Chapter 2

Warped Geometry: The
Randall-Sundrum model and the radion

Scenarios where extra space-like compact dimensions exist, with implications in TeV scale physics, have been widely explored over the last two decades. Scenarios with flat extra dimensions include the so called ADD models [58] as well as those with UED models [59]. However, such scenarios require large compactification radii, whose stability introduces a fresh naturalness problem. Such problems are largely avoided in theories of warped extra dimensions. The Randall-Sundrum(RS) model is the most popular example of such theories.

2.1 The minimal Randall-Sundrum model

Randall and Sundrum proposed a scenario where the hierarchy between the Planck scale and the weak scale is generated from a five dimensional non-factorizable geometry. The model elides on a slice of $AdS_5$ space-time. The additional space-like dimension is $S_1/Z_2$ orbifolded. One further postulates the existence of two 3-branes
with opposite tensions, which are placed at the orbifold fixed points, namely $\phi = 0, \pi$, where $\phi$ is the angular coordinate parameterizing the extra dimension. One also expresses the co-ordinate along this dimension as $y = r_c \phi$, where $r_c$ is the radius of the compactification. Gravitation propagates in the entire 5-dimensional 'bulk', peaking at the 'hidden' or 'Planck' brane ($\phi = 0$), whereas the standard model (SM) fields are confined to the 'visible' brane ($\phi = \pi$). The action for the aforementioned configuration is given by [36]

$$S = S_{\text{gravity}} + S_v + S_h,$$

$$S_{\text{gravity}} = \int d^4x \int_{-\pi}^{\pi} d\phi \sqrt{-G} \left(-\Lambda + 2M_5^3 R\right),$$

$$S_v = \int d^4x \sqrt{-g_v} (L_v - V_v) \text{ and}$$

$$S_h = \int d^4x \sqrt{-g_h} (L_h - V_h) \tag{2.1.1}$$

where the subscripts $v$ and $h$ refer to the visible and the hidden branes respectively, $G$ is the determinant of the five dimensional metric $G_{MN}$. The metrics on the visible and hidden branes are given by

$$g^v_{\mu \nu}(x^\mu) \equiv G_{\mu \nu}(x^\mu, \phi = \pi), \quad g^h_{\mu \nu}(x^\mu) \equiv G_{\mu \nu}(x^\mu, \phi = 0), \tag{2.1.2}$$

where the greek indices are the representation of $(1+3)$ dimensional coordinates on the visible (hidden) brane. $M_5$ is the five dimensional Planck mass and $\Lambda$ is the bulk cosmological constant. $V_v$ and $V_h$ are the brane tensions of the visible and the hidden branes respectively.

If the solution respects four dimensional Poincare symmetry, then the metric is

$$ds^2 = e^{-2\sigma(\phi)} \eta_{\mu \nu} dx^\mu dx^\nu - r_c^2 d\phi^2. \tag{2.1.3}$$

On plugging the ansatz form of the metric in the Einstein’s equation (2.1.2), we get
\[ \frac{6\sigma''^2}{r_c^2} = \frac{-\Lambda}{4M_5^3} \]

\[ 3\frac{\sigma''}{r_c^2} = \frac{V_h}{4M_5^3r_c} \delta(\phi) + \frac{V_v}{4M_5^3 r_c} \delta(\phi - \pi). \tag{2.1.4} \]

On applying the orbifolding condition and solving (2.1.4), we get

\[ \sigma = r_c |\phi| \sqrt{-\frac{\Lambda}{24M_5^3}}. \tag{2.1.5} \]

Thus, the bulk metric becomes

\[ ds^2 = e^{-2k|y|} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2 \tag{2.1.6} \]

where, \( k = \sqrt{-\frac{\Lambda}{24M_5^3}} \) and

\[ V_h = -V_v = 24M_5^3 k. \tag{2.1.7} \]

On comparing the curvature term of the effective four dimensional action, we get the following relation between the five dimensional Planck mass \( M_5 \) and the four dimensional reduced Planck mass \( \bar{M}_{Pl} \)

\[ \bar{M}_{Pl}^2 = \frac{M_5^3}{k} [1 - e^{-2kr_c\pi}] \tag{2.1.8} \]

where, \( \bar{M}_{Pl} = M_{Pl}/\sqrt{8\pi} \).

