Chapter 8

Bose-glass to vortex-liquid phase transition in the vortex state of the annealed Ti$_{0.7}$V$_{0.3}$ alloy

8.1 Introduction

In type-II superconductors, magnetic field penetrates the bulk of the material in the form of superconducting vortices or flux lines when the applied magnetic field is higher than the lower critical field $H_{C1}$. In defect free type-II superconductors, the repulsive interaction among the flux lines tend to drive the flux lines to get them arranged in a hexagonal array called the Abrikosov lattice [22]. The existence of such an ordered Abrikosov lattice has been observed in the superconducting mixed state of clean Nb samples through neutron scattering experiment [243, 244]. However, flux-line pinning at quenched disorders can prevent the emergence of the long-range order of the Abrikosov lattice [32]. The weak random pinning of the flux lines perturbs the transla-
tional invariance of the Abrikosov lattice, giving a quasi-ordered Bragg-glass phase which has been observed experimentally in the mixed state of several type-II superconductors [33-37]. On the other hand, strong pinning of the flux lines gives rise to the formation of the disordered vortex-glass [38, 39] or the Bose-glass phase [40, 41] depending on the nature and strength of the quenched disorders present in a sample. Generally, a vortex-glass phase exists in disordered materials involving point disorders while a Bose-glass phase is observed in materials with correlated disorders like grain boundaries and/or twin boundaries, and/or heavy ion-induced columnar tracks [245, 246]. The vortex-glass phase continues to exist up to the glass transition temperature $T_G$, where it transforms into a vortex-liquid through a second order phase transition [38]. In theoretical ground [38-41], it was predicted that when the temperature is increased toward $T_G$, electrical resistivity in the vortex-liquid phase vanishes following a power law relation $\rho \propto |T - T_G|^s$, where $s$ is the critical exponent of the glass transition. For high-$T_C$ superconductors, $T_G$ is generally found to be substantially lower than the superconducting transition temperature $T_C$ due to the relatively soft vortex matter and the enhanced role of thermal fluctuations in these materials [38]. Consequently, for high-$T_C$ superconductors, an appreciable temperature regime exists between the $T_G(H)$- and $T_C(H)$-line, where the description of the vortex-glass transition in term of vortex-liquid resistivity is possible [39]. Therefore, the study of the resistive transition in the presence of magnetic field continues to be an active experimental method for investigating vortex-glass transition in high-$T_C$
superconductors.

Generally, thermal fluctuations are less effective in bulk low-$T_C$ superconductors, causing the $T_G(H)$-line to lie very close to the $T_C(H)$-line. Consequently, it is not possible to study of the vortex-glass transition in bulk low-$T_C$ superconductors through resistive transition measurements. Ideally, one has to rely on the small-angle neutron scattering (SANS), and/or scanning tunnelling microscopy (STM) and/or Bitter decoration (BD) for observing the vortex-glass phase in bulk low-$T_C$ superconductors. Though the experiment felicities like STM and BD can be useful for the real space imaging of the vortex-glass phase and SANS for mapping of the vortex-glass phase in reciprocal space, the dynamical nature of the vortex-glass phase remains completely unrevealed in these experiments. Experimental study of the vortex-glass phase in bulk low-$T_C$ superconductors through resistive transition measurement is therefore necessary for a comparative study of the vortex-glass phase in low-$T_C$ and high-$T_C$ superconductors, and also in the point of view of the dynamical nature of the vortex-glass phase in a bulk low-$T_C$ superconductor. Dimitriv et al. [37] have recently found the existence of the Bragg-glass phase in the mixed state of a disordered Ti$_{0.21}$V$_{0.79}$ superconductor through SANS experiments. In Ti-V alloy system, the amount of disorder is enhanced significantly with increasing Ti concentration because of the formation of the $\omega$ phase and the martensite $\alpha$ or $\alpha'$ phase within the major $\beta$ phase matrix of these alloys [42, 51]. In chapter 3, we have found that the above mentioned secondary phases are indeed present in the
samples of Ti$_{0.8}$V$_{0.2}$ and Ti$_{0.7}$V$_{0.3}$ alloys. Hence, the presence of a highly disordered vortex-glass and/or Bose-glass phase is likely in the Ti-rich Ti-V alloys. Moreover, we have found in the previous chapters that the thermal fluctuations become increasingly important in Ti-rich Ti-V alloys. This gives rise to an appreciable magnetic field-temperature regime between the $H_{C2}(T)$- and $H_{Ir}(T)$-line [$T_G(H)$-line is equivalent to $H_{Ir}(T)$-line], where a vortex-liquid phase exists with non-ohmic resistivity [39]. We have therefore explored the possibility of such vortex-liquid to vortex-glass phase transition in Ti-rich Ti-V alloys through electrical resistivity measurements, and indeed observed the signatures of the stated phase transition in the sample of annealed Ti$_{0.7}$V$_{0.3}$ alloy. These results are presented in this chapter.

