Chapter 5

THE HADRON SHOWER DIRECTION RESOLUTION OF ICAL

The upward going neutrinos passing through the Earth’s core and experiencing significant matter effects and MSW resonances would be very crucial for the physics sensitivity of the ICAL. Thus it is required to be able to distinguish between the up-going and down-going neutrinos at ICAL. The incident neutrino direction may be retraced from the final state particles using four–momentum conservations, if the momenta of those final state particles are reconstructed. The muon momentum is reconstructed using a Kalman–Filter based algorithm. As has been shown in section 3.3.2, the muon direction can be reconstructed with a good precision (< 1°) [9]. In the case of the hadrons, since ICAL is not capable of distinguishing between individual hadrons on the basis of the hits recorded, it is not possible to reconstruct the direction of each hadron. However, an estimation of the average direction of the hadrons can be done using the shape of the hadron shower. Two methods namely the centroid technique and the orientation matrix technique have been taken up for this purpose. Both of the techniques use the position vectors of the shower hits with respect to the neutrino interaction vertex. The vertex can be reconstructed in the CC

\footnote{This chapter is based on [12], which is in preparation.}
events using the muon track reconstruction algorithm.

In addition, since the RS interaction events typically contain a single pion and the DIS events contain multiple hadrons, the shower shape for this two interactions are expected to be different to some extent. Thus shower shape can also be used to differentiate between these two interaction processes.

In this chapter the two approaches towards the shower direction reconstruction are discussed. The resolution of the shower direction has also been quantified here. The separation of the DIS and RS events on the basis of the shower shape has also been reported.

5.1 THE RECONSTRUCTION METHODOLOGY

The average direction of a shower like feature would depend on its shape and extent. For the hadron showers in ICAL, the position vectors of the hits in an event with respect to the interaction vertex, have been used to extract the relevant information. Two different techniques are tested for the shower direction reconstruction.

5.1.1 CENTROID TECHNIQUE

For each simulated event, the vertex position and the positions of hits forming the shower are obtained. Each event is required to have at least one hit for analysis. At $E_{\text{had}} = 1$ GeV, 10% of the events fail to meet this constraint. However as energy increases, the fraction of the useful events increases and at 5 GeV, only 0.5% events are rejected. The direction of the centroid of the shower is found by summing over the position vectors of each hit in that event [17]. For a shower constituting of $N$ hits with position coordinates $(x_i, y_i, z_i; i = 1, ..., N)$, if the vertex is $(x_0, y_0, z_0)$, the direction of the centroid of the shower can be expressed as the vector $P \left( X, Y, Z \right)$
with

\[
X = \sum_{i=1}^{N} (x_i - x_0),
\]

\[
Y = \sum_{i=1}^{N} (y_i - y_0),
\]

\[
Z = \sum_{i=1}^{N} (z_i - z_0).
\]

The position vectors along the Z–axis are normalized to one layer to avoid overestimation due to hits by the same particle in multiple layers. The direction of the vector \(P\) is taken to be the reconstructed shower direction from the centroid technique.

### 5.1.2 ORIENTATION MATRIX TECHNIQUE

Additional information on the shower spread can be included by take into account the higher order moments through the orientation matrix technique [17].

For a collection of unit vectors \((x_i, y_i, z_i), i = 1, \ldots, N\), the symmetric orientation matrix \(T\) is defined as

\[
T = \begin{pmatrix}
\sum_{i=1}^{N} x_i^2 & \sum_{i=1}^{N} x_i y_i & \sum_{i=1}^{N} x_i z_i \\
\sum_{i=1}^{N} x_i y_i & \sum_{i=1}^{N} y_i^2 & \sum_{i=1}^{N} y_i z_i \\
\sum_{i=1}^{N} x_i z_i & \sum_{i=1}^{N} y_i z_i & \sum_{i=1}^{N} z_i^2
\end{pmatrix}.
\]

The eigenanalysis of this symmetric matrix gives an idea of the shape of the underlying distribution.
If a unit mass is assumed to be placed at each point, then the moment of inertia of the \( N \) points about any arbitrary axis \((x_{arb}, y_{arb}, z_{arb})\) is

\[
N - \begin{pmatrix} x_{arb} & y_{arb} & z_{arb} \end{pmatrix} T \begin{pmatrix} x_{arb} \\ y_{arb} \\ z_{arb} \end{pmatrix}
\]  

(5.3)

As the choice of axis varies, the variation of this moment of inertia gives information about the scatter of the points around that axis. The axis about which the moment is least, is the principal axis and it is defined to be the shower direction. The MIGRAD and SIMPLEX minimizer algorithms, inbuilt in the TMinuit class in ROOT, are used for this calculation [16].