The five dimensional curvature of the metric is \( R_5 = 20k^2 \). The classical solution for the bulk metric can be trusted for \( R_5 < M_5 \). As a result, \( k/\bar{M}_{Pl} \) cannot be too large \((k/\bar{M}_{Pl} \sim 0.2)\).

### 2.2 Hierarchy problem revisited

Let us consider the Higgs field on the visible brane \((\phi = \pi)\),
\[ S_{\text{vis}} = \int d^4x \sqrt{-g_v} \left( g^{\mu\nu}_{\text{vis}} D_\mu H H^\dagger D_\nu H - \lambda (H^2 - v_0^2)^2 \right). \] (2.2.9)

On using the expression for the induced metric \( g^v_{\mu\nu} = e^{-kr_v r} \bar{g}_{\mu\nu} \) on the visible brane given in (2.1.2), (2.2.9) takes the form of

\[ S_{\text{vis}} = \int d^4x \sqrt{-\bar{g}} e^{-4kr_v r} \left( \bar{g}^{\mu\nu} e^{2kr_v r} D_\mu H H^\dagger D_\nu H - \lambda (H^2 - e^{-2kr_v r} v_0^2)^2 \right). \] (2.2.10)

After redefining \( H (H \rightarrow H e^{-kr_v r}) \), we get

\[ S_{\text{vis}} = \int d^4x \sqrt{-\bar{g}} \left( \bar{g}^{\mu\nu} D_\mu H H^\dagger D_\nu H - \lambda (H^2 - e^{-2kr_v r} v_0^2)^2 \right). \] (2.2.11)

Thus, effectively the mass parameter \( v_0 \) in (2.2.9) is replaced by \( v_0 e^{-kr_v r} \) in (2.2.11). The five dimensional metric consists solely of mass parameters whose values are around the Planck scale. For the choice of \( kr_v \simeq 12 \), which requires barely an order of disparity between the scales \( k \) and \( 1/r_v \), the mass parameters on the visible brane are suppressed with respect to the Planck scale by the exponential factor \( e^{kr_v r} \simeq 10^{16} \), thus offering a rather appealing explanation of the hierarchy between the Planck and TeV scales.

### 2.3 Signatures of the RS model

When we are studying 4-dimensional physics on the visible brane, the extra dimension is integrated out, or compactified. Consequently, a tower of Kaluza-Klein (KK) states of any bulk field should appear on the visible brane. The first excitation of the RS graviton \( (G^*) \) is the readiest object of investigation in the quest of phenomenological signatures of this scenario. Another particle that can be of interest is the radion, arising out of a modulus field introduced to stabilise the radius of the fifth dimension. Both the graviton excitation(s) and the radion may lead to signals of the RS model. Such potential signals have been probed extensively in the recent and present high-energy
colliders. They are reviewed in the following sections.

## 2.4 KK graviton

The Kaluza-Klein (KK) decomposition of the graviton on the visible brane leads to a discrete tower of states, with one massless graviton and a series of TeV-scale spin-2 particles \([60, 61, 62]\).

Let us parametrize the tensor fluctuation \(h_{\alpha\beta}\) by considering the linear expansion of the flat metric about its Minkowski value,

\[
G_{\alpha\beta} = e^{-2\sigma} (\eta_{\alpha\beta} + K^* h_{\alpha\beta})
\]  

(2.4.12)

where \(K^*\) is the expansion parameter.

One can expand \(h_{\alpha\beta}\) in terms of the KK modes of the graviton,

\[
h_{\alpha\beta}(x, \phi) = \sum_{n=0}^{\infty} h_{\alpha\beta}^n(x) \frac{\chi^n(\phi)}{\sqrt{r_c}}
\]

(2.4.13)

where \(h_{\alpha\beta}^n(x)\) corresponds to the \(n^{th}\) KK mode of the graviton about the Minkowski space on the 3-brane.