Figure 8.1: (a) Temperature dependence of electrical resistivity of annealed Ti$_{0.7}$V$_{0.3}$ sample measured at low temperatures and in different magnetic fields. (b) The same curves plotted in a reduced temperature scale to highlight the field induced broadening of the superconducting transition.
8.2 Results and discussion

8.2.1 Temperature dependence of electrical resistivity in different constant magnetic fields: Evidence of a glass to liquid phase transition in the flux-line system of the annealed Ti$_{0.7}$V$_{0.3}$ sample

Fig. 8.1 (a) shows the temperature dependence of resistivity $\rho$ measured for the annealed Ti$_{0.7}$V$_{0.3}$ sample at low temperatures and in the presence of various constant applied magnetic fields ranging from zero to 5 T. In zero magnetic field, the sample enters into the superconducting state at the superconducting transition temperature $T_C = 6.69$ K with a transition broadening of $\Delta T_C \sim 0.14$ K. Here, $T_C$ is defined as the temperature at which a steep increase in the temperature derivative of $\rho$ first appears on the higher temperature side. The superconducting transition broadening $\Delta T_C$ is defined as the temperature interval where resistivity drops from 90 % to 10 % of its the normal state value across the superconducting transition. In the presence of magnetic field, $T_C$ shifts to lower temperatures and the superconducting transition becomes more broadened. This latter effect is clearly visible in Fig. 8.1 (b), where the resistivity is plotted against the reduced temperature $t = T/T_C(H)$. It is well known that the broadening of the superconducting transition in the presence of magnetic field could be resulted from thermally-activated-flux-flow (TAFF). The temperature dependence of resistivity in such a case is generally analysed with the help of the Arrhenius relation given as [249]
\[
\rho(T, H) = \rho_0 \exp \left[ \frac{-U(T, H)}{k_B T} \right],
\]

where, \( k_B \) is the Boltzmann constant, \( \rho_0 \) is a pre-exponential factor (constant), and \( U(T, H) \) is the activation energy associated with the flux-flow. The Arrhenius plots (\( \ln \rho \) versus \( T^{-1} \) plots) in different applied magnetic fields are shown in Fig. 8.2 (a). It can be seen from this figure that except at temperatures in the close vicinity of \( T_C \), the Arrhenius plots are linear down to a characteristic temperature \( T^* \), indicating that the observed magnetic field-induced broadening of the resistive transition is caused by the TAFF at temperatures above \( T^* \). At temperatures below \( T^* \), the Arrhenius plots deviate from showing the linear behaviour and exhibits a strong downward bend. This suggests that the activation energy for flux motion increases more rapidly at temperatures below \( T^* \) than in the TAFF regime. This becomes more evident in Fig. 8.2 (b), where we present the plots of the function \( F = -d(\ln \rho)/dT^{-1} \) against temperature for different values of the applied magnetic field. According to the Arrhenius relation, the function \( F \) is proportional to the activation energy \( U \). It can be seen from this figure that the activation energy tends to diverge as the temperature is decreased below \( T^* \). Such a diverging behaviour of the activation energy at low temperatures has previously been observed in many high-\( T_C \) superconductors and is attributed to a crossover to a critical region associated with the vortex-liquid to vortex-glass phase transition [250-253].
Figure 8.2: (a) The Arrhenius plots obtained for the annealed Ti$_{0.7}$V$_{0.3}$ sample in different magnetic fields. These plots exhibit a change of slope at the characteristic temperature $T^*$. (b) Temperature dependence of the activation energy $U$ for this sample in various constant magnetic fields.

