nu\), would thus vary in the \(E_\nu – \theta_\nu\) plane. The parameter

\[
\Delta P_{\mu\mu} = P^{\text{NH}}_{\mu\mu} - P^{\text{IH}}_{\mu\mu},
\]

would provide information on the potential of determining the mass hierarchy through the Earth matter effects. Fig. 7.2 shows the variation of \(\Delta P_{\mu\mu}\) in the \(E_\nu – \cos \theta_\nu\) plane for the neutrinos as well as the antineutrinos. It can be observed from the figure that the magnitude and sign of \(\Delta P_{\mu\mu}\) fluctuates, which indicate the regions in the \(E_\nu – \cos \theta_\nu\) plane which would contribute significantly to the mass hierarchy determination. A detector with good energy and zenith angle resolution would be
able to explore the regions with strong matter effects. Also, since the matter effects in neutrinos and the antineutrinos are complementary to each other, separate observation of these two would improve the scope of mass hierarchy identification, particularly for the experiments which would expect relatively low statistics.

7.2 THE HADRON CONTRIBUTION IN CC $\nu_\mu$ EVENTS

As has been discussed in Chapter 2, the three main processes namely the quasi-elastic (QE), resonance scattering (RS) and deep inelastic scattering (DIS) contribute to the CC $\nu_\mu$ interactions in the ICAL detector. The QE process, with no hadrons produced in the final state, dominates in the sub–GeV range. With an increase in the incident neutrino energy to multi–GeV range, the Hadronic showers start appearing in resonance (RS) and deep-inelastic scattering (DIS) processes. The final hadronic state in the RS process mostly consists of a single pion, though multiple pions may contribute in a small fraction of events.

![Figure 7.3: The number of (a) neutrino and (b) antineutrino events in ICAL, produced with QE, RS and DIS processes as functions of neutrino energy, with an exposure of 500 kt-yr, in the absence of oscillations. The total number of events is also shown. Note that this figure has been plotted with the event information at the generator level, without the detector response. [20]](image)

In the DIS process, multiple hadrons are produced in the final state, which carry a large fraction of the incoming neutrino energy. Fig. 7.3 shows the relative contributions of these three processes to the total number of events in the absence of
oscillations, obtained using the event generator NUANCE, which have been used in the analysis presented in this chapter. In the neutrino energy range $5 - 10$ GeV, which is expected to have significant matter effects that will enhance the mass hierarchy identification, the contribution of DIS events is significant. Therefore, the information on hadrons produced in these DIS events is crucial.

![Figure 7.4](image)

Figure 7.4: The average inelasticities $\langle y \rangle$, for (a) neutrinos and (b) antineutrinos, in QE, RS, DIS processes as a function of neutrino or antineutrino energies. The black line indicates the $\langle y \rangle$ for all processes combined. [20]

![Figure 7.5](image)

Figure 7.5: The distribution of inelasticity $y$ in QE, RS and DIS events, for (a) neutrinos and (b) antineutrinos, with neutrino energies in the range $4 - 7$ GeV, with an exposure of 500 kt-yr, in the absence of oscillations. The distribution of $y$ for all events combined is shown by the black histogram. [20]

The fraction of the neutrino energy that is carried by hadrons in an event can be indicated by the inelasticity, defined as $y \equiv (E_\nu - E_\mu)/E_\nu = E'_{\text{had}}/E_\nu$. In Fig. 7.4 the average inelasticities $\langle y \rangle$ in the three interaction processes are shown, separately for the neutrinos and the antineutrinos, as a function of the incident energy of the
neutrino/antineutrino. The significant $\langle y \rangle$ of the DIS events imply that a large fraction of the energy of the incoming neutrino goes to the hadrons in the energy range of interest for MH determination. Though the average inelasticity in this energy range does not fluctuate much, the inelasticities in individual events have a wide distribution as shown in Fig. 7.5 for a sample $E_\nu$ range (4 – 7 GeV). This implies the importance to treat the $y$ values in individual events separately. Therefore, in this analysis the energies of hadrons and muons obtained in each event are treated separately, so that the correlation between them is not lost. A similar analysis has been done in [131], which shows significant enhancement. The procedure of the numerical analysis has been discussed in the next section.

7.3 THE ANALYSIS PROCEDURE

In this work, the oscillation analysis is carried out with the muon energy ($E_\mu$), muon direction ($\cos \theta_\mu$), and the hadron energy ($E'_\text{had} = E_\nu - E_\mu$), in a neutrino interaction event as separate observables. Only the CC interaction events are used. It is assumed that the reconstruction of the hadrons is 100% in an event where the muon is reconstructed. Thus the reconstruction and charge identification efficiency of the neutrino event are determined by the muon reconstruction. Also the muon and hadron hits are assumed to be separable with 100% efficiency. The lookup tables for the muon and hadron responses of the ICAL as given by the INO collaboration has been used. The background hits coming from other sources such as the NC events, CC $\nu_e$ events, cosmic muons, and the noise, have not been taken into account, and assumed to be of marginal effect. Note that, at a magnetized iron neutrino detector (MIND) which is similar to ICAL, these background can be reduced to the level of about a per cent by using the cuts on track quality and kinematics [132].