In the gauge where \(\eta^{\alpha\beta} \partial_\alpha h_{\beta\gamma}^n(x) = \eta^{\alpha\beta} h_{\alpha\beta}^n(x)_{\alpha\beta} = 0\), the equation of motion of the \(n^{th}\) mode of the KK graviton having mass \(m_n\) is given by

\[
(n_{\alpha\beta} \partial^\alpha \partial^\beta - m_n^2) h_{\mu\nu}(x) = 0.
\]

(2.4.14)

On plugging the KK expansion of \(h_{\alpha\beta}^n(x, y)\) in the Einstein’s equation and using the equation of motion (2.4.14), we obtain the following differential equation for \(\chi^n(\phi)\),

\[
\frac{-1}{r_c^2} \frac{d}{d\phi} \left( e^{-4\sigma} \frac{d}{d\phi} \chi^n \right) = m_n^2 e^{-2\sigma} \chi^n.
\]

(2.4.15)

The solutions for \(\chi^n(\phi)\) are given by
\[ \chi^n(\phi) = \frac{e^{2\sigma}}{N_n} [J_2(z_n) + \alpha_n Y_2(z_n)] \]  

(2.4.16)

where \( J_2 \) is the Bessel function of order 2 and \( Y_2 \) is the Neumann function of order 2, \( z_n(\phi) = m_n e^{\sigma(\phi)/k}, N_n \) is the wave function normalization and \( \alpha_n \) are constant coefficients.

The continuity of the first derivative of \( \chi(\phi) \) at the orbifold fixed points leads to \( J_1(z_n(\pi)) = 0 \) and \( \alpha_n \sim x_n^2 e^{-2kr_c\pi} \). Thus, \( z_n(\pi) = x_n \), where \( x_n \) are the roots of the Bessel functions of order 1.

As a result, the mass of the \( n^{th} \) KK mode of the graviton is given by

\[ m_n = k x_n e^{-kr_c\pi}. \]  

(2.4.17)

For \( x_n << e^{kr_c\pi} \), the second term of (2.4.15) is negligible. Hence, the normalization, \( N_n \) becomes

\[ N_n \simeq \frac{e^{kr_c\pi}}{\sqrt{kr_c}} J_2(x_n); \quad n > 0, \quad (2.4.18) \]

and the normalization of the zero mode is \( N_0 = 1/\sqrt{kr_c} \).

Using the solutions for \( \chi^n \), we can derive the interactions of the KK graviton \( h_{\alpha\beta}^n \) with the matter fields on the visible brane. The interaction Lagrangian in the 4-D effective theory has the form of

\[ \mathcal{L} = -\frac{1}{M_5^{3/2}} T^{\alpha\beta}(x) h_{\alpha\beta}(x, \phi = \pi) \]  

(2.4.19)

where \( T^{\alpha\beta} \) is the energy momentum tensor of the matter fields.

Expanding the graviton wavefunction in terms of the KK towers and using the normalization of (2.4.18), we obtain the interaction of the KK modes of graviton with the
matter fields on the visible brane

\[ \mathcal{L} = \frac{-1}{\bar{M}_{Pl}} T^{\alpha\beta}(x) h(x)^{0}_{\alpha\beta} - \frac{1}{\Lambda_\pi} T^{\alpha\beta}(x) \sum_{n=1}^{\infty} h(x)^n_{\alpha\beta}(x) \]  \hspace{1cm} (2.4.20)

where \( \Lambda_\pi = \bar{M}_{Pl} e^{-kr_{cr,\pi}} \). The massless graviton couples to all matter fields with strength \( \sim 1/\bar{M}_{Pl} \), whereas the corresponding couplings for the massive modes receive an exponential enhancement and is suppressed by TeV scale. As a result, the possibility of observing signals of the massive gravitons in TeV-scale experiments opens up.

### 2.4.1 Status of the KK gravitons at the colliders

The first KK excitation of RS graviton (G*) is produced resonantly at hadron colliders through the process of quark-antiquark annihilation or gluon-gluon fusion. Due to small gluon parton distribution function at large momentum fraction, the quark-antiquark annihilation dominates over the gluon fusion production cross-section for the KK graviton. Inspite of its universal coupling to all SM particles, it decays predominantly to quarks and gluons because of their high multiplicity to color, spin and flavor states. However, these channels are difficult to probe at the hadron colliders due to large QCD backgrounds. Although the decay width of KK gravitons to dilepton is suppressed because of its spin-2 nature, the dilepton channel offers a clean signal for KK graviton search.