Following the power law relation $\rho \propto |T - T_G|^s$ for the vortex-liquid resistivity, one can expect the plot of the inverse of the logarithmic derivative of resistivity $[d(ln\rho)/dT]^{-1}$ against $T$ to be linear in the critical region of the vortex-liquid to vortex-glass transition. In Fig. 8.3, the plots of $[d(ln\rho)/dT]^{-1}$ against temperature are shown for the annealed Ti$_{0.7}$V$_{0.3}$ sample for 0.5 and 5 T applied magnetic fields. In agreement with the theory, these plots are linear at temperatures below $T^*$, indicating the existence of a glassy vortex phase in this sample at temperatures below $T_G$. We then estimate the $T_G$ using the Vogel-Fulcher relation: $[d(ln\rho)/dT]^{-1} = (T - T_G)/s$ [254-256]. According to this relation, $T_G$ is obtained by finding the temperature where the linear portion of $[d(ln\rho)/dT]^{-1}$ versus $T$ plot extrapolates to $[d(ln\rho)/dT]^{-1} = 0$. Additionally, the critical exponent $s$ at different applied
magnetic fields is estimated from the inverse of the slope of the linear section of these plots, and is shown in Fig. 8.4 (a). The value of \( s \) is found to be \( \sim 1.8 \), and is almost independent of magnetic field. This value is smaller than the values of \( s \sim 6-8 \) generally obtained in the vortex-liquid to vortex-glass phase transition in several high-\( T_C \) superconductors where point disorders act as the major flux-line pinning centres [254, 257]. However, similar small values of \( s \) as estimated for the present sample have been obtained for phase transition from vortex-liquid to Bose-glass in materials involving correlated disorders, such as twined \( \text{YBa}_2\text{Cu}_3\text{O}_{7-\delta} \) \( (s \sim 2) \) [254, 258] and YBCO with columnar defects \( (s \sim 2.4) \) [259].

Figure 8.3: The plots of \( [d(\ln \rho)/dT]^{-1} \) against temperature for the annealed \( \text{Ti}_{0.7}\text{V}_{0.3} \) sample for 5 and 0.5 T magnetic fields.

We present in Fig. 8.4 (b)-(d) few selected micrographs obtained in scanning electron microscopy (SEM) and optical metallography experiments on the annealed samples of the \( \text{Ti}_{0.7}\text{V}_{0.3} \) alloy. These micrographs reveal that the crystal disorders which can lead to the flux-line pinning in the present
sample consist of mainly grain boundaries and martensitic $\alpha$ phase. The X-ray diffraction result indicates that the amount of the martensitic $\alpha$ phase in this alloy is about 28% (refer to chapter 3). These extended disorders are correlated over mesoscopic length scales, and the presence of these correlated disorders gives rise to the Bose-glass phase in the annealed Ti$_{0.7}$V$_{0.3}$ sample, which is consistence with the estimate of the low value of the critical exponent $s$ for this sample.

Figure 8.4: (a) Critical exponent $s$ for the annealed Ti$_{0.7}$V$_{0.3}$ sample in various magnetic fields. (b), (c) SEM images of this sample showing the presence of martensite $\alpha$ phase in the main $\beta$ phase matrix this sample. (d) Optical micrograph showing the grain structures in this sample.
8.2.2 The modified vortex-glass model

In the previous section, we have obtained the experimental evidence for the existence of a glassy vortex phase in the mixed state of annealed Ti$_{0.7}$V$_{0.3}$ sample based on the prediction of the vortex-glass theory [38-41]. Recent works [247, 248] have made modification in the vortex-glass theory to give a consistent description of vortex-liquid resistivity in the critical region of the vortex-liquid to vortex-glass transition in the high-$T_C$ oxide superconductors. This modified vortex-glass model has been extensively used in the recent times to ascertain both the vortex-liquid to vortex-glass as well as the vortex-liquid to Bose-glass transition in various high-$T_C$ oxide and Fe-based superconductors [247, 248, 255, 256, 260-263]. This modified vortex-glass model takes the effective pinning energy $U_0$ into consideration while analysing the vortex-liquid to vortex-glass transition. This is done by replacing the temperature difference $(T - T_G)$ by an energy difference $(k_B T - U_0)$. In this model the temperature dependence of resistivity in the critical region of vortex-liquid to vortex-glass transition is expressed as: [247, 248]

$$\rho(T) = \rho_n \left| \frac{k_B T}{U_0(H,T)} - 1 \right|^s. \quad (8.2)$$

The modified vortex-glass model assumes an empirical effective pinning energy $U_0(H,T)$ for the high-$T_C$ oxide superconductors, which has the form [247, 248]