The analysis procedure consists of various steps, like, the neutrino event generation, inclusion of the oscillation effects, incorporation of the detector response and the statistical analysis. These steps are described in the following subsections.
CHAPTER 7. ENHANCING THE ICAL PHYSICS REACH WITH HADRONS

7.3.1 EVENT GENERATION AND THE INCLUSION OF OSCILLATIONS

Using the NUANCE event generator, $\nu_\mu$ and $\bar{\nu}_\mu$ interaction events are generated, with the ICAL detector specifications given as input. The atmospheric neutrino fluxes were provided by HONDA et al [133] for the Superkamiokande location. The differential cross sections for all the possible processes for both the charged-current (CC) and the neutral-current interactions, for all the nuclear constituents of the materials of ICAL is incorporated in NUANCE. The event rates for all scattering processes and possible target nuclei are calculated by multiplying the neutrino fluxes with the interaction cross sections. Event kinematics are generated based on the differential cross-sections. The total number of $\nu_\mu$ events comes from the channels $\nu_\mu \to \nu_\mu$, and $\nu_e \to \nu_\mu$. For each event, the NUANCE output consists of the 4-momentum of the initial, intermediate and final state particles.

To reduce the Monte Carlo (MC) fluctuations in the event sample, interactions for an exposure of 1000 year $\times$ 50 kt are generated and scaled down to the various required exposures. Generation of such a large event-set for each set of oscillation parameters is very much time consuming and would have been practically impossible within a reasonable time frame. Therefore, we have generated one set of events for no oscillations and the oscillation effects are imposed by applying a re-weighting algorithm. In order to calculate the oscillation probability, the path traveled between the production point and the detector, for a given $\cos \theta_\nu$, is given by

$$L = \sqrt{(R + L_0)^2 - (R \sin \theta_\nu)^2} - R \cos \theta_\nu,$$

(7.6)

where $R$ is the radius of the Earth (6378 km) and $L_0$ is the average height of the atmospheric neutrino production ($\sim 15$ km). The re-weighting is then done by generating a random number between 0 – 1 for each event and comparing it to the relevant oscillation probability [18].
7.3.2 INCORPORATION OF THE DETECTOR RESPONSE

The detector response (reconstruction and charge identification efficiencies and resolutions) are then incorporated upon the raw binned data one by one. First, the $\mu^-$ events ($N_{\mu^-}$) after applying the reconstruction efficiency in a given ($E'_{\text{had}}$, $E_{\mu}$, $\cos \theta_{\mu}$) bin is obtained by multiplying the true number of events with the corresponding reconstruction efficiency

$$N_{\mu^-}(E_{\mu}, \cos \theta_{\mu}, E'_{\text{had}}) = \epsilon_R(\mu^-) \times N_{\mu^-}^{\text{true}}(E_{\mu}, \cos \theta_{\mu}, E'_{\text{had}}).$$  \hspace{1cm} (7.7)$$

Then the charge identification efficiency is applied as in the following.

$$N_{\mu^-}^{\text{CID}}(E_{\mu}, \cos \theta_{\mu}, E'_{\text{had}}) = \epsilon_{\text{CID}}(\mu^-) \times N_{\mu^-}^{\text{true}}(E_{\mu}, \cos \theta_{\mu}, E'_{\text{had}}) + (1 - \epsilon_{\text{CID}}^+) \times N_{\mu^+}^{\text{true}}(E_{\mu}, \cos \theta_{\mu}, E'_{\text{had}}),$$  \hspace{1cm} (7.8)

where $\epsilon_{\text{CID}}^-$ and $\epsilon_{\text{CID}}^+$ are the CID efficiencies for $\mu^-$ and $\mu^+$ respectively, and are functions of $E_{\mu}$ and $\cos \theta_{\mu}$.

Finally the muon energy and direction resolutions and the hadron energy resolution are applied as

$$\langle N_{\mu^-}^{D}(E_{\mu}, \cos \theta_{\mu}, E'_{\text{had}}) \rangle_{ijk} = \sum_l \sum_m \sum_n N_{\mu^-}^{\text{CID}}(E_{\mu}, \cos \theta_{\mu}, E'_{\text{had}}) \times K_i^{l}(E_{\mu}) \times M_j^{m}(\cos \theta_{\mu}) \times N_k^{n}(E_{\text{had}}), \hspace{1cm} (7.9)$$

where $\langle N_{\mu^-}^{D}(E_{\mu}, \cos \theta_{\mu}, E'_{\text{had}}) \rangle_{ijk}$ denotes the number of muon events in the $i^{th}$ $E_{\mu}$ – bin, the $j^{th}$ $\cos \theta_{\mu}$ – bin and $k^{th}$ $E'_{\text{had}}$ – bin, after applying the energy and angle resolutions. Here $E_{\mu}$, $\cos \theta_{\mu}$ and $E'_{\text{had}}$ are the measured muon energy, muon zenith angle and hadron energy respectively. The summation is over the true $E_{\mu}$ bin $l$, the true $\cos \theta_{\mu}$ bin $m$ and true $E'_{\text{had}}$ bin $n$, with $E_{\mu}^{l}$, $\cos \theta_{\mu}^{m}$ and $E_{\text{had}}^{n}$ being the respective central values of the $i^{th}$ true $E_{\mu}$ bin, $m^{th}$ true $\cos \theta_{\mu}$ bin and $n^{th}$ true $E_{\text{had}}$ bin. The quantities $K_i^{l}$, $M_j^{m}$ and $N_k^{n}$ are the integrals of the detector resolution functions over the bins of the
measured values of $E_{\mu}$, $\cos \theta$ and $E'_{\text{had}}$. They are calculated as

$$K^i_l(E^i_{\mu}) = \int_{E^i_{\mu} - \Delta E_{\mu}}^{E^i_{\mu} + \Delta E_{\mu}} dE_{\mu} \frac{1}{\sqrt{2\pi} \sigma E^i_{\mu}} \exp \left( -\frac{(E^m_{\mu} - E)^2}{2 \sigma^2 E^i_{\mu}} \right),$$  \hspace{1cm} (7.10)