\( k/\bar{M}_{Pl} \) and the mass of the first KK mode of graviton \( (m_1) \) are considered as free parameters in the KK graviton sector. The mass of \( n^{th} \) KK mode can be expressed in terms of \( m_1 \).

\[ m_n = \left( \frac{x_n}{x_1} \right) m_1 \]  \hspace{1cm} (2.4.21)

For the first KK mode, the total decay width is given by

\[ \Gamma_1 = \rho m_1 x_1^2 (k/\bar{M}_{Pl})^2 \]  \hspace{1cm} (2.4.22)
where $\rho$ is a constant that depends on number of SM channels available.

Thus, limits at 95\% CL on total cross-section for $G^*$ decaying to dilepton are compared to the theoretical value for the total cross-section. This results are used to set lower limits on $m_1$ at a given value of $k/M_{Pl}$. Current experimental limits from the LHC rule out any mass for the lowest graviton excitation below 1.15(2.47) TeV for $k/M_{Pl} \leq 0.01(0.1)$ [63].

### 2.5 The radion

How does the chosen value of $r_c$, the radius of compactification arise, and why is it stable at $r_c \sim 12/k$? In order to answer this question, there has been an attempt to view $r_c$ as the vacuum expectation value (vev) of a $\phi$-independent field, known as modulus field or the radion. This field is parametrized by a scalar fluctuation about the background geometry. Thus, the metric is

$$ ds^2 = e^{-2k|\phi|^2} \bar{g}_{\mu
u}(x) dx^\mu dx^\nu - T^2(x) d\phi^2. $$  \hspace{1cm} (2.5.23)

The KK reduction of the five dimensional Einstein-Hilbert action using the metric of (2.5.23) leads to the following effective action for $T(x)$ and $\bar{g}_{\mu\nu}(x)$

$$ S = 2M_5^3 \int d^4 x d\phi \sqrt{-\bar{g}} e^{-2k|\phi|^2} \left[ 6k |\phi| \partial_\mu T \partial^\mu T - 6k^2 |\phi|^2 T \partial_\mu T \partial^\mu T + R \right] $$ \hspace{1cm} (2.5.24)

where $R$ is the Ricci scalar constructed from $\bar{g}_{\mu\nu}(x)$.

After integrating out the additional coordinate $\phi$, we have the following four dimensional effective action involving the modulus field $T(x)$ and the induced metric $\bar{g}_{\mu\nu}(x)$
\[ S = \frac{2M_5^3}{k} \int d^4x \sqrt{-\bar{g}}(1 - e^{-2k\pi T})R + \frac{3M_5^3}{k} \int d^4x \sqrt{-\bar{g}}\partial_{\mu}(e^{-k\pi T})\partial^{\mu}(e^{-k\pi T}). \] (2.5.25)

To make the kinetic term of the (2.5.25) canonical, the modulus field is redefined to

\[ \varphi(x) = \Lambda \varphi e^{-k \pi (T(x) - r_c)} \] with \( \Lambda = \sqrt{6M_5^3/ke^{-kr_c\pi}}. \) Hence, we arrive at

\[ S = \frac{2M_5^3}{k} \int d^4x \sqrt{-\bar{g}}(1 - (\varphi/f)^2)R + \frac{1}{2} \int d^4x \sqrt{-\bar{g}}\partial_{\mu}\varphi\partial^{\mu}\varphi \] (2.5.26)

where \( f = \sqrt{6M_5^3/ke}. \)

The effective four dimensional action (2.5.26) contains a massless scalar \( \varphi(x). \) However, to solve the hierarchy problem, there should be some additional dynamics that will stabilize the \( \varphi(x) \) and give \( T(x) \) its desired vev, \( r_c. \) The vev of the field \( \varphi(x) \) is \( \Lambda \varphi. \) The stabilization procedure was first proposed by Goldberger-Wise [37, 38] and is briefly reviewed here.