$$U_0(H,T) = U_H \left[ 1 - \frac{T}{T_C} \right]; U_H = k_B T_C H^{-\beta}. \quad (8.3)$$
In the above relation, $U_H$ is a function of field only and $\beta$ is a constant independent of temperature and magnetic field. The model also assumes that the transition from vortex-liquid to vortex-glass occurs at $T_G$, where the thermal energy becomes equal to the pinning energy, i.e., $U_0(H, T_G) = k_B T_G$. With this assumption the magnetic field dependent part of $U_0(H, T)$ in Eqn. (8.3) is obtained as a unique function of $T_G$ as

$$U_H(H) = \frac{k_B T_C T_G}{T_C - T_G}. \quad (8.4)$$

Using the above form of $U_H(H)$, the empirical pinning energy given in Eqn. (8.3) is then reformulated as:

$$U_0(H, T) = \frac{k_B T_G \left[ 1 - \frac{T}{T_C} \right]}{\left[ 1 - \frac{T_G}{T_C} \right]} \cdot \quad (8.5)$$

When the effective pinning energy given in Eqn. (8.5) is substituted back in Eqn. (8.2), the expression for the temperature dependence of resistivity in the critical region associated with the vortex-liquid to vortex-glass transition is obtained as

$$\rho(H, T) = \rho_n \left| \frac{T(T_C - T_G)}{T_G(T_C - T)} - 1 \right|^s \quad (8.6)$$

The resistivity given in Eqn. (8.6) depends on the magnetic field explicitly through the field dependence of $T_G$. Eqn. (8.6) predicts a scaling behaviour between the normalized resistivity $\rho/\rho_n$ and the scaling temperature.
$T_s = \left[ T(T_C - T_G)/T_G(T_C - T) - 1 \right]$ in the critical region of the vortex-liquid to vortex-glass transition [247, 248]. Such a scaling behaviour between $\rho/\rho_n$ and $T_s$ has been found to exist in various disordered high-$T_C$ superconductors such as YBCO single crystals [247, 248], Ho-doped (Bi, Pb)-2212 [255], BaFe$_2$As$_2$ single crystal [256], Ba$_{0.55}$K$_{0.45}$Fe$_2$As$_2$ [262], and $\text{C}^{4+}$-irradiated BaFe$_{0.19}$Ni$_{0.11}$As$_2$ single crystal [263] etc., and this scaling behaviour has been used to estimate the critical exponent $s$ of the vortex-glass phase transition in these materials. Another important aspect of the modified vortex-glass model is that it provides consistent description of the effective pinning energy $U_0$ in the critical region of the vortex-glass transition. The effective pinning energy $U_0$ is important for the understanding of the flux-line pinning properties of a superconductor. We shall now use this modified vortex-glass model to study the vortex-liquid to Bose-glass transition in the annealed Ti$_{0.7}$V$_{0.3}$ sample.

8.2.3 Temperature and field dependence of effective pinning energy

In the modified vortex-glass model, temperature and field dependence of $U_0(H, T)$ may be obtained from Eqn. (8.2) as

$$U_0(H, T) = k_B T \left[ 1 - \left( \frac{\rho}{\rho_n} \right)^s \right],$$  \hspace{1cm} (8.7)

provided $s$ and $\rho_n$ are known. The values of $U_0(H, T)$ are estimated using Eqn. (8.7) for the annealed Ti$_{0.7}$V$_{0.3}$ sample. For the estimation of $U_0(H, T)$,
Figure 8.5: Temperature dependence of the effective pinning energy $U_0$ for the annealed Ti$_{0.7}$V$_{0.3}$ sample in different magnetic fields. (b) The field dependence of $U_H$ of the same sample, which is calculated using Eqn. (8.4). (c) The $U_0(H,T)/U_H$ versus $T$ curves showing the temperature dependent part of the pinning energy.
we have taken \( \rho_n \) as the resistivity value measured at 15 K, and we have used the values of \( s \) presented in Fig. 8.4 (a). The estimated values of \( U_0 \) in different magnetic fields are shown as a function of temperature in Fig. 8.5 (a). Only the data points within the critical region of the vortex-liquid to Bose-glass transition, i.e. in the temperature regime \( T_G \leq T \leq T^* \), are shown here for the sake of clarity. In this figure, it is observed that these \( U_0(H,T) \) curves exhibit linear temperature dependence. In the modified vortex-glass model, \( T_G \) value is determined from the point of intersection between the \( U_0(H,T) \) curve and the \( k_B T \)-line as indicated in Fig. 8.5 (a) [247, 248, 255, 256]. \( T_G \) values determined at various magnetic fields using this procedure match with those determined in the previous section. For \( H \geq 2 \) T, the straight lines obtained from the extrapolation of these linear \( U_0(H,T) \) curves meet at a common point \( T = T_C \), where \( U_0 \) becomes zero. This confirms that for \( H \geq 2 \) T, \( U_0 \) in the critical region does vary with temperature following \( (1 - T/T_C) \) dependence. However, for \( H < 1 \) T, we find that the straight lines obtained by the extrapolation of these linear \( U_0(H,T) \) curves meet at a common point \( T \sim 7.56 \) K on \( U_0(H,T) = 0 \) line. On the other hand, for \( H = 1 \) T, this extrapolated straight line meets \( U_0(H,T) = 0 \) line at a temperature slightly higher than \( T_C \) but lower than 7.56 K. These experimental facts indicate that for \( H < 2 \) T, \( U_0 \) in the critical region varies with temperature following a temperature dependence other than \( (1 - T/T_C) \) law.