and

$$M^m_j(\cos \theta^m_{\mu}) = \int_{\cos \theta^m_{\mu} - \Delta \cos \theta_{\mu}}^{\cos \theta^m_{\mu} + \Delta \cos \theta_{\mu}} d \cos \theta_{\mu} \frac{1}{\sqrt{2\pi} \sigma \cos \theta^m_{\mu}} \exp \left( -\frac{(\cos \theta^m_{\mu} - \cos \theta)^2}{2 \sigma^2 \cos \theta^m_{\mu}} \right),$$  \hspace{1cm} (7.11)

$$N^m_k(E^m_{\text{had}}) = \int_{E^m_{\text{had}} - \Delta E_{\text{had}}^m}^{E^m_{\text{had}} + \Delta E_{\text{had}}^m} dE_{\text{had}} P_{\text{Vavilov}}(P_0, P_1, P_2, P_3),$$  \hspace{1cm} (7.12)

where $\sigma E^i_{\mu}$ and $\sigma \cos \theta^m_{\mu}$ are the resolutions of muon energy and zenith angle, respectively, in these bins. The $P_{\text{Vavilov}}$ is the modified Vavilov probability distribution function, as defined in section 4.2.1, with the $P_i$ ($i = 0, 1, 2, 3$) being the parameters which describe the reconstructed $E'_{\text{had}}$ distributions. The integrations have been performed between the lower and upper boundaries of the measured values of the observables.

### 7.3.3 THE $\chi^2$ ANALYSIS

The Poissonian definition of $\chi^2$ for $\mu^-$ events as given below is used

$$\chi^2 = \min \sum_{i=1}^{N_{E_{\mu}}'} \sum_{j=1}^{N_{\cos \theta_{\mu}}} \sum_{k=1}^{N_{E'_{\text{had}}}} \left[ 2(N_{ij,k}^\text{theory} - N_{ij,k}^\text{data}) - 2N_{ij,k}^\text{data} \ln \left( \frac{N_{ij,k}^\text{theory}}{N_{ij,k}^\text{data}} \right) \right] + \sum_{l=1}^{5} \xi_l^2 ,$$  \hspace{1cm} (7.13)

where

$$N_{ij,k}^\text{theory} = N_{ij,k}^0 \left( 1 + \sum_{l=1}^{5} a_{ij,k}^l \xi_l \right).$$  \hspace{1cm} (7.14)

Here $N_{ij,k}^\text{theory}$ and $N_{ij,k}^\text{data}$ are the expected and observed number of $\mu^-$ events in a given $(E_{\mu}, \cos \theta_{\mu}, E'_{\text{had}})$ bin. $N_{ij,k}^0$ are the number of events without systematic errors. Here $N_{E_{\mu}}, N_{\cos \theta_{\mu}},$ and $N_{E'_{\text{had}}}$ are the number of bins in $E_{\mu}, \cos \theta_{\mu}$ and $E'_{\text{had}}$, respectively. To simulate $N_{ij,k}^\text{data}$, the oscillation parameters as given in Table 7.1 are used as the true
values. These are benchmark values used in our analysis, and are consistent with those allowed by the global fit. The effective mass-squared difference is related to the $\Delta m_{31}^2$ and $\Delta m_{21}^2$ mass-squared differences through the expression [134, 135]

$$\Delta m_{\text{eff}}^2 = \Delta m_{31}^2 - \Delta m_{21}^2 (\cos^2 \theta_{12} - \cos \Delta_{\text{cp}} \sin \theta_{13} \sin 2\theta_{12} \tan \theta_{23}) \, .$$  \hspace{1cm} (7.15)$$

The following five systematic errors ($\xi_i$) are included in the analysis using the method of pulls: (i) Flux normalization error (20%), (ii) cross-section error (10%), (iii) tilt error (5%), (iv) zenith angle error (5%), and (v) overall systematics (5%). Here, $\pi_{ijk}^l$ is the change in the number of events in the $(ijk)^{th}$ bin caused by varying the value of $l$ th pull variable $\xi_i$ by $\sigma_l$ [26].

The $\chi^2$ for $\mu^+$ events is also obtained by using an identical procedure. The total $\chi^2$ is obtained by adding the individual contributions from $\mu^-$ and $\mu^+$ events. A prior of 8% (at 1\sigma) on $\sin^2 2\theta_{13}$ is added, since this quantity is currently known to this accuracy. No prior is added on $\theta_{23}$ or $\Delta m_{32}^2$ since these parameters will directly be measured at the ICAL detector. Thus

$$\chi^2_{\text{ICAL}} = \chi^2_1 + \chi^2_2 + \chi^2_{\text{prior}} \, ,$$ \hspace{1cm} (7.16)$$

where

$$\chi^2_{\text{prior}} \equiv \left( \frac{\sin^2 2\theta_{13} - \sin^2 2\theta_{13}(\text{true})}{\sigma(\sin^2 2\theta_{13})} \right)^2 \, ,$$ \hspace{1cm} (7.17)$$

and

$$\sigma(\sin^2 2\theta_{13}) = 0.08 \times \sin^2 2\theta_{13}(\text{true}) \, .$$ \hspace{1cm} (7.18)$$

In the minimization procedure, $\chi^2_{\text{ICAL}}$ is first minimized with respect to the pull variables $\xi_i$, and then marginalized over the ranges of oscillation parameters $\sin^2 \theta_{23}$, $\Delta m_{\text{eff}}^2$ and $\sin^2 2\theta_{13}$ as given in Table 7.1, wherever appropriate. We do not marginalize over $\delta_{\text{CP}}$, $\Delta m_{21}^2$ and $\theta_{12}$ since they have negligible effects on the relevant oscillation probabilities at ICAL [136]. The best-fit values of $\Delta m_{21}^2$ and $\theta_{12}$ from the global
fit references \[23, 24, 25\] are used, while \(\delta_{CP}\) is taken to be zero.