Let us consider a scalar field \( \Phi \) propagating in the bulk, having interaction terms on the visible and the hidden branes (\( v_v \) and \( v_h \) respectively).

\[ S_b = \frac{1}{2} \int d^4x \int_{-\pi}^{\pi} d\phi \sqrt{G}(G^{AB}\partial_{A}\Phi\partial_{B}\Phi - m^2\Phi^2) \]
\[ S_h = -\int d^4x \sqrt{-g_h} \lambda_h(\Phi^2 - v_h^2)^2 \]
\[ S_v = -\int d^4x \sqrt{-g_v} \lambda_v(\Phi^2 - v_v^2)^2. \] (2.5.27)

The terms on the branes cause the scalar field \( \Phi \) to develop a \( \phi \)-dependent vev which is determined classically by solving the equation of motion of the action in (2.5.27). Inserting the general solution of \( \Phi(\phi) \) into the bulk scalar field action and

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integrating over $\phi$ yields an effective potential for $\varphi(x)$ of the form

$$
V_\varphi(r_c) = k\epsilon v_h^2 + 4ke^{-4kr_c\pi}(v_v - v_he^{-\epsilon kr_c\pi})^2(1 + \epsilon/4)
- k\epsilon v_he^{-(4+\epsilon)kr_c\pi}(2v_v - v_he^{-\epsilon kr_c\pi})
$$

(2.5.28)

where we have assumed $\epsilon \equiv m^2/4k^2 \ll 1$. After neglecting the terms of order $\epsilon$ and working in large $\lambda$ limit, we get

$$
V(\varphi) = \frac{k^3}{144M_5^6}\varphi^4(v_v - v_h(\varphi/f)^\epsilon)^2.
$$

(2.5.29)

The potential of (2.5.29) has a minima at

$$
<\varphi> = \frac{v_v}{f} = \left(\frac{v_v}{v_h}\right)^{1/\epsilon}
$$

(2.5.30)

or

$$
kr_c = k < T > = \frac{1}{\pi\epsilon} ln(v_h/v_v).
$$

(2.5.31)

From the relation given in (2.5.31) we can see that $kr_c \sim 12$ can be generated without any kind of extreme fine tuning of the parameters. Also, (2.5.29) gives us the mass of the radion

$$
m_\varphi^2 = \frac{\partial^2 V}{\partial \varphi^2}|_{\varphi<} = \frac{k^2 v_v^2}{3M_5^2}\epsilon^2 e^{-2kr_c\pi}.
$$

(2.5.32)

If we assume that the effective potential is generated by a light bulk scalar, then, due to the suppression by $\epsilon$, the mass of radion is smaller than TeV. As a result, it becomes the first clear signal of RS model.

As the radion arises from the gravitational degree of freedom, its couplings to the matter fields on the visible brane should be governed by the principle of general covariance. The induced metric on the visible brane is defined by $g_{\mu\nu} = (\varphi/L_\varphi)^2\eta_{\mu\nu}$. This term generates a direct coupling of the radion to the matter fields on the visible brane. The radion does not couple directly to the matter fields present in the hidden
brane. The linear coupling of the radion to the matter \([64, 65]\) on the visible brane can be obtained from

\[
\varphi \frac{\partial S_{SM}}{\partial \varphi} \bigg|_{\Lambda_{\varphi}} = \varphi \frac{\partial S_{SM}}{\partial g_{\mu \nu}} \frac{\partial}{\partial \varphi} \bigg|_{\Lambda_{\varphi}} = \frac{\varphi}{\Lambda_{\varphi}} T_{\mu}^{SM}. \tag{2.5.33}
\]

where \(T_{\mu}^{SM}\) is the trace of energy momentum tensor of the SM and is given by

\[
T_{\mu} = m_{f}^{2} h^{2} + \Sigma f m_{f} \bar{f} f - 2 M_{W}^{2} W_{\mu}^{+} W_{\mu}^{-} - M_{Z}^{2} Z_{\mu} Z_{\mu}. \tag{2.5.34}
\]

