CHAPTER 3

STUDIES OF ELECTRON-ELECTRON INTERACTION WITHIN A BUNCH

3.1 Introduction

In an electron storage ring, circulating electrons are scattered and lost through collision with residual gas atoms present in the vacuum chamber. In addition, the electrons within a bunch collide with each other via transverse betatron oscillations and longitudinal synchrotron oscillations [25, 26]. As explained in section 1.8.3 of chapter 1, the beam loss due to electron-electron scattering within a bunch is due to Touschek effect, it was first explained by Bruno Touschek [41] after observations of the beam lifetime on Frascati storage ring ADA at Orsay, France. The Touschek effect is one of the limiting beam loss mechanisms in present day low emittance and high brilliance [52, 53] synchrotron radiation sources.

To understand the beam loss mechanism [42, 54] we consider the motion of electrons in a frame which moves with them. The betatron motion in this frame is purely transverse and a collision will transfer momentum into the longitudinal plane. Transforming back to the laboratory frame the transferred momentum is boosted by a factor $\gamma$. The process is shown in the Figure 3.1.

Fig.3.1. Coulomb scattering of two electrons in centre of mass and laboratory system
### 3.2 Expression of Touschek lifetime

The loss rate of electrons due to Touschek scattering \(1/\tau_{\text{tous}}\) between electrons within a bunch is given as [25, 42, 54]

\[
\frac{1}{\tau_{\text{tous}}} = -\frac{N}{p^2} \frac{r_0^2 c}{8 \pi \sigma_x \sigma_z \sigma_r} \left(\frac{\Delta p}{p}\right)^2 D(\xi)
\]

where

\[
D(\xi) = \xi^{1/2} \left[ -\frac{3}{2} \xi^{-\xi} + \frac{\xi}{2} \ln u \frac{e^{-u}}{u} du + \frac{1}{2} \left(3\xi - \xi \ln\xi + 2\right) \frac{e^{-u}}{u} du \right]
\]

\[
\xi = \left(\frac{\Delta p}{p} \frac{\beta_x}{\gamma \sigma_x}\right)^2
\]

The integral \(D(\xi)\) was evaluated numerically, the variation in \(D(\xi)\) with \(\xi\) is shown in Figure 3.2. The sign marked in Figure 3.2 is the value of \(D(\xi)\) for Indus-2.

![Graph](image)

**Fig.3.2. The variation of \(D(\xi)\) with \(\xi\)**

From Figure 3.5, we see that the function \(D(\xi)\) varies slowly with parameter \(\xi\). \(\tau_{\text{tous}}\) given by the relation 3.12 gives half lifetime \(\tau_{1/2}(s)\) due to Touschek scattering at one point \(s\) in the ring. So the overall Touschek lifetime is obtained by taking the average over the whole
circumference of ring $r_{1/2} = \langle r_{1/2}(s) \rangle$. To convert it to $r_{1/2}$ required for the estimation of total beam lifetime, the value of $r_{1/2}$ is divided by $\ln 2$.

3.3 Methods for the enhancement of Touschek lifetime

In a low emittance electron storage ring operating at an average vacuum pressure $1 \times 10^{-9}$ Torr, the beam lifetime is limited due to the electron-electron scattering within a high density electron bunch. As seen from relation 3.12 and 3.13, the Touschek lifetime is

a) Proportional to the cube of beam energy ($\tau_{\text{tous}} \propto \gamma^3$).

b) Proportional to the square of the momentum acceptance ($\tau_{\text{tous}} \propto (\Delta p/p)^2$).

c) Proportional to beam sizes in horizontal ($\sigma_x$), vertical ($\sigma_z$) and longitudinal ($\sigma_s$) plane ($\tau_{\text{tous}} \propto \sigma_x \sigma_z \sigma_s$).

d) Inversely proportional to the number of electrons in a bunch $N$ ($\tau_{\text{tous}} \propto (1/N)$).

To increase the Touschek lifetime, we have to increase the momentum acceptance. The momentum acceptance in transverse plane is optimised while designing the lattice and the RF acceptance is increased by increasing the RF cavity voltage. There are alternate methods for the enhancement of Touschek lifetime which are described in next section.

3.3.1 Vertical beam size in Indus-2

Another way to reduce the Touschek scattering is by increasing the vertical beam size because in an electron storage ring the beam is nearly flat. As we know, in a normal quadrupole, the force experienced by the electron displaced in horizontal plane is in horizontal direction and similarly for electron displaced in vertical plane, the force is in vertical direction. If there is rotational error in normal quadrupole along the longitudinal axis then the electron displaced in horizontal plane experience a force in vertical plane and for electron displaced in vertical plane experience a force in horizontal plane. So, due to the rotational error in normal quadrupoles about the beam direction, there is coupling between
horizontal and vertical motion and it give rise to vertical emittance in storage ring. The rotated qudrupoles in dispersion region \( (\eta_z \neq 0) \) will give rise to vertical dispersion and due to this vertical dispersion, there is also vertical emittance in ring. So, the vertical size of the electron beam in Indus-2 is mainly defined by two processes, first is betatron coupling between vertical and horizontal motion and second is due to the residual vertical dispersion function.