Table 7.1: Benchmark oscillation parameters used in this analysis. The true values of the oscillation parameters, are used to simulate the observed data set. The range, corresponding to the 3\(\sigma\) allowed values of the parameter in the global fit \[23, 24, 25\], over which the parameter values are varied while minimizing the \(\chi^2\) are also listed. Note that, while performing the analysis for precision measurements in Sec. 7.5, \(\Delta m_{eff}^2\) or \(\sin^2 \theta_{23}\) are not marginalized, and \(|\Delta m_{32}^2(\text{true})|\) is taken as \(2.4 \times 10^{-3} \text{ eV}^2\).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>True value</th>
<th>Marginalization range</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sin^2 2\theta_{13})</td>
<td>0.09, 0.1, 0.11</td>
<td>[0.07, 0.11]</td>
</tr>
<tr>
<td>(\sin^2 \theta_{23})</td>
<td>0.4, 0.5, 0.6</td>
<td>[0.36, 0.66]</td>
</tr>
<tr>
<td>(\Delta m_{eff}^2/\text{eV}^2)</td>
<td>(\pm 2.4 \times 10^{-3})</td>
<td>([2.1, 2.6] \times 10^{-3} \text{ (NH)}) (-[2.6, 2.1] \times 10^{-3} \text{ (IH)})</td>
</tr>
<tr>
<td>(\sin^2 2\theta_{12})</td>
<td>0.84</td>
<td>Not marginalized</td>
</tr>
<tr>
<td>(\Delta m_{21}^2/\text{eV}^2)</td>
<td>(7.5 \times 10^{-5})</td>
<td>Not marginalized</td>
</tr>
<tr>
<td>(\Delta_{CP})</td>
<td>0°</td>
<td>Not marginalized</td>
</tr>
</tbody>
</table>

7.3.4 THE BINNING SCHEME IN \((E_{\mu} - \cos \theta_{\mu} - E'_{\text{had}})\) SPACE

For an exposure of 500 kt – yr in ICAL, about 6200 events with a \(\mu^-\) and 2800 events with a \(\mu^+\) are expected, after incorporating the reconstruction efficiencies and resolutions for muons and hadrons, in the absence of oscillations. With oscillations, these numbers would decrease further. For the analysis using the information on the muons only, i.e., in the \((E_{\mu} - \cos \theta_{\mu})\) space, the excellent energy and angular resolutions of muon in ICAL \[9, 19\] make it possible to use a fine binning scheme. For example, the analysis presented in \[18\], 20 uniform \(E_{\mu}\) bins in the range 1 to 11 GeV and 80 uniform \(\cos \theta_{\mu}\) bins in the range \([-1, +1]\) were used for each polarity of muon. However, in such a fine scheme, a number of bins are left without a significant statistics. Including \(E'_{\text{had}}\) as an additional observable for binning would increase the total number of bins further, which would reduce the statistical strength of each bin significantly. In order to avoid such a situation, a coarser binning scheme that is suitable for the three observables \(E_{\mu}, \cos \theta_{\mu},\) and \(E'_{\text{had}}\) has been used in this
7.3. THE ANALYSIS PROCEDURE

analysis. This scheme ensures that most of the bins have sufficient number of events, without affecting the results much.

An optimized binning scheme would depend on the parameters to be measured. In particular, it could be different for the mass hierarchy identification and precision measurements of atmospheric neutrino mixing parameters. For this analysis, the regions in the 3-dimensional parameter space \((E_\mu - \cos \theta_\mu - E'_\text{had})\) that are sensitive to the mass hierarchy has been identified, and finer bins of the three observables in those regions are used. These regions roughly span in \(E_\mu = 4\) to \(7\) GeV, \(\cos \theta_\mu = -1\) to \(-0.4\), and \(E'_\text{had} = 0\) to \(4\) GeV. In the rest of the observable space, a coarser binning has been used. The atmospheric neutrino flux follows a steep power law, resulting in a smaller number of events at higher muon and hadron energies. Therefore, in general, finer bins at low energies and wider bins at higher energies, for both muons and hadrons have been considered to ensure sufficient statistics in each bin. This is also consistent with larger uncertainties in energy measurement at higher energies.

The binning scheme used is given in Table 7.2. For each polarity of muons, 10 bins for \(E_\mu\), 21 bins for \(\cos \theta_\mu\), and 4 bins for \(E'_\text{had}\) are used, resulting into a total of \((4 \times 10 \times 21) = 840\) bins per polarity.

Table 7.2: The binning scheme for the reconstructed observables \(E_\mu\), \(\cos \theta_\mu\), and \(E'_\text{had}\) for each muon polarity.