Thus, the couplings of radion with massive gauge bosons, fermions and Higgs \([3]\) are given by

\[
\Gamma(\varphi \rightarrow f \bar{f}) = \frac{N_{c} m_{f}^{2} m_{\varphi}^{2}}{8 \pi \Lambda_{\varphi}^{2}} (1 - x_{f})^{3/2},
\]

\[
\Gamma(\varphi \rightarrow W^{+} W^{-}) = \frac{m_{\varphi}^{2}}{16 \pi \Lambda_{\varphi}^{2}} \sqrt{1 - x_{W}} \left(1 - x_{W} + \frac{3}{4} x_{W}^{2}\right),
\]

\[
\Gamma(\varphi \rightarrow ZZ) = \frac{m_{\varphi}^{2}}{32 \pi \Lambda_{\varphi}^{2}} \sqrt{1 - x_{Z}} \left(1 - x_{Z} + \frac{3}{4} x_{Z}^{2}\right),
\]

\[
\Gamma(\varphi \rightarrow hh) = \frac{m_{\varphi}^{2}}{32 \pi \Lambda_{\varphi}^{2}} \sqrt{1 - x_{h}} \left(1 + \frac{1}{2} x_{h}\right)^{2}. \tag{2.5.35}
\]

The symbol \(f\) denotes all quarks and leptons. The variable \(x_{i}\) is defined as \(x_{i} = 4 m_{i}^{2} / m_{\varphi}^{2} (i = t, f, W, Z, h)\).

Since, the gluon and the photon are massless. The radion does not have any tree level coupling to them. The couplings of the radion with the gluon(photons) are generated from the following terms:

- There are triangle diagrams involving the W-boson and the top quark. These terms are analogous to that of the SM Higgs.

- The running of the gauge couplings in QCD and QED breaks the scale invariance and generates the following trace anomaly term,

\[
T_{\mu}^{\text{anomaly}} = \frac{\beta_{a}(g_{a})}{2 g_{a}} F_{\mu \nu}^{a} F_{\mu \nu}^{a}. \tag{2.5.36}
\]
where

\[
\begin{align*}
\frac{\beta_{\text{QCD}}}{2g_s} &= -\frac{\alpha_s}{8\pi} b_3 \\
\frac{\beta_{\text{QED}}}{2g_f} &= -\frac{\alpha_e}{8\pi} (b_2 + b_Y)
\end{align*}
\] (2.5.37)

where \( b_3 = 7 \) is the QCD \( \beta \)-function coefficient and \( b_2 = 19/6 \) and \( b_Y = -41/6 \) are the SM \( SU(2)_L \times U(1)_Y \) \( \beta \)-function coefficients. The trace anomaly contribution enhances the \( \varphi gg \) and \( \varphi \gamma \gamma \) amplitudes, with the same Lorentz structures as for loop diagrams and these terms are responsible for supplementing rates in \( \gamma \gamma \) and \( gg \) channels. On adding the above contributions, we get

\[
\begin{align*}
\Gamma(\varphi \rightarrow gg) &= \frac{\alpha_s^2 m_\varphi^3}{32\pi^3 \Lambda_\varphi^2} \left| b_3 + x_t \left\{ 1 + (1 - x_t) f(x_t) \right\} \right|^2, \\
\Gamma(\varphi \rightarrow \gamma \gamma) &= \frac{\alpha_{\text{EM}}^2 m_\varphi^3}{256\pi^3 \Lambda_\varphi^2} \left| b_2 + b_Y - \left\{ 2 + 3x_W + 3x_W(2 - x_W) f(x_W) \right\} \\
&\quad + \frac{8}{3} x_t \left\{ 1 + (1 - x_t) f(x_t) \right\} \right|^2, \\
\Gamma(\varphi \rightarrow Z\gamma) &= \frac{\alpha_{\text{EM}}^2 m_\varphi^3}{128\pi^4 s_W^2 \Lambda_\varphi^2} \left( 1 - \frac{m_Z^2}{m_\varphi^2} \right)^3 \\
&\quad \times \left| \sum_f N_f \frac{Q_f}{e_W} \hat{v}_f A_{1/2}^\varphi(x_f, \lambda_f) + A_1^\varphi(x_W, \lambda_W) \right|^2
\end{align*}
\] (2.5.38, 2.5.39, 2.5.40)

where \( x_i = 4m_i^2/m^2_\varphi \) (\( i = t, f, W, Z, h \)), and \( \lambda_i = 4m_i^2/m^2_Z \) (\( i = f, W \)). The gauge couplings for QCD and QED are given by \( \alpha_s \) and \( \alpha_{\text{EM}} \), respectively. The factor \( N_f \) is the number of active quark flavors in the 1-loop diagrams and \( N_\ell \) is 3 for quarks and 1 for leptons. \( Q_f \) and \( \hat{v}_f \) denote the electric charge of the fermion and the reduced vector coupling in the \( Zf \bar{f} \) interactions \( \hat{v}_f = 2I_3^f - 4Q_f s_W^2 \), where \( I_3^f \) denotes the weak isospin and \( s_W^2 = \sin^2 \theta_W, \ c_W^2 = 1 - s_W^2 \).
The form factors $A_{1/2}^\phi(x, \lambda)$ and $A_1^\phi(x, \lambda)$ are given by