Main sources of vertical beam emittance [55-64] is

a) Betatron coupling due to rotation error in normal quadrupoles along beam axis and vertical closed orbit distortion at sextupole location

b) Residual Vertical dispersion (generated due to dipole rotation error, quadrupole transverse displacement, quadrupole rotation in dispersion region, sextupole transverse displacement)

### 3.3.1.1 Measurement of betatron coupling in Indus-2

In the presence of betatron coupling, the horizontal oscillatory motion of a beam can be transferred to the vertical motion, thereby increasing the vertical beam size. It is necessary to measure the degree of coupling in the storage ring.

Under the influence of linear coupling, the ratio of vertical beam emittance \( (\varepsilon_z) \) to the horizontal beam emittance \( (\varepsilon_x) \) known as coupling ratio is given by [65]

\[
\kappa = \frac{\varepsilon_z}{\varepsilon_x} = \frac{[\Delta \nu_{\text{min}}/\Delta I]^2}{2 + [\Delta \nu_{\text{min}}/\Delta I]^2} \quad 3.14
\]

where \( \Delta \nu_{\text{min}} \) is the minimum tune separation at coupling resonance and \( \Delta I = \nu_x - \nu_z - p \).

The coupling ratio \( \kappa \) is measured by driving the tunes across the coupling resonance. By changing current in one of the quadrupole power supplies, the betatron tunes are changed and when the tunes are close to the coupling resonance, the horizontal motion is transferred to the
vertical plane and vice versa which leads to a coupling between horizontal and vertical motion. In Indus-2, the strength of Q3D family of quadrupoles was changed gradually at the same rate and betatron tune in both horizontal and vertical plane was measured. Q3D family quadrupoles were chosen for measurement because it is in a non-dispersion region and the change in betatron tune in both planes with change in quadrupole current is less as compared to the other quadrupoles like Q1D and Q2F which are also in the non dispersion region. Figure 3.3 shows the measured horizontal and vertical betatron tunes as a function of the quadrupole current in Q3D family of quadrupoles [66].

![Fig.3.3. Measured horizontal and vertical fractional betatron tunes with change of current in Q3D family of quadrupoles](image)

From Figure 3.6, the value of minimum tune separation at coupling resonance is $\Delta \nu_{\text{min}} \sim 0.01$. From relation 3.14, the coupling ratio is 0.0049 (0.49%).

3.3.1.2 Measurement of vertical dispersion in Indus-2

By changing the RF frequency up to 8 kHz in steps of 2 kHz, change in vertical closed orbit at all beam position indicators was observed. If with frequency change $\Delta f_f$, there is change in orbit $\Delta z$ in vertical plane, then the vertical dispersion is given as $\eta_z = \Delta z / (\Delta p/p)$ where
change in momentum or energy is \[ \Delta p = \frac{1}{\alpha} \frac{\Delta f_{rf}}{f_{rf}} \] where \( \alpha \) is the momentum compaction factor and \( f_{rf} \) is the RF frequency and \( \Delta f_{rf} \) is the change in RF frequency.

Change in energy was estimated considering \( \alpha = 0.00732 \). The measured value of vertical dispersion at BPIs location with RF frequency change \( \Delta f_{rf} = 8 \text{ KHz} \) is shown in Figure 3.4.

![Figure 3.4](image)

**Fig.3.4.** Measured vertical dispersion at all BPI locations in Indus-2

The vertical beam size \( \sigma_z \) is given by

\[
\sigma_z(s) = \sqrt{\varepsilon \beta_z(s) + \eta_z^2(s) \left( \frac{\sigma_z}{E_0} \right)^2}
\]

### 3.3.1.3 Beam size measurement at beamline-16

The coupling measurement was complemented with the technique of beam size measurement at one of the dipole beamlines. The beam sizes were measured at micro focus X-ray fluorescence beamline (XRF-microprobe beamline-16) [67] which is connected at 5º port of one of the dipole magnets. The measured horizontal and vertical beam sizes are \( \sigma_x(\text{FWHM}) = 7.5 \mu m \) and \( \sigma_z(\text{FWHM}) = 4.3 \mu m \) respectively. These beam sizes were measured at ~17 m away from the source point in storage ring. After applying the demagnification factors of 118 in horizontal plane and 54 in vertical plane, the horizontal and vertical beam size at the source point are 885 \( \mu m \) and 232.2 \( \mu m \) respectively.
The electron beam sizes in horizontal and vertical planes at the source point are given by

$$\sigma_x = \sqrt{\varepsilon_x \beta_x + \eta_x^2 \left( \sigma_x / E_0 \right)^2}$$

$$\sigma_z = \sqrt{\varepsilon_z \beta_z + \eta_z^2 \left( \sigma_z / E_0 \right)^2}$$