<table>
<thead>
<tr>
<th>Observable</th>
<th>Range</th>
<th>Bin width</th>
<th>Total bins</th>
</tr>
</thead>
<tbody>
<tr>
<td>(E_\mu) (GeV)</td>
<td>([1, 4])</td>
<td>0.5</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>([4, 7])</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>([7, 11])</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>(\cos \theta_\mu)</td>
<td>([-1.0, -0.4])</td>
<td>0.05</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>([-0.4, 0.0])</td>
<td>0.1</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>([0.0, 1.0])</td>
<td>0.2</td>
<td>5</td>
</tr>
<tr>
<td>(E'_\text{had}) (GeV)</td>
<td>([0, 2])</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>([2, 4])</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>([4, 15])</td>
<td>11</td>
<td>1</td>
</tr>
</tbody>
</table>


7.4 THE NEUTRINO MASS HIERARCHY IDENTIFICATION

In this section, the sensitivity of the ICAL detector in identifying the neutrino mass hierarchy (MH), as obtained from the analysis with the muons and the hadrons are presented. The statistical significance to rule out the wrong hierarchy is quantified by

\[
\Delta \chi^2_{\text{ICAL-MH}} = \chi^2_{\text{ICAL}}(\text{false MH}) - \chi^2_{\text{ICAL}}(\text{true MH}).
\]  

(7.19)

Here \( \chi^2_{\text{ICAL}} \) (true MH) and \( \chi^2_{\text{ICAL}} \) (false MH) are obtained by performing a fit to the observed data assuming true and false mass hierarchy, respectively. Here, with the statistical fluctuations suppressed, \( \chi^2_{\text{ICAL}} \) (true MH) \( \approx 0 \). The statistical significance is also represented in terms of \( n\sigma \), where \( n \equiv \sqrt{\chi^2_{\text{ICAL-MH}}} \). This represents the median sensitivity in the frequentist approach of hypothesis testing \([137]\).

The inclusion of the hadron energy information in the analysis enhance the results significantly. In section 7.4.1, the extent to which the capability of the ICAL is improved with hadrons has been explored by studying the \( \Delta \chi^2 \), in the individual bins. The MH sensitivity results are then shown in section 7.4.2.

7.4.1 THE ENHANCEMENT IN THE BIN–BY–BIN \( \chi^2 \) WITH HADRONS

In Fig. 7.6 the distribution of \( \Delta \chi^2_{\pm} \equiv \chi^2_{\pm} \) (IH) \( - \chi^2_{\pm} \) (NH) in the reconstructed \( E_\mu - \cos \theta_\mu \) plane are shown. The left panels show the results for the analysis that does not use the hadron energy information. The right panels show the analysis where events are further divided into four sub-bins of \( E'_{\text{had}} \) and for each \( E_\mu - \cos \theta_\mu \) bin, the \( \Delta \chi^2_{\pm} \) has been summed over the hadron energy bins. Note that, the constant contribution in \( \chi^2 \) coming from the term involving the five pull parameters \( \xi_l^2 \) in Eq. (7.13) has not been considered. Also, the marginalization over the oscillation parameters in the fit has not been performed here. The final MH results shown in the coming section, the
7.4. THE NEUTRINO MASS HIERARCHY IDENTIFICATION

full pull contributions and marginalizations have been taken care of.

Figure 7.6: (Top) The distribution of $\Delta \chi^2$ per unit area, in the $(E_\mu - \cos \theta_\mu)$ plane, (a) without and (b) with hadron information. (Bottom) $\Delta \chi^2$ per unit area, (c) without and (d) with hadron information. Here, the NH is assumed to be the true hierarchy, and 500 kt-yr of ICAL exposure is used. [20]

The upper (lower) panels in Fig. 7.6 present the distribution of $\Delta \chi^2$ ($\Delta \chi^2_\pm$) arising from $\mu^-$ ($\mu^+$) events. It can be observed that with the addition of the hadron energy information, the area in the $E_\mu - \cos \theta_\mu$ plane that contributes significantly to $\Delta \chi^2_\pm$ increases, which in turn improves the net $\Delta \chi^2_\pm$. This increase in $\chi^2_\pm$ is contributed by not only the information contained in the hadron energy measurement, but also the correlation between the hadron energy and muon momentum.

Another important fact to be noted is that the increase in the sensitivity is not simply due to the events with low $E'_{\text{had}}$, where the muon energy $E_\mu$ could be expected to closely match the original neutrino energy $E_\nu$. This may be illustrated by Table 7.3, where, the total $\Delta \chi^2$ contributions from $\mu^-$ ($\mu^+$) events for the four
individual hadron bins are shown.

Table 7.3: The contributions of various $E'_\text{had}$-bins to the net $\Delta \chi^2$. The events in the last row without $E'_\text{had}$ information have true hadron energies up to 100 GeV.

<table>
<thead>
<tr>
<th>$E'_\text{had}$ (GeV)</th>
<th>events</th>
<th>$\Delta \chi^2$</th>
<th>$\Delta \chi^2$/events</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 1</td>
<td>3995</td>
<td>5.8</td>
<td>0.0014</td>
</tr>
<tr>
<td>1 - 2</td>
<td>1152</td>
<td>1.9</td>
<td>0.0017</td>
</tr>
<tr>
<td>2 - 4</td>
<td>742</td>
<td>1.7</td>
<td>0.0023</td>
</tr>
<tr>
<td>4 - 15</td>
<td>677</td>
<td>1.2</td>
<td>0.0018</td>
</tr>
<tr>
<td>0 - 15 (with $E'_\text{had}$ information)</td>
<td>6566</td>
<td>10.7</td>
<td>0.0016</td>
</tr>
<tr>
<td>without $E'_\text{had}$ information</td>
<td>6775</td>
<td>6.3</td>
<td>0.0009</td>
</tr>
</tbody>
</table>

As can be seen from Table 7.3, while the $\Delta \chi^2$ contribution from the lowest $E'_\text{had}$ bin is more than half the total $\Delta \chi^2$, this bin also has a large statistics. Indeed, the normalized $\Delta \chi^2$ per event is slightly higher for larger $E'_\text{had}$ bins. This indicates that the hadron energy information from even the higher $E'_\text{had}$ bins would be significant for identifying the true hierarchy.