In order to find out the exact temperature dependence of \( U_0 \) in the applied magnetic field regime \( H < 2 \) T, we compare the experimental \( U_0(H,T) \) data
for the present Ti$_{0.7}$V$_{0.3}$ sample with the more general expression of $U_0(H,T)$
given as $U_0(H,T) = U_H(1 - T/T_C)^n(H)$, where the values of $n$ depends on
the dimensionality of the flux-line system [264]. In the present case, we find
that $n$ is unity for $H \geq 2$ T while it assumes some values other than unity for
$H < 2$ T. Binomial expansion of the above form of effective pinning energy
gives $U_0(H,T) \sim U_H[1 - n(H)T/T_C] + D(T/T_C)$. Here, the term $D(T/T_C)$
represents the deviation from the linear behaviour of $U_0(H,T)$, which arises
from the higher order term in $(T/T_C)$. However, the observed linearity of
the $U_0(H,T)$ curves for $H < 2$ T suggests that the term $D(T/T_C)$ can be
considered to be negligible in the present case. Therefore, we may write
$U_0(H,T) \sim U_H[1 - n(H)T/T_C]$. Then the extrapolated $U_0(H,T)$ curves will
give a threshold at $T = T_C/n(H)$. $U_0(H,T)$ curves for $H < 1$ T give a
threshold at 7.56 K, which implies $n \sim 0.87$ in this magnetic field regime.

For $H \geq 2$ T, we can estimate the $U_H$ values in different magnetic fields
from the slope of the linear $U_0(H,T)$ curves (the slope being $-U_H/T_C$).
Alternatively, we can also estimate $U_H$ from Eqn. (8.4) using the exper-
imentally obtained value of $T_G$. We find that the $U_H$ values estimated using
both these procedures nicely agree with each other. On the other hand,
for $H < 2$ T, the slope of the linear $U_0(H,T)$ curves is $-nU_H/T_C$. Hence,
the values of $U_H$ for $H < 2$ T can be determined from the slope of the linear
$U_0(H,T)$ curve and the corresponding $n$ value. The alternative route for finding $U_H$ for $H < 2$ T is obtained from the fact that in the modified vortex-glass
model, the glass transition temperature $T_G$ is defined as $k_B T_G = U_0(H,T_G)$. 
Since \( U_0(H, T_G) = U_H(1 - T_G/T_C)^n(H) = k_B T_G \) for \( H < 2 \) T, we can write \( U_H = k_B T_G/(1 - T_G/T_C)^n(H) \). Therefore, the values of \( U_H \) at different magnetic fields below 2 T can be estimated from the corresponding values of \( T_G \) and \( n \) using the above relation. A good agreement is obtained between the values of \( U_H \) estimated using these two procedures, indicating that the values of \( n(H) \) obtained previously are quite reasonable.

In Fig. 8.5 (b), we present the magnetic field dependence of \( U_H \) in log-log scales covering the magnetic field range \( 0.1 \leq H \leq 5 \) T. In this figure, we observed that \( U_H(H) \) exhibits a power law relation \( U_H(H) \propto H^{-0.13} \) for \( H \leq 1 \) T. On the other hand, the field dependence of \( U_H \) can be roughly described by another power law relation \( U_H(H) \propto H^{-0.8} \) for \( H \geq 2 \) T. Having obtained the field dependence of \( U_H \) from our experimental data, we are now in a position to extract the temperature dependent part of \( U_0(H, T) \) as \( U_T(T) = U_0(H, T)/U_H \), and is shown in Fig. 8.5 (c). This figure shows a linear relationship in \( U_0(H, T)/U_H \) versus \( T \) curves. For \( H \geq 2 \) T, these curves fall onto the \( (1 - T/T_C) \)-line. On the other hand, the corresponding curves for \( H \leq 1 \) T fall onto the \( (1 - 0.87T/T_C) \)-line. These observations are consistent with our inferences made above.

