Chapter 6

THE RECONSTRUCTION OF NEUTRINO ENERGY AND ANGLE

The reconstruction techniques and the resolutions of the muon momentum and its direction, and those of the hadron energy and the average direction of the hadron shower, have been discussed in Chapters 3, 4 and 5 respectively. These reconstructed parameters are then used to reconstruct the energy and direction of the incident neutrino in the CC interaction events.

To reconstruct CC $\nu_\mu$ events, events with a reconstructed muon momentum are considered. Note that this analysis is done with the events fully contained (FC) in the ICAL detector. It is assumed that, the muon track and the hadron shower can be separated with 100% efficiency. Applying the energy and momentum conservation, the energy and direction of the incident neutrino have been reconstructed. The efficiency of reconstruction of the neutrino events and their leptonic charge identification, the neutrino energy and direction resolution in ICAL are discussed and quantified in this chapter.

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This chapter is based on [12], which is in preparation.
6.1 THE RECONSTRUCTION EFFICIENCY

The reconstruction efficiency ($\epsilon^\nu_{\text{rec}}$) of the $\nu_\mu$ events is defined as

$$\epsilon^\nu_{\text{rec}} = \frac{N_{\text{rec}}}{N_{\text{total}}},$$

with error

$$\delta \epsilon^\nu_{\text{rec}} = \frac{\epsilon^\nu_{\text{rec}}(1 - \epsilon^\nu_{\text{rec}})}{N_{\text{total}}}.$$  \hspace{1cm} (6.1)

where $N_{\text{rec}}$ is the number of events with reconstructed muon momentum irrespective of the charge, and $N_{\text{total}}$ is the total events. Here the criteria on the muon track fitting has been taken as $\chi^2/ndf < 10$.

The neutrino event reconstruction efficiency, as a function of the input neutrino energy ($E_\nu$) and at certain $\cos \theta_\nu$ bins, is shown in Fig. 6.2. It ranges between 40% to 90% in various ($E_\nu, \cos \theta_\nu$) bins.

6.2 THE CHARGE IDENTIFICATION EFFICIENCY

The neutrinos and antineutrinos are differentiated by the separation of the muon and antimuon tracks by virtue of their direction of bending in the magnetic field in the ICAL. The (leptonic) charge reconstruction efficiency ($\epsilon^\nu_{\text{cid}}$) of the CC neutrino
events is defined as
\[ \epsilon'^{\nu}_{\text{cid}} = \frac{N_{\text{tc}}}{N_{\text{rec}}}, \]
with error
\[ \delta \epsilon'^{\nu}_{\text{cid}} = \frac{\epsilon'^{\nu}_{\text{cid}}(1 - \epsilon'^{\nu}_{\text{cid}})}{N_{\text{rec}}}, \]
(6.2)

where \( N_{\text{rec}} \) is the number of events with reconstructed muon momentum irrespective of the charge, and \( N_{\text{tc}} \) is the number events with correct charge identification.

The charge identification efficiency of the neutrinos, as a function of the input neutrino energy \( (E_\nu) \) and at certain \( \cos \theta_\nu \) bins, is shown in Fig. 6.2. It varies between 65% to 90% in various \( (E_\nu, \cos \theta_\nu) \) bins.

Figure 6.2: The charge identification efficiency of the (a) CC \( \nu_\mu \) events, and, (b) CC \( \bar{\nu}_\mu \) events in ICAL, as a function of the neutrino energy for the \( | \cos \theta_\nu | \) bins, 1 – 0.8, 0.8 – 0.5, 0.5 – 0.2 and 0.2 – 0.

6.3 THE NEUTRINO ENERGY RESOLUTION

The muon energy \( (E_\mu) \) is reconstructed using a Kalman filter based algorithm, as discussed in section 3.2. The hadron energy parameterized by \( E'_\text{had} \) is estimated using the calibration of \( E'_\text{had} \) against hadron hits (as stated in section 4.5). The neutrino energy is reconstructed as
\[ E^{\text{rec}}_\nu = E^{\text{rec}}_\nu + E'_{\text{had}}, \]
(6.3)

Fig 6.3 shows a typical distribution of \( (E^{\text{rec}}_\nu - E^{\text{true}}_\nu) \). Those distributions at various \( (E_\nu, \cos \theta_\nu) \) bins are fitted to the Vavilov distribution function. The relative neutrino
energy resolution, defined to be $\sigma_{\text{Vavilov}}/E_\nu$, at those bins are shown in Fig. 6.4. The energy resolution is found to be flat in the $E_\nu$ range $1 \to 10$ GeV, in the range $(22 \to 26)\%$. The reason is, though both the muon and the hadron energy resolution show sharper variation with energy, the neutrinos in any of the bins are composed of muons and hadrons in a wider energy range.

![Graph showing energy resolution](image)

**Figure 6.3:** The distribution of $(E_\nu^{\text{rec}} - E_\nu^{\text{true}})$ for $E_\nu = 3.5 \to 4$ GeV, and $|\cos \theta_\nu| = [0.8, 1]$.

![Graph showing energy resolution](image)

**Figure 6.4:** The energy resolutions of (a) $\nu_\mu$ and (b) $\bar{\nu}_\mu$ in ICAL, as a function of the neutrino energy for the $|\cos \theta_\nu|$ bins, $1 \to 0.8$, $0.8 \to 0.5$, $0.5 \to 0.2$ and $0.2 \to 0$.