### 7.4.2 THE MASS HIERARCHY SENSITIVITY OF ICAL

The sensitivity of 50 kt ICAL for identifying the neutrino mass hierarchy are shown in Fig. 7.7 as a function of the run–time of the experiment. It can be seen that after including the hadron energy information, 10 years of running can rule out the wrong hierarchy with $\chi^2_{ICAL-MH} \approx 9.7$ (for true NH), and $\chi^2_{ICAL-MH} \approx 9.1$ (for true IH). Thus, the sensitivity to discard the wrong hierarchy is about $3\sigma$ for either hierarchy.

This figure also shows that for the same run-time the value of $\chi^2_{ICAL-MH}$ increases by about 40% when the correlated hadron energy information is added. Note that for comparison purpose, the same binning scheme in $(E_\mu, \cos \theta_\mu)$ has been used for both analyses, as shown in Table 7.2. As compared to the finer binning scheme in the muon–only analysis, as presented in [18], the improvement is about 35%.
Figure 7.7: The $\chi^2$ as a function of the run-time assuming (a) NH and (b) IH as true hierarchy. The results obtained using hadron energy information is compared to the results from the analysis that is done without hadron information. Here, $\sin^2 2\theta_{13} \text{ (true)} = 0.1$ and $\sin^2 2\theta_{23} \text{ (true)} = 0.5$. [20]

Figure 7.8: The variation of $\chi^2$ for different true values of $\sin^2 \theta_{23}$ assuming (a) NH and (b) IH as the true hierarchy. The value of $\sin^2 2\theta_{13} \text{ (true)}$ is taken to be 0.1.[20]

Fig. 7.8 and Fig. 7.9 show the variation of the MH identification sensitivity for three benchmark values of $\sin^2 \theta_{23}$ and $\sin^2 2\theta_{13}$, respectively, in the allowed ranges of these parameters. The higher values of these two parameters increase the matter effects in neutrino oscillations and thus, better hierarchy sensitivity is obtained. This is expected since the leading matter effect terms in the probability expressions of $P_{\mu\mu}$ and $P_{e\mu}$ are proportional to these parameters as shown in Eq. 7.1 and Eq. 7.2.

Depending on the range of the true values of these parameters and the true choice of MH, the ICAL detector can identify the MH with a $\chi^2_{\text{ICAL-MH}}$ in the range of 7 – 12 for 10 years of running of the 50 kt ICAL.
Figure 7.9: The variation of $\chi^2_{ICAL-MH}$ for different true values of $\sin^2\theta_{13}$ assuming (a) NH and (b) IH as the true hierarchy. The value of $\sin^2\theta_{23}(\text{true})$ is 0.5. [20]

Figure 7.10: The variation of $\chi^2_{ICAL-MH}$ with respect to $\delta_{CP}$, assuming (a) NH and (b) IH as the true hierarchy, for an exposure of 500 kt–year at ICAL. Here, $\sin^2\theta_{23}(\text{true})$ is taken to be 0.5, and $\sin^2\theta_{13}(\text{true})$ is taken to be 0.1.

Fig. 7.10 show the variation of $\chi^2_{ICAL-MH}$ with respect to $\delta_{CP}$. It can be seen that, the projected ICAL atmospheric data is not sensitive to $\delta_{CP}$. This may be explained by the fact that in the full expression of $P_{\mu\mu}$, the $\delta_{CP}$ dependent term is suppressed by a factor of $\alpha \equiv \Delta m^2_{21}/\Delta m^2_{31}$ [136].

7.5 PRECISION MEASUREMENT OF THE ATMOSPHERIC PARAMETERS

The precision in the measurements of a parameter $\lambda$ (where $\lambda$ may be either $\sin^2\theta_{23}$
or \( |\Delta m_{32}^2| \) can be quantified in terms of the following,

\[
\Delta \chi^2_{\text{ICAL-PM}}(\lambda) = \chi^2_{\text{ICAL-PM}}(\lambda) - \chi^2_0,
\]

(7.20)

where \( \chi_0^2 \) is the minimum value of \( \chi^2_{\text{ICAL-PM}} \) in its allowed parameter range. Here, with the statistical fluctuations suppressed, \( \chi^2_0 \approx 0 \). The significance may be denoted by \( n\sigma \) where

\[
n \equiv \sqrt{\Delta \chi^2_{\text{ICAL-PM}}}.
\]

In terms of these quantities, the relative precision achieved on the parameter \( \lambda \) at \( 1\sigma \) is given by [138]

\[
p(\lambda) = \frac{\lambda(\text{max}) - \lambda(\text{min})}{\lambda(\text{true})}.
\]

(7.21)

where \( \lambda(\text{max}) \) and \( \lambda(\text{min}) \) are the maximum and minimum allowed values of \( \lambda \) at \( 2\sigma \) respectively, and \( \lambda(\text{true}) \) is its true choice.