### 8.2.4 Scaling of the vortex-liquid resistivity

We now look into the scaling behaviour of the \( \rho/\rho_n \) versus \( T_S \) curves for the annealed Ti\(_{0.7}\)V\(_{0.3}\) sample in the light of the above discussion. We have observed that, for \( H \geq 2 \) T, \( U_0(H, T) \) in the critical region of the vortex-
Figure 8.6: The log-log plots of normalized resistivity against the scaling temperature for the annealed Ti$_{0.7}$V$_{0.3}$ sample. For $H \geq 2$ T, we use the scaling temperature proposed in Ref. [247] while for low magnetic field regime we use the scaling temperature derived by us (see the relevant text).

Liquid to Bose-glass transition follows the empirical relation given in Eqn. (8.3). Consequently, the $\rho/\rho_n$ versus $T_S$ curves for $H \geq 2$ T overlap with each other in the critical region of Bose-glass transition, i.e. in the temperature regime $T_G \leq T \leq T^*$. This is shown in Fig. 8.6 (a). For $H < 2$ T, both the magnetic field and temperature dependence of $U_0(H,T)$ is different from that observed for $H \geq 2$ T. Consequently, the $\rho/\rho_n$ versus $T_S$ curves for $H < 2$ T do not scale with each other and also with the corresponding curves for $H \geq 2$. However, for $H < 1$ T, it is found that $U_0$ varies with temperature as $U_0(T) \propto (1-T/T_C)^n$, where $n \sim 0.87$. Moreover, in this magnetic field regime, the magnetic field dependent part of $U_0(H,T)$ can be obtained in term of $T_G$ as: $U_H = k_B T_G/(1-T_G/T_C)^n$. Hence, the overall effective pinning energy in
This magnetic field regime can be expressed as

\[ U_0(H, T) = U_H(H) \left[ 1 - \frac{T}{T_C} \right]^n = \frac{k_B T_G \left[ 1 - \frac{T}{T_C} \right]^n}{\left[ 1 - \frac{T_G}{T_C} \right]^n}. \]  

(8.8)

This relation for \( U_0(H, T) \) is evidently different from one given in Eqn. (8.5), and accordingly, the scaling relation between \( \rho/\rho_n \) and the scaling temperature will also be different for \( H < 1 \) T. Substituting Eqn. (8.8) in Eqn. (8.2), we obtain the temperature and magnetic field dependence of resistivity in the critical region of the Bose-glass transition as

\[ \rho(H, T) = \rho_n \left| \frac{T \left[ 1 - \frac{T_G}{T_C} \right]^n}{T_G \left[ 1 - \frac{T}{T_C} \right]^n} - 1 \right|^s. \]  

(8.9)

Eqn. (8.9) predicts a scaling behaviour between the normalized resistivity \( \rho/\rho_n \) and a modified scaling temperature \( T'_S = [T(T_C-T_G)^n/T_G(T_C-T)^n-1] \).

In agreement with this, a good scaling behaviour between \( \rho/\rho_n \) and \( T'_S \) is obtained for \( H < 1 \) T, and these scaled curves are shown in Fig. 8.6 (b). In the modified vortex-glass model, the critical exponent \( s \) is determined from the slope of these scaled curves. In this method, the value of \( s \) comes out to be \( \sim 1.7 \), which is close to the value obtained in Sec. 8.2. Hence, we have found that the scaling behaviour of the electrical resistivity in the critical region of vortex-liquid to Bose-glass transition remains valid in annealed Ti_{0.7}V_{0.3} sample. Since both temperature and magnetic field dependence of \( U_0(H, T) \)
in the annealed Ti$_{0.7}$V$_{0.3}$ sample are distinctly different in the magnetic field regimes below and above 2 T, the scaling behaviour of the resistivity are also found to be different in these magnetic field regimes.