$$A_{1/2}^\phi(x, \lambda) = I_1(x, \lambda) - I_2(x, \lambda) ,$$

$$A_1^\phi(x, \lambda) = c_W \left\{ 4 \left( 3 - \frac{s_W^2}{c_W^2} \right) I_2(x, \lambda) + \left[ (1 + \frac{2}{x}) \frac{s_W^2}{c_W^2} - \left( 5 + \frac{2}{x} \right) \right] I_1(x, \lambda) \right\} .$$

The functions $I_1(x, \lambda)$ and $I_2(x, \lambda)$ are

$$I_1(x, \lambda) = \frac{x \lambda}{2(x - \lambda)} + \frac{x^2 \lambda^2}{2(x - \lambda)^2} [f(x^{-1}) - f(\lambda^{-1})] + \frac{x^2 \lambda}{(x - \lambda)^2} [g(x^{-1}) - g(\lambda^{-1})] ,$$

$$I_2(x, \lambda) = - \frac{x \lambda}{2(x - \lambda)} [f(x^{-1}) - f(\lambda^{-1})] ,$$

where the loop functions $f(x)$ and $g(x)$ in ((2.5.38)), ((2.5.39)) and ((2.5.42)) are given by

$$f(x) = \begin{cases} \left\{ \sin^{-1} \left( \frac{1}{\sqrt{x}} \right) \right\}^2 , & x \geq 1 \\ -\frac{1}{4} \left( \log \frac{1 + \sqrt{1 - x}}{1 - \sqrt{1 - x} - i\pi} \right)^2 , & x < 1 \end{cases} ,$$

$$g(x) = \begin{cases} \sqrt{x^{-1} - 1} \sin^{-1} \sqrt{x} , & x \leq 1 \\ \frac{\sqrt{1 - x^{-1}}}{2} \left( \log \frac{1 + \sqrt{1 - x^{-1}}}{1 - \sqrt{1 - x^{-1}} - i\pi} \right) , & x > 1 \end{cases} .$$

The radion mass $m_\phi$ and the vev $\Lambda_\phi$ constitute the set of free parameters of the theory in the radion sector, which now has the distinction of ‘naturally’ generating a TeV-scale vev on the visible brane. Since, the radion mass is below a TeV, the detection of radion becomes somewhat easier than that of the first KK mode of the graviton [37, 38].
2.5.1 Phenomenology of the radion

The radion can be produced at hadron colliders in the gluon fusion channel, vector boson fusion channel and in association with the vector bosons and $t\bar{t}$. However, because of the enhancement factor coming from the trace anomaly term, the gluon fusion becomes the prime channel for production of radion. The other production channels are relatively suppressed by the vev of radion. The production of radion in gluon fusion channel has two times larger cross-section than the other production channels. The cross-section of the radion produced via gluon fusion channel at a hadronic collider with centre of mass energy ($\sqrt{s}$) is given by

$$\sigma(s) = \int_{m^2}^{1} \frac{dx}{x} g(x) g \left( \frac{m^2}{s x} \right) \frac{\alpha_s^2}{256 \pi} \frac{m^2}{s} |b_3 + x_i (1 - x_i) f(x_i)|^2 \tag{2.5.45}$$

where $g(x)$ is the gluon parton distribution function at a given momentum fraction $x$.

At electron-positron collider, the radion can be produced in association with $W, Z$ bosons or in association with $\nu\bar{\nu} (e^+e^-)$ via $W(Z)$ fusion.