Figure 7.11: (a) The \( \Delta \chi^2_{\text{ICAL-PM}}(\sin^2 \theta_{23}) \) and (b) \( \Delta \chi^2_{\text{ICAL-PM}}(|\Delta m_{32}^2|) \), assuming NH as true hierarchy. The results obtained using hadron energy information is compared to the results from the analysis that is done without hadron information. [20]

In Fig. 7.11a and Fig. 7.11b the sensitivity of ICAL to the two parameters \( \sin^2 \theta_{23} \) and \( |\Delta m_{32}^2| \) are shown separately, where the other parameter has been marginalized over. The parameter \( \theta_{13} \) and the two possible choices of mass hierarchies have also been marginalized over. The figures show the results assuming NH to be the true hierarchy. It has been checked that, the results with true IH are almost identical. It may be observed from the figures that the inclusion of hadron energy information would make it possible to measure \( \sin^2 \theta_{23} \) to a relative \( 1\sigma \) precision of 12% and
\[ |\Delta m^2_{32}| \] to 2.9\%, for 10 years of running of the ICAL. The muon-only analysis with the same binning scheme in \( E_\mu \) and \( \cos \theta_\mu \), the same relative precisions are obtained to be 13.7\% and 5.4\%, respectively. Note that, the muon-only analysis with finer binning (20 \( E_\mu \) bins and 80 \( \cos \theta_\mu \) bins) would yield the relative precisions in \( \sin^2 \theta_{23} \) and \( |\Delta m^2_{32}| \) as, 13.5\% and 4.2\%, respectively.

The precision in \( \sin^2 \theta_{23} \) is governed mainly by the statistics available from the experiment. The addition of the hadron energy information does not change the statistics, and therefore makes only a small difference in the two analyses. However, the independent measurements of \( E_\mu \) and \( E_{\text{had}} \) corresponds to a better estimation of \( E_\nu \), which appears in the oscillation expression as \( \sin^2(\Delta m^2 L/E_\nu) \). A better measurement of \( E_\nu \) thus leads to a better measurement of \( \Delta m^2 \), resulting in the significant improvement in the precision on \( |\Delta m^2_{32}| \).

**Figure 7.12:** The \( \Delta \chi^2_{\text{ICAL-PM}} \) contours at 68\%, 90\%, and 99\% confidence levels (2 dof) in (a) \( \sin^2 \theta_{23} - |\Delta m^2_{32}| \) plane and (b) \( \sin^2 2\theta_{23} - |\Delta m^2_{32}| \) plane, after including the hadron energy information. Here, NH is assumed as the true hierarchy. The true choices of the parameters have been marked with a dot. [20]

Fig. 7.12a and Fig. 7.12b show the \( \Delta \chi^2_{\text{ICAL-PM}} \) contours at 68\%, 90\%, and 99\% confidence levels in the \( \sin^2 \theta_{23} - |\Delta m^2_{32}| \) plane and in the \( \sin^2 2\theta_{23} - |\Delta m^2_{32}| \) plane, respectively, with the inclusion of the hadron energy information. Here, the true value of \( \theta_{23} \) has been taken to be maximal, so the contours in the left panel are almost symmetric in \( \sin^2 \theta_{23} \). The comparison of the projected 90\% C.L. precision reach of ICAL (500 kt–yr exposure) in the \( \sin^2 \theta_{23} - |\Delta m^2_{32}| \) plane with other experiments is shown in
Fig. 7.13: The comparison of the projected 90% C.L. precision reach of ICAL (500 kt-yr exposure) in the $\sin^2 \theta_{23} - |\Delta m^2_{32}|$ plane with the results from SK [27], T2K [29] and MINOS [28] experiments. [20]

Using hadron energy information, the ICAL will be able to achieve a precision in $\sin^2 \theta_{23}$ which is comparable to the current precision for Super-Kamiokande [27] or T2K [29], and the $|\Delta m^2_{32}|$ precision comparable to the MINOS reach [28]. Note that, some of these experiments would have collected much more statistics by the time ICAL would have an exposure of 500 kt-yr. The ICAL will therefore not be competing with these experiments for the precision measurements of these mixing parameters, however the ICAL measurements will serve as complementary information for the global fit of world neutrino data. As compared to the atmospheric neutrino analysis at Super-Kamiokande, the ICAL precision on $|\Delta m^2_{32}|$ is far superior. This is a consequence of the better precision in the reconstruction of the muon momentum and direction at ICAL.

The 68%, 90%, and 99% C.L. contours in the $\sin^2 \theta_{23} - |\Delta m^2_{32}|$ plane, for two non-maximal choices of the mixing angle $\theta_{23}$ ($= 0.37, 0.63$) are presented in Fig. 7.14. It can be seen that the precisions obtained are similar, though the shapes of the contours are more complicated. For $\theta_{23}$ in the lower octant, the maximal mixing can be ruled out with 99% C.L. with 500 kt-yr of ICAL data. However, if $\theta_{23}$ is closer to the maximal mixing value, or in the higher octant, then the ICAL sensitivity to exclude maximal mixing would be much smaller. These contours also motivate the attempt to resolve the $\theta_{23}$ octant degeneracy, which is discussed in the next section.
Figure 7.14: The $\Delta \chi^2_{\text{ICAL-PM}}$ contours at 68%, 90%, and 99% confidence levels (2 dof) in $\sin^2 \theta_{23} - |\Delta m^2_{32}|$ plane, for (a) $\sin^2 \theta_{23}\text{(true)} = 0.37$ and (b) $\sin^2 \theta_{23}\text{(true)} = 0.63$, after including the hadron energy information. Here, NH is assumed to be the true hierarchy. The true choices of the parameters have been marked with a dot. [20]