8.2.5 Vortex matter phase diagram

The field-temperature ($H - T$) phase diagram for the annealed Ti$_{0.7}$V$_{0.3}$ sample is shown in Fig. 8.7. In this phase diagram, $T_G(H)$-, $T_C(H)$-, and $H_{Irr}(T)$-line respectively represent the glass transition line, the upper critical field line and the irreversibility field line. $T_G(H)$-line and $T_C(H)$-line [or equivalently $H_{C2}(T)$-line] are constructed from the $\rho(T)$ curves in various applied magnetic fields, and the procedures employed to determine $T_C$ and $T_G$ have been discussed above. The irreversibility field $H_{Irr}$ is determined from the field dependence of magnetically measured critical current density $J_C$ for this sample using a criterion that $J_C$ falls to zero (within the limit of experimental accuracy) at $H_{Irr}$. In this phase diagram, both $T_G(H)$-line and $H_{Irr}(T)$-line lie distinctly below the $T_C(H)$-line. Since both $T_G(H)$-line obtained from resistivity measurements and $H_{Irr}(T)$-line obtained from magnetic measurements represent the magnetic field limit up to which a superconductor can carry currents without dissipation, they should coincide with each other, and this is indeed observed for $H \geq 2$ T. However, for $H < 2$ T, $T_G(H)$-line splits strongly away from the $H_{Irr}(T)$-line towards lower temperatures. Consequently, there exists a finite magnetic field and temperature region between $T_G(H)$-line and $H_{Irr}(T)$-line, where finite role of flux-line
pinning still survives. We have also found that both temperature and field dependences of \( U_0 \) in this sample undergo a change of behaviour at almost the same magnetic field value where the splitting of the \( T_C(H) \)-line from the \( H_{Ir}(T) \)-line starts to occur, and thereby suggesting a common origin for all of these observed phenomena. In the following sections we will investigate for the probable origin leading to these observed phenomena.

![Field-temperature phase diagram](image)

Figure 8.7: The field-temperature \((H - T)\) phase diagram for the annealed Ti\(_{0.7}V_{0.3}\) sample

### 8.2.6 Crossover from individual flux-line pinning to collective pinning regime and its manifestations

The weak magnetic field dependence of \( U_0 \) observed in low magnetic fields suggests that individual pinning of the flux lines coexists with the collective flux-creep phenomenon in this magnetic field regime [265]. For higher applied magnetic fields, the flux lines are very large in number, and hence, the flux-line spacing becomes significantly smaller than the magnetic field
penetration depth. In such case, we expect, particularly in the high-\(\kappa\) materials like Ti-V alloys, a crossover to a new kind of pinning regime due to collective behaviour of the flux lines, where \(U_0\) becomes strongly dependent on magnetic field \([210]\). Qualitatively very similar change of behaviour of the field dependences of \(U_0\) is also observed in various superconductors such as BaFe\(_2\)As\(_2\) single crystal \([256]\), Fe\(_{1+y}\)(Te\(_{1+x}\)Se\(_x\)) \([266]\), Nd(O,F-)-FeAs single crystal \([267]\) superconductors etc. Due to the collective behaviour of the flux lines in higher magnetic field, the entire flux-line system will undergo a phase transition from Bose-glass to vortex-liquid at \(T_G\). Since flux-line pinning and hence \(J_C\) vanishes within the vortex-liquid phase, \(T_G(H)\)-line will be identical to \(H_{Irr}(T)\)-line, and this is indeed observed for \(H \geq 2 \text{ T}\).

However, in low magnetic field regime, the collective behaviour among the flux lines becomes weak. In such case, when the flux-line system undergoes a phase transition from Bose-glass to vortex-liquid phase, few flux lines may still be remained pinned at some stronger pinning sites available in the sample via the individual flux-line pinning mechanism, which is consistent with the observed slow variation of \(U_H(H)\) in this magnetic field regime. Hence, in low magnetic field regime, \(J_C\) may not vanish at \(T_G(H)\)-line due to the existence of individual pinning of few flux lines at temperatures above \(T_G\). However, the line tension of these pinned flux lines will eventually vanish at \(H_{Irr}(T)\)-line \([25]\), and thereby the pinned flux lines above the \(T_G(H)\)-line will be capable of escaping from the pinning sites to give rise to a zero pinning state of the sample at and above the \(H_{Irr}(T)\)-line. Hence, in the \(H - T\) phase...
diagram, $T_G(H)$-line will be located below the $H_{Ir}(T)$-line in low magnetic field regime. This is what we observe in Fig. 8.7. Based on this argument, we will now explain the change of behaviour of the temperature dependence of $U_0$ observed below and above a crossover over magnetic field $H_D \sim 2$ T.