## 5.2 THE RESULTS

The results of the hadron shower reconstruction using the two methods described in section 5.1 for CC \( \nu_\mu \) events propagating through a 200 cm \( \times \) 200 cm \( \times \) 200 cm volume in the central region of the detector are presented here. The resolution of two reconstructed quantities, the average shower direction vector \( (\alpha_{had}) \) and the average zenith angle \( (\theta_{had}) \) has been obtained.

### 5.2.1 THE \( \alpha_{had} \) RESOLUTION

The parameter \( \alpha_{had} \) is defined as the angle between the reconstructed shower direction and the true shower direction. The true shower direction is obtained from the net momentum of the hadrons constituting the shower, as available from the NUANCE generator. Fig. 5.1a shows the comparison of the \( \alpha_{had} \) distributions obtained from both the centroid and the orientation matrix methods, at \( E'_{had} = 4.5 - 5 \) GeV and \( \cos \theta_{had}^{true} = [1, 0.8] \). Clearly, the latter method gives a marginally better result.
5.2. THE RESULTS

Figure 5.1: (a) The $\alpha_{\text{had}}$ distributions from the centroid and orientation matrix methods, at $E'_{\text{had}} = 4.5 - 5 \text{ GeV}$ and $\cos \theta_{\text{had}}^{\text{true}} = [1, 0.8]$. (b) The $\alpha_{\text{had}}$ distribution from the orientation matrix technique, fitted to Eqn. (5.4). Here, the parameters $p0 = A$, and $p1 = B$. 

In Fig. 5.1b the $\alpha_{\text{had}}$ distribution from the orientation matrix technique is fitted with the function

$$f(\alpha_{\text{had}}) = A \cdot \alpha_{\text{had}} \cdot \exp(-B \cdot \alpha_{\text{had}}),$$

(5.4)

where $A$ and $B$ are the two parameters using which the mean and the standard deviation of the distribution can be calculated. Note that, the fraction of events within 1σ, 2σ and 3σ confidence level of the actual $\alpha_{\text{had}}$ value are roughly 82%, 94% and 98% respectively. It is different from the typical significance levels in the case of normal distributions. The direction resolution $\sigma_{\alpha_{\text{had}}}$ is defined as

$$\sigma_{\alpha_{\text{had}}} = \sqrt{\langle(\alpha_{\text{had}})^2\rangle - \langle\alpha_{\text{had}}\rangle^2},$$

(5.5)

where $\langle\alpha_{\text{had}}\rangle$ and $\langle(\alpha_{\text{had}})^2\rangle$ are calculated using the fit parameters A and B. In terms of the fit parameters,

$$\sigma_{\alpha_{\text{had}}} = \frac{\sqrt{2}}{B},$$

(5.6)
Figure 5.2: The direction resolution $\sigma_{\alpha_{\text{had}}}$ as a function of the $E'_{\text{had}}$, in the $|\cos \theta_{\text{had}}|$ bins $[1, 0.8], [0.8, 0.5], [0.5, 0.2], \text{and } [0.2, 0]$. In Fig. 5.2 the direction resolution $\sigma_{\alpha_{\text{had}}}$ is shown as a function of the $E'_{\text{had}}$, in the $|\cos \theta_{\text{had}}|$ bins $[1, 0.8], [0.8, 0.5], [0.5, 0.2], \text{and } [0.2, 0]$. In the $|\cos \theta_{\text{had}}|$ bin $[0.2, 0]$, the direction resolution is worse. It is due to the geometry, since the thickness of the iron plate traversed increases with increasing zenith angle, and it affects the number of hits recorded.

### 5.2.2 THE $\theta_{\text{had}}$ RESOLUTION

The quantity $\Delta \theta_{\text{had}}$ is defined as

$$\Delta \theta_{\text{had}} = \theta_{\text{rec}} - \theta_{\text{true}}.$$  \hspace{1cm} (5.7)

The distributions of $\Delta \theta_{\text{had}}$, obtained from the two methods, are shown in Fig. 5.3a. As expected, the orientation matrix gives a improved estimation of $\theta_{\text{had}}$. In Fig. 5.3b the $\Delta \theta_{\text{had}}$ distribution from orientation matrix technique, is fitted to the Breit–Wigner distribution. The functional form of the Breit–Wigner distribution is given by

$$L(x) = \frac{1}{\pi} \frac{\Gamma}{\left( x - x_0 \right)^2 + \frac{\Gamma^2}{2}},$$  \hspace{1cm} (5.8)

where $\Gamma$ is the FWHM, i.e., the full width at half maximum and $x_0$ is the mean. The
resolution of $\theta_{\text{had}}$ is defined as
\[ \sigma_{\theta_{\text{had}}} = \frac{\Gamma}{2.35}. \] (5.9)