7.6 OCTANT OF $\theta_{23}$

The Earth matter effects in the $P_{\mu\mu}$ channel may be used to resolve the octant ambiguity of $\theta_{23}$ [139]. In analogy with the mass hierarchy discovery sensitivity of ICAL, the statistical significance of the analysis to rule out the wrong octant is quantified by

$$\Delta \chi^2_{\text{ICAL-OS}} = \chi^2_{\text{ICAL}}(\text{false octant}) - \chi^2_{\text{ICAL}}(\text{true octant}).$$

(7.22)

Where $\chi^2_{\text{ICAL}}(\text{true octant})$ and $\chi^2_{\text{ICAL}}(\text{false octant})$ are obtained by performing a fit to the observed data assuming the true octant and wrong octant, respectively. Here with the statistical fluctuations suppressed, $\chi^2_{\text{ICAL}}$ (true octant) $\approx 0$. For each given value of $\theta_{23}\text{(true)}$, $\theta_{23}$ has been marginalized over all the allowed values in the opposite octant, including the maximal mixing value. $\Delta \chi^2_{\text{ICAL-OS}}$ has also been marginalized over the true choices of mass hierarchy. The statistical significance for ruling out the wrong octant is represented in terms of $n\sigma$, where $n \equiv \sqrt{\Delta \chi^2_{\text{ICAL-OS}}}$.

Figure 7.15 shows the sensitivity of ICAL to the identification of the $\theta_{23}$ octant, with and without including the hadron energy information. This figure indicates the possibility of a $2\sigma$ identification of the octant with the 500 kt–yr ICAL data alone for NH as the true hierarchy and the lower octant to be the true octant. In this case,
without using the hadron energy information one can get a $2\sigma$ identification only when $\sin^2 \theta_{23} \text{(true)} < 0.375$, which is almost close to the present $3\sigma$ bound. With the addition of hadron energy information, this task is possible as long as $\sin^2 \theta_{23} \text{(true)} < 0.395$. If the higher octant is the true one, or the true mass hierarchy is inverted, then the discrimination of $\theta_{23}$ octant with the ICAL data alone becomes rather difficult. In case of NH (IH), neutrino (antineutrino) events are mostly affected by the Earth’s matter effect and give crucial information about the octant of $\theta_{23}$. Since the statistical strength of atmospheric neutrino events is higher compared to antineutrino events, the octant sensitivity is better for NH compared to IH. These observations are not much sensitive to the true value of $\theta_{13}$. A variation of $\sin^2 2\theta_{13} \text{(true)}$ in the range $0.09 - 0.11$ changes the values of $\Delta \chi^2_{ICAL-OS}$ quite marginally. Clearly, the octant discrimination becomes more and more difficult as the true value of $\sin^2 \theta_{23}$ moves close to the maximal mixing. Combining of the atmospheric and long–baseline experiments is, however, an effective approach, in which the ICAL contribution would also be significant [140, 141, 142].

Figure 7.15: The $\Delta \chi^2_{ICAL-OS}$ for octant discovery potential as a function of true $\theta_{23}$ for (a) NH and (b) IH as true hierarchy, for the ICAL exposure of 500 kt–yr. The results obtained using hadron energy information is compared to the results from the analysis that is done without hadron information. [20]
7.7 REMARKS

In this chapter we have discussed the methodology and results of a statistical analysis which is used to assess the ICAL physics potentials with the inclusion of the hadron energy. Though ICAL is primarily optimized for muon detection, its capability of detecting hadrons and estimations their energy has been an additional advantage. The enhancement is not only due to the hadron energy, but also due to the correlation between the hadron energy and the muon momentum in an event. The analysis presented in this chapter uses hadron energy, muon energy and muon direction as separate observables in each event.

Significant improvements have been observed in the ICAL sensitivities, using this analysis. After including the $E_{\text{had}}'$ information, 10 years of running can rule out the wrong hierarchy with $\Delta \chi^2_{\text{ICAL-MH}} \approx 9.5$ (for true NH), and $\Delta \chi^2_{\text{ICAL-MH}} \approx 8.7$ (for true IH), which mark an improvement of about 40% over the muon-only analysis. It is observed that with the inclusion of $E_{\text{had}}'$ information, 500 kt–yr of ICAL exposure would be able to measure $\sin^2 \theta_{23}$ to a relative 1σ precision of 12% and $|\Delta m_{32}^2|$ to 2.9%. However, that the potential of distinguishing the $\theta_{23}$ octant with the ICAL data alone is rather weak. A 2σ identification of the octant is possible with the 500 kt–yr ICAL data alone only when, the true hierarchy is NH and the true octant is LO ($\sin^2 \theta_{23} \text{(true)} < 0.395$).

Note that, due to the present status of understanding of the ICAL detector response as obtained from the simulations, certain assumptions had to be made in the course of this analysis. For example, it was assumed that the muon track and the hadron shower can be separated neatly in all events. The background hits and noise are neglected, and are assumed that they do not affect the hadron response of the detector. This analysis procedure is expected to be the preferred one for the ICAL physics reach. It is therefore crucial to look into the effects due to various assumptions. Also, improvement in the reconstruction of the observables would provide
further enhancement in this study. Various attempts on improving the reconstruction algorithms are being taken up.

The muon track reconstruction depends on the time information of the hits, to reconstruct the vertex. This also affects the hadron shower reconstruction, where the vertex information is used. One possible way of improving the time information is to use a detector with a better time resolution. In order to probe this as well as other applications, the Multigap RPC (MRPC) detectors [30, 31] have been developed. These detectors, due to the presence of a number of sub-gaps, are much faster than the RPC detectors. In Chapter 8, the development and performance of six-gap MRPC detectors, which achieve a time resolution of about 60 ps, will be discussed.