For the operating lattice, the optical functions $\beta_x, \beta_z, \eta_x$ at the source point are 0.636m, 6.362m, 0.00123m respectively, for which $\eta_x^2 \left( \sigma_x / E_0 \right)^2 << \varepsilon_x \beta_x$ and measured value of vertical dispersion at source is ~1.2cm, so $\eta_z^2 \left( \sigma_z / E_0 \right)^2 \approx 7 \times 10^{-11}$ is very small. It leads to simple formula for the coupling ratio $\kappa$ as

$$\kappa = \frac{\varepsilon_z \beta_z \sigma_z^2}{\varepsilon_x \beta_x \sigma_x^2} \quad 3.15$$

Using the measured beam sizes and the lattice function in relation 3.15, the coupling ratio comes out to be 0.0066 (0.66%). This value is very close to the coupling ratio obtained using minimum tune separation method. So by increasing betatron coupling, Touschek lifetime in an electron storage ring can be increased.

3.3.2 RF phase modulation

Touschek lifetime can be enhanced by decreasing the density of electron bunches. This can be done by applying RF phase modulation in main RF which has been applied in other synchrotron radiation sources such as KEK photon factory [68, 69], ASTRID [70], LNLS [71], PLS [72], TLS [73] ring for the improvement in Touschek lifetime. A theoretical simulation study of the application of RF phase modulation of nearly one and two times of synchrotron oscillation frequency in main RF of Indus-2 ring at beam energy 2.5 GeV was carried out.

3.3.2.1 Longitudinal beam dynamics under RF Phase modulation

Considering the longitudinal motion of a single electron under RF phase modulation and neglecting the beam loading effect, let $\tau$ be the longitudinal time advance of an electron
away from the position of synchronous particle and $\delta$ be the relative energy deviation from that of the synchronous particle, the equations of motion [68] for electron are given as

$$\frac{d\tau}{dt} = -\alpha_e \delta$$

$$\frac{d\delta}{dt} = e V_{rf} \cos(\phi_i - \omega \tau + \phi_m) \frac{U_0}{T_0 E_0} - 2\gamma_e \delta$$

where $\alpha_e$ is the momentum compaction factor, $e$ is the electron charge, $V_{rf}$ is the RF cavity peak voltage, $\phi_i$ is the synchronous phase, $\omega$ is the angular RF frequency, $U_0$ is the synchrotron radiation loss per turn for the synchronous particle, $T_0$ is the revolution time, $E_0$ is the energy of the synchronous particle, $\gamma_e$ is the radiation damping rate in longitudinal plane and $\phi_m$ is the phase modulation. The simulation is carried out with RF phase modulated at the one and twice of the synchrotron oscillation frequency $\omega_s$.

$$\phi_m = \phi_{m0} \cos(\omega_s t) \text{ and } \phi_m = \phi_{m0} \cos(2\omega_s t)$$

where $\phi_{m0}$ is the modulation amplitude and $\omega_s$ is the unperturbed synchrotron frequency which is given as

$$\omega_s = \sqrt{\frac{\alpha_e \omega e V_{rf} \sin \phi_i}{T_0 E_0}}$$

3.3.2.2 Criterion for RF phase modulation

In order that the cavity field can follow any modulation in the input RF wave, the bandwidth of the cavity which is the ratio of cavity resonant frequency to the loaded quality factor should be comparable to or wider than the modulation frequency. In Indus-2, cavity bandwidth is about $\sim$50 kHz, so the cavity can follow the phase modulation up to about this frequency with certain amplitude and phase response due to the cavity impedance.
3.3.2.3 Effect of the RF phase modulation on distribution of electrons in a bunch