In the case of the extended defect such as grain boundaries and martensitic $\alpha$ phase, the effective pinning energy $U_0$ arises from the loss of condensation energy within the generalized pinning volume, and is expressed as $U_0(T, H) \propto H_C(T)^2 \xi(T)2l_P$ [268]. Here, $H_C$ is the thermodynamic critical field, $\xi$ is the coherence length, and $l_P$ is the length of pinning sites along the direction of the applied magnetic field. Thus, the temperature dependence of $H_C$ and $\xi$ governs the temperature dependence of $U_0$. Since $\xi(T) \propto (1 - T/T_C)^{-1/2}$ and $H_C(T) \propto (1 - T/T_C)$ near $T_C$ [269], the temperature dependence of $U_0$ will be obtained as $U_0(T) \propto (1 - T/T_C)$, which is indeed observed experimentally for $H \geq 2$ T. In annealed Ti$_{0.7}$V$_{0.3}$ sample, different orientations and sizes of the extended disorders like grain boundaries and martensitic $\alpha$ phase lead to a distribution of $l_P$ and hence $U_0$ over the sample volume. However, in high magnetic field regime, the flux-line system can be described by a single value of $U_0$ rather than a distribution of $U_0$ because of the collective behaviour of the flux-line system. On the other hand, in low magnetic field, the single flux-line pinning occurring above $T_G(H)$-line at some mesoscopic strong pinning sites like $\alpha$ phase may give rise to Bose-glass islands having relatively higher $U_0$ and hence $T_G$ values within the interstitial vortex-liquid phase [270, 271]. The pinned flux lines within these
Bose-glass islands are then expected to be de-localized and thereby forming the vortex-liquid at a relatively higher temperature [270-272]. Consequently, in low magnetic field regime, when temperature is increased keeping the magnetic field constant, there are two independent ways which govern the temperature dependence of $U_0$. Firstly, $U_0$ decreases following $(1 - T/T_C)$ dependence which arises from the temperature dependence of the intrinsic superconducting parameters. Secondly, an increase in temperature in presence of a constant magnetic field involves the melting transitions of the Bose-glass islands having higher and higher $U_0$ values. As a result of this, the pre-factor $U_H$, which was presumed to be independent of temperature in high magnetic field regime, also evolves with temperature in low magnetic field regime. Such a temperature evolution of $U_H$ along with the usual $(1 - T/T_C)$ dependence gives rise to an overall slow variation of $U_0$ in low magnetic field regime than observed in high magnetic field regime. Hence, we suggest that in Ti$_{0.7}$V$_{0.3}$ sample, the co-existence of the Bose-glass islands within the interstitial vortex-glass slows down the dynamic of the flux lines in low magnetic field regime.

### 8.3 Summary and conclusions

In summary, we have investigated the resistive transition of Ti$_{0.7}$V$_{0.3}$ sample in presence of magnetic fields up to 5 T. We have found experimental evidences for the existence of a Bose-glass vortex state in the superconducting
mixed state of this sample. In the magnetic field regime $H \geq 2$ T, the vortex-liquid resistivity exhibits a scaling behaviour as predicted for vortex-glass and/or Bose-glass scenario, proving further evidence of a Bose-glass transition in the studied sample. However, the vortex-liquid resistivity does not follow the same scaling relation in the magnetic field regime $H < 2$ T. This is due to the fact that both the temperature as well as magnetic field dependencies of the effective pinning energy $U_0$ are distinctly different in the magnetic field regimes below and above 2 T. We have formulated a new scaling relation to describe the vortex-liquid resistivity for $H < 2$ T by taking into account the experimental temperature dependence of $U_0$ in this magnetic field regime. Another important result of this study is that although the Bose-glass transition line [$T_G(H)$-line] obtained from the resistive transitions overlaps with the magnetically measured irreversibility line [$H_{Irr}(T)$-line] for $H \geq 2$ T, they strongly split from each other for $H < 2$ T. We have suggested that all observed phenomena i.e. the change of behaviour of both the temperature and magnetic field dependencies of $U_0$ and the splitting of the $T_G(H)$-line from the $H_{Irr}(T)$-line, all occurring at a crossover field $H_D \sim 2$ T, are manifestations of the combined effects of the spatial variation of $U_0$ over the sample volume and the crossover in the pinning behaviour from individual to collective pinning regime with increasing magnetic field.