The striking difference between the decay of low mass radion and SM Higgs is in their decay to pair of gluons(figure 2.1). A light radion, due to the boost coming from trace anomaly, decays mostly to gluon-gluon whereas a SM Higgs of the same mass decays to pair of bottom quarks. After the radion crosses $WW$ threshold, it decays predominantly to a pair of $W$s. If kinematically allowed, a heavier radion can also decay to a pair of SM Higgs. A heavier radion ($m_\varphi > 350$ GeV) decays to $WW^*$ mostly, whereas an SM-like second Higgs if exist, decays mostly into $t\bar{t}$.

2.5.2 Status of the radion at the colliders

The radion has been studied extensively in the LEP, Tevatron and LHC. As most of the decays of radion are similar to the SM Higgs, so searches of the SM Higgs has been used to constraint the parameter space of the radion. Apart from the SM Higgs search,
Figure 2.1: Branching ratio of the radion for all possible decay channels [3]. $\phi$ denotes the radion.

searches on the RS graviton excitations also put a lower bound on the vev of radion.

$$\Lambda_\phi = \frac{\sqrt{6} m_1}{k/M_{Pl} x_1}$$

(2.5.46)

As the RS solution is valid for $k/M_{Pl} < 1$, absence of the first excitation of RS graviton(G*) till 2.6 TeV excludes radion vev till 1.8 TeV. The radion can mix with the SM Higgs because of the presence of a curvature term. After the discovery of the scalar at 125 GeV at LHC, speculations have been made on whether the observed 125-126 GeV state, instead of being a pure SM Higgs, could instead be the radion, or a mixture of the two. A number of studies have already taken place in this direction, based on both the ‘pure radion’ and ‘radion-Higgs mixing’ hypotheses [66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89].

The Higgs-radion mixing scenario has been discussed thoroughly in chapter 3. The next chapter comprises of the study that we carried out in the paper [90]. We have restricted the available parameter space of the Higgs-radion mixing using the results of LHC run-1. We found that the LHC run-1 data has constrained the space made up of mass of the mixed scalar and the mixing parameter with vev of the order of 1-5 TeV.
Once the vev of the radion increases beyond 5 TeV, the production cross-section of the radion becomes too small and hence, run-1 data is unable to put any limit on it. We showed that the discovered scalar can not be a radion. However, the scenario where a very heavy ($m_\phi > 500$ GeV) or a very light radion ($m_\phi < 100$ GeV) exist along with the SM Higgs, is still allowed by the LHC run-1 data. The gluon fusion production cross section of the heavy radion decreases. The cross section gets further suppression once $\Lambda_\phi$ increases. Thus, a heavy radion with vev of the order of 3-5 TeV is still allowed by the LHC data. The LHC run-2 will be able to shed light on the allowed regions in the parameter space of the scenario involving Higgs-radion mixing. The phenomenology of a light radion ($\sim 100$ GeV) is also very interesting. At LEP, a light radion could have been produced via $e^+ e^- \rightarrow Z \phi$. The production mode in this channel is however found to be suppressed for $\Lambda_\phi > 1.0$ TeV and hence, a radion as light as 50 – 100 GeV with $\Lambda_\phi \simeq 2 – 3$ TeV, is still allowed by the LEP data as well as by the LHC searches [91, 92, 93]. The distinct signature of a light radion is its decay to two gluon. However, due to immense QCD background at the LHC, the gluonic channel is impossible to study. We have found that the diphoton channel can be used as a probe for discovering the radion [94]. Because of the limited luminosity at 8 TeV run of LHC, signals coming from the light radion were masked by huge diphoton background. However, with proper signal background analysis one can discover such a light radion at 14 TeV run of LHC with high luminosity. The detailed phenomenology of such a light radion is the topic of discussion in chapter 4.

There exist various modifications over the minimal RS model, for example, including gauge fields and fermions in the bulk [95, 96, 97, 98, 99, 100, 101, 102], that explain the SM flavor structure as well. As the SM fermions receive masses from the Higgs, one can explain the fermion mass hierarchy by localizing the light fermions relatively away from the TeV brane or ‘visible’ brane. The third family fermions, on the other hand, peak close to the ‘visible’ brane. The SM Higgs is localized on the TeV brane. All the fermions have $O(1)$ Yukawa couplings in the 5D theory. The varying degrees
of overlap affect Yukawa coupling on the TeV brane and generate the flavor hierarchy. In this chapter, I have discussed the minimal version of the RS model, where the SM particles are on the visible brane and only gravity propagates in the bulk.