A simulation study of the effect of RF phase modulation on the distribution of electrons in a bunch was carried out using particle tracking code ELEGENT [32]. The particle tracking equation is given as

\[
\tau_{n+1} = \tau_n - \alpha \delta_n T_0
\]

\[
\delta_{n+1} = (1 - 2\gamma_s T_0)\delta_n + \frac{\left[ eV_{\text{rf}} \cos(\phi_n - \omega \tau_n + \phi_0) - U_0 \right]}{E_0}
\]

where \( n \) is the number of turns of electron motion

Particle tracking considering 5000 electrons in a bunch was carried out in longitudinal plane for 10,000 turns with nearly one and two times of synchrotron frequency and with different modulation amplitude. With applied RF modulation frequency of 20.5 kHz, 41 kHz and modulation amplitude of 3˚, the longitudinal phase space of electrons at the start of tracking i.e. 0 turn, after 1000 turns and 10000 turns are shown in Figure 3.5(a)-(c) and 3.6(a)-(c) respectively, where \( \gamma \) on y-axis is the relativistic factor. The tracking results show that by applying the RF phase modulation of nearly two times of synchrotron oscillation frequency, the distribution of electrons in a bunch in phase space changes. In low emittance storage ring, the density of electrons in a Gaussian bunch is higher at the centre. Due to high density of electron at the bunch centre, there is a large scattering which causes loss of electrons and decrease in beam lifetime. As seen in Figure 3.6(c), the density of the electrons at the centre of the bunch become lesser and the distribution is divided in two parts by applying RF phase modulation so there is less scattering. The electrons execute two states of stable oscillations, the phases of which are opposite to each other, there arises a quadrupole mode longitudinal oscillation of the electrons and it leads to increase in bunch length \[68, 74\] which causes the increase in Touschek lifetime. The studies show that it is not happening if we apply phase modulation of frequency near to synchrotron frequency as shown in Figure 3.5(c).
Modulation with synchrotron frequency

Modulation with two times of synchrotron frequency

Fig. 3.5(a). Electron distribution at the start of tracking

Fig. 3.5(b). Electron distribution after 1000 turns

Fig. 3.5(c). Electron distribution after 10,000 turns

Fig. 3.6(a). Electron distribution at the start of tracking

Fig. 3.6(b). Electron distribution after 1000 turns

Fig. 3.6(c). Electron distribution after 10,000 turns
3.3.2.4 Effect of RF phase modulation on longitudinal beam parameters

To study the effect of RF phase modulation on bunch length $\sigma_x$ and energy spread $\sigma_\delta$, particle tracking of 5000 electrons in a bunch was carried out for 10,000 turns. The tracking results are shown in Figure 3.7(a, b) and 3.8(a, b) respectively.

The bunch length and energy spread in Indus-2 at beam energy 2.5 GeV and cavity voltage 1200 kV without RF phase modulation are 1.53 cm and $9\times10^{-4}$ respectively. The tracking results show that by applying RF phase modulation of nearly two times of synchrotron frequency, the bunch length is increased on an average from 1.53 cm to 2.0 cm and increase in energy spread is from $9\times10^{-4}$ to $1.1\times10^{-3}$. 

![Figure 3.7(a). Bunch length variation](image)

![Figure 3.7(b). Energy spread variation](image)

![Figure 3.8(a). Bunch length variation](image)

![Figure 3.8(b). Energy spread variation](image)
Touschek lifetime is proportional to the bunch length. The increase in bunch length by the application of RF phase modulation [74] gives rise to increase in Touschek lifetime. If the beam lifetime is Touschek limited then by the application of RF phase modulation, the overall beam lifetime will increase.

3.3.2.5 Implementation of RF phase modulation in Indus-2

A schematic diagram of RF phase modulation implemented in Indus-2 ring is shown in Figure 3.9. An experiment was conducted to study the effect of RF phase modulation in present operating condition. The applied RF cavity peak voltage was ~1200 kV. Beam spectrum was observed on a spectrum analyzer and a signal of main RF was observed at operating frequency ~505.812 MHz. The estimated synchrotron frequency at RF voltage 1200 kV is 20.5 kHz.

RF phase modulation of frequency ~41 kHz was made ON with 0° modulation amplitude. No signal of modulation frequency was observed on beam spectrum. By increasing the modulation amplitude, the modulated frequency signal was seen at a separation of ~41 kHz from the main RF signal. As the modulation amplitude was increased, the modulated signal amplitude on beam spectrum also increased. The beam spectrum without applying RF phase modulation and for RF modulated frequency ~41 kHz and modulation amplitude 3° is shown in Figure 3.10(a) and (b) respectively.
Fig. 3.9. RF phase modulation in Indus-2, AFG is arbitrary frequency generator, $f_s$ is synchrotron frequency, LLRF is Low Level RF system, SSA is solid state amplifier.

The stored beam current in the ring at the time of experiment was ~100 mA at beam energy 2.5 GeV. From the beam spectrum showing the phase modulated signal, we found that the beam has undergone the RF phase modulation. The beam lifetime at stored current 100 mA before and after applying the phase modulation was observed and found nearly same. It shows that the beam lifetime is not Touschek lifetime limited in Indus-2.