## 6.4 THE NEUTRINO DIRECTION RESOLUTION

The direction of the incident neutrino can be calculated via the four–momentum conservation of the neutrino interaction. The reconstructed parameters (energy and
momentum, direction) of the muon and the hadrons are used here. The resolution obtained through this method might be slightly coarser as it includes the uncertainties in the measurements of the parameters of both the muon and the hadrons. An alternative approach is the indirect estimation of the neutrino direction from the scatter angle between the incident neutrino and the muon, which does not include the the hadron reconstruction. Both of the methods are discussed in the following subsections.

6.4.1 THE $\theta_\nu$ RESOLUTION USING HADRONS

The $(X, Y, Z)$ projections of the neutrino momentum, in terms of the muon and hadrons, are given by

\[ P_{\nu X} = P_\mu \cdot \sin \theta_\mu \cdot \cos \phi_\mu + P_{\text{had}} \cdot \sin \theta_{\text{had}} \cdot \cos \phi_{\text{had}}, \]  
\[ P_{\nu Y} = P_\mu \cdot \sin \theta_\mu \cdot \sin \phi_\mu + P_{\text{had}} \cdot \sin \theta_{\text{had}} \cdot \sin \phi_{\text{had}}, \]  
\[ P_{\nu Z} = P_\mu \cdot \cos \theta_\mu + P_{\text{had}} \cdot \cos \theta_{\text{had}}. \] 

The neutrino zenith angle $\theta_\nu$ can then be calculated as

\[ \tan \theta_\nu = \frac{\sqrt{P_{\nu X}^2 + P_{\nu Y}^2}}{P_{\nu Z}}. \] 

The reconstructed $P_\mu$, $\theta_\mu$, and $\phi_\mu$ are obtained from the Kalman track fit algorithm. $P_{\text{had}}$ is reconstructed using the calibration of $P_{\text{had}}^{\text{true}}$ against the hadron hit number, where $P_{\text{had}}^{\text{true}}$ is the sum of the momentum of all the hadrons in an event. The angles $\theta_{\text{had}}$ and $\phi_{\text{had}}$ are reconstructed from the hadron hit information using orientation matrix technique.

Fig. 6.5 shows a sample distribution of the difference between $\theta_{\nu}^{\text{true}}$ and $\theta_{\nu}^{\text{rec}}$. These distributions are fitted to the Breit–Wigner distribution function. The func-
The functional form of the Breit–Wigner distribution is

\[ L(x) = \frac{1}{\pi} \frac{\Gamma}{(x - x_0)^2 + \Gamma^2/4}. \]  

(6.6)

The Lorentzian fit is parameterized with the mean \((x_0)\) and the FWHM \((\Gamma)\). The HWHM \((\Gamma/2)\) is defined as the direction resolution which is shown in Fig. 6.6. It is in the range \(19^\circ - 7.5^\circ\) depending on the neutrino energy and \(\cos \theta_\nu\).

Figure 6.5: The distribution of \((\theta_{\nu_{\text{true}}} - \theta_{\nu_{\text{rec}}})\) for \(E_\nu = 4 - 5 \text{ GeV}\), and \(|\cos \theta_\nu| = [0.8, 1]\).

Figure 6.6: The zenith angle resolutions of (a) \(\nu_\mu\), and (b) \(\bar{\nu}_\mu\).
6.4.2 THE SCATTER ANGLE BETWEEN NEUTRINO AND MUON

The angle between the direction of the incident neutrino and the direction of the secondary muon, $\alpha_{\nu}$, also gives a naive estimation of the neutrino direction, when the muon direction is reconstructed. Since the muon direction reconstruction is quite precise in ICAL, this scatter angle would also provide information on the direction of the incident neutrino. However, note that, $\alpha_{\nu}$ would provide an indirect hint of the neutrino direction.

![Graphs showing $\alpha_{\nu}$ distributions](image)

Figure 6.7: The $\alpha_{\nu}$ distributions at (a) $E_{\nu} = 1-2$ GeV, and (b) $E_{\nu} = 4-5$ GeV.

Fig. 6.7 shows two sample $\alpha_{\nu}$ distributions at $E_{\nu} = 1-2$ GeV, and, $E_{\nu} = 4-5$ GeV. The mean $\alpha_{\nu}$, as a function of the neutrino energy is shown in Fig. 6.8. This information, combined with other reconstructed parameters, can be used to extract the true direction of the incident neutrino.

![Graph showing mean $\alpha_{\nu}$](image)

Figure 6.8: The distribution of $(\theta_{\text{true}} - \theta_{\text{rec}})$ for $E_{\nu} = 4-5$ GeV and $|\cos \theta_{\nu}| = [0.8, 1]$. 
6.5 REMARKS

In this chapter, the reconstructions of the neutrino energy and direction are discussed. The energy and direction of the incident neutrino have been estimated from the reconstructed momentum of the final state particles through the application of the 4–momentum conservation. The neutrino energy is reconstructed by adding up the reconstructed energies of the muon and the hadrons in each event. For the events fully confined in ICAL, the energy resolution of neutrinos in the ICAL is in the range \((22–26)\%\). The muon and hadron momentum information is used to calculate the \(X\), \(Y\) and \(Z\) projections of the neutrino momenta and eventually the neutrino direction. The resolution of the neutrino zenith angle at the ICAL is in the range \((19^\circ – 7.5^\circ)\). In an alternative approach, the neutrino direction has also been estimated from the average scatter angle between the incident neutrino and the reconstructed final state muon.

The coarse neutrino energy and direction resolution, mentioned in this chapter, are still preliminary and would not be able to produce an improved physics sensitivity. It is due to the fact that, the naive addition of the reconstructed parameters of the muons and the hadrons dilute the advantage of the precise reconstruction of the muon momentum. Instead of this the use of the muon and the hadron resolutions separately, boosts the ICAL sensitivity towards its physics goals. This will be discussed in Chapter 7.