Figure 5.3: (a) The $\Delta\theta_{\text{had}}$ distributions from the centroid and orientation matrix methods, at $E'_{\text{had}} = 4.5 - 5$ GeV and $\cos\theta_{\text{true}}^{\text{had}} = [1, 0.8]$. (b) The $\Delta\theta_{\text{had}}$ distribution from the orientation matrix technique, fitted to Eqn. (5.4). Here, the parameters $p_0 = x_0$, and $p_1 = \Gamma$.

Figure 5.4: The direction resolution $\sigma_{\alpha_{\text{had}}}$ as a function of the $E'_{\text{had}}$, in the $|\cos\theta_{\text{had}}|$ bins [1, 0.8], [0.8, 0.5], [0.5, 0.2], and [0.2, 0].

In Fig. 5.4 the direction resolution $\theta_{\text{had}}$ is shown as a function of the $E'_{\text{had}}$ in the $|\cos\theta_{\text{had}}|$ bins [1, 0.8], [0.8, 0.5], [0.5, 0.2], and [0.2, 0]. A lookup table in terms of the fit parameters and resolutions as a function of energy can further be used to include $\theta_{\text{had}}$ in a statistical analysis of the ICAL sensitivity. The shape of the shower can also be used to separate the RS and DIS events, which is discussed in section 5.3.
5.3 SEPARATION OF RS AND DIS EVENTS

The hadron showers produced in the resonance scattering (RS) and the deep inelastic scattering (DIS) interactions of $\nu_\mu$ are expected to differ in shape. The RS processes usually produce a single pion, whereas the DIS processes produce multiple hadrons. Thus the shower spreads in the case of the DIS events are expected to be larger than those in the RS events.

A quantity $P_t$ has been defined to study the transverse extent of the hits in a shower. For a shower with $N$ hits, it is given by

$$P_t = \sum_{i=1}^{N} p_i,$$  \hspace{1cm} (5.10)

where $p_i$ is the perpendicular distance of the $i^{th}$ hit on the average direction vector of the shower.

![Graphs showing the comparison of the distribution of the transverse spread $P_t$ for the RS and DIS interaction events at (a) $E'_{\text{had}} = (2 - 3)$ GeV and (b) $E'_{\text{had}} = (5 - 6)$ GeV.](image)

**Figure 5.5**: The comparison of the distribution of the transverse spread $P_t$ for the RS and DIS interaction events at (a) $E'_{\text{had}} = (2 - 3)$ GeV and (b) $E'_{\text{had}} = (5 - 6)$ GeV.

In Fig. 5.5 the distributions of the transverse spread $P_t$ for the RS and DIS interaction events are compared at $E'_{\text{had}} = (2 - 3)$ GeV and $E'_{\text{had}} = (5 - 6)$ GeV. As expected, the spread in the RS events is much narrower than that in the DIS events. Thus the quantity $P_t$ is a useful tool to separate out the two different interaction processes.
5.4 REMARKS

In this chapter the methodology and results of the reconstruction of the mean direction of the hadron shower are discussed. It is difficult to reconstruct the average direction of the hadron showers as accurately as that of muons, however even an approximate estimation is also useful to obtain additional information on the hadron showers. The hadron information of the hadron hits, in terms of their position vectors with respect to the interaction vertex, are used. In the first approach, the direction of the centroid of the hadron shower is obtained. The accuracy of reconstruction is slightly increased in the second approach, by taking into account the higher order moments by the use of an orientation matrix. A resolution in the range $20^\circ - 7^\circ$ is obtained for the average direction vector of the hadron shower. Similar resolutions are also obtained for the mean zenith angle of the hadrons. These results may be used to have the hadron direction as an observable for the ICAL physics studies.

The shower shape can also be used to differentiate between different types of interactions. The RS events produce a single pion, and the shower is expected to be narrower. While, the DIS events produce multiple hadrons, and the showers are broader. The net transverse extent of the hits with respect to the shower direction vector is a useful quantity, and can be used to separate these interactions.

The reconstructed hadron energy is then used to reconstruct the neutrino direction. This will be described in Chapter 6.