Chapter 7

Quantification of Web Quality Model using Graph Theory

Chapter Objectives

This chapter aims to develop digraph of factors of developed web quality model showing interactions with each other. The interaction among website quality factors is represented through digraph, matrix and a multinomial in order to quantify the proposed web quality model.

7.1 Introduction

The purpose of developed web quality model is to showcase the quality of a website based on defined factors within the scope of model. But defining web quality of a website may not provide the expected result as qualitative nature of the constituent factors makes it difficult to point out the strong and the weak factor within the web quality model. Whenever the web quality model shall be applied to a website, ascertaining the desired effect of each factor will be difficult to measure. The need of converting the existing qualitative web model into quantitative web model arises to identify the weak links in the web model when applied.

The above said problem of quantifying the web quality model can be easily solved using graph theory and matrix methods. Graph theory is the method of converting the effect of factors onto each other into a digraph. This digraph can then be converted into a matrix. The matrix also known as permanent is solved just like determinant except that only positive signs are used to avoid loss of any data.

The key website quality factors affecting the overall website quality has been identified in web quality model. The effect of interaction of these factors among themselves and the resulting overall effect help attain a better managed website. This chapter attempts to represent the overall effect of key website quality factors quantitatively by developing a mathematical model using graph theoretic approach. In this approach, interaction among identified key website quality factors is represented through digraph, matrix model and a multinomial. The extent of key website quality factors affecting a website, representing its popularity among users and indicating total quality concept in website management is represented in terms of the "website quality index". It provides an insight into the website quality factors at system and subsystem level.
Figure 7.1 Cause Effect Diagram for web quality system
7.2 Development of graph theoretic model

Graph theoretic and matrix model consists of digraph representation, matrix representation and permanent representation. Digraph representation is useful for visual analysis. Matrix model is useful for computer processing. Permanent multinomial function characterizes abstract web quality uniquely. Permanent value of multinomial represents the effect of factors on web quality uniquely by a single number/index, which is useful for comparison, ranking and optimum selection. We have tried to enlist some of the fields in which graph theory has been applied successfully in Table 7.1. The systematic application of graph theoretic methodology is discussed further in this thesis.

Table 7.1 List of some of the field in which graph theory has been employed

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Field</th>
<th>Paper Title</th>
<th>Reference (year)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Reconfigurable Manufacturing System</td>
<td>Graph theory-based approach to optimize single-product flow-line configurations of RMS</td>
<td>Dou et al. (2009)</td>
</tr>
<tr>
<td>2.</td>
<td>Biochemical</td>
<td>Graph models and mathematical programming in biochemical network analysis and metabolic engineering design</td>
<td>Lanzeni et al. (2008)</td>
</tr>
<tr>
<td>4.</td>
<td>Supply Chain</td>
<td>Quantification of risk mitigation environment of supply chains using graph theory and matrix methods</td>
<td>Faisal et al. (2007)</td>
</tr>
<tr>
<td>6.</td>
<td>Power Plants</td>
<td>Selection of power plants by evaluation and comparison using graph theoretical methodology</td>
<td>Garg et al. (2006)</td>
</tr>
<tr>
<td></td>
<td>Title</td>
<td>Description</td>
<td>Author(s)</td>
</tr>
<tr>
<td>---</td>
<td>----------------------------------------------------------------------</td>
<td>-------------------------------------------------------------------------------------------------------</td>
<td>-----------------------------------------------</td>
</tr>
<tr>
<td>8</td>
<td>Human Resource</td>
<td>Human resource performance index in TQM environment</td>
<td>Grover et al. (2005)</td>
</tr>
<tr>
<td>11</td>
<td>FMEA</td>
<td>FMEA - A digraph and matrix approach</td>
<td>Gandhi and Agrawal (1992)</td>
</tr>
</tbody>
</table>

### 7.3 Website Quality System Model digraph

A digraph is used to represent the governing attributes and their interdependencies within the system. A Website quality system digraph represents the qualitative measure of the characteristics or the attributes $w_i$ through its nodes and edges related to the interdependencies of the attributes $w_{ij}$. A qualitative website quality system attributes digraph models the quality attributes of the web systems and their relative importance. The digraph consists of a set of nodes $w_i$, where $i=1,2,3,\ldots, M$ and a set of directed edges $w_{ij}$. A node $w_i$ represents the $i$th qualitative attribute and the edges represent the relative importance between such quality attributes of the website quality system. The number of nodes in the digraph represents the total number of quality attributes considered for the qualitative evaluation of the website quality systems. In the present digraph method, if a node `$i$' exhibits relative importance over node `$j$' during the qualitative evaluation of the website quality system, then a directed edge is represented from node `$i$' to node `$j$' (i.e. $w_{ij}$). If a node `$j$' exhibits relative importance over node `$i$' then a directed edge is drawn from node `$j$' to node `$i$' (i.e. $w_{ji}$).

In the figure 7.2, $w_1$ node representing ‘quality attributes’ exhibits relative importance over $w_2$ representing ‘website features’ and vice versa, hence the directed edge from
each other. Similarly, the directed edge is drawn between each website quality dimensions where \( w_1 \) represents ‘quality attributes’, \( w_2 \) represents ‘website features’, \( w_3 \) represents ‘lifecycle processes’ and \( w_4 \) represents ‘cognition factors’.

![Figure 7.2 Digraph of website quality system dimensions](image)

### 7.4 Website quality system matrix

Matrix representation of the Website quality system's digraph presents a one-to-one representation. A matrix called website quality matrix is defined. This matrix is an MxM matrix and considers all the attributes \( (w_i) \) and their relative importance (i.e: \( w_{ij} \)) with respect to each other for the website quality system. Consider a general case of M factors leading to M order symmetric matrix \( A = [w_{ij}] \) where \( w_{ij} \) represents the interaction of i th factor with the j th factor. The value of \( w_{ij} \) is 1 if factor i is connected to factor j else 0. The matrix of the website quality system digraph is represented by equation (7.1).

\[
A = \begin{bmatrix}
1 & 2 & 3 & 4 \\
0 & 1 & 1 & 1 \\
1 & 0 & 1 & 1 \\
1 & 1 & 0 & 1 \\
1 & 1 & 1 & 0 \\
\end{bmatrix}
\]

(7.1)

Off diagonal elements in above matrix represent the interdependency of website quality factors with value 0 or 1. The diagonal elements are 0 since effect of website quality factors is not taken into consideration. Another matrix is defined as website quality characteristic matrix to consider this effect.

### 7.5 Website quality characteristic matrix (CM-W)

The characteristic matrix already used in mathematics is used to characterize factors affecting website quality. Let I be the identity matrix and W the variable representing
website quality factors. Then website quality characteristic matrix is represented as $C = [W - A]$.

$$
C = \begin{bmatrix}
W & -1 & -1 & -1 \\
-1 & W & -1 & -1 \\
-1 & -1 & W & -1 \\
-1 & -1 & -1 & W
\end{bmatrix}
$$

(7.2)

The value of all diagonal elements is same in the above matrix. It simply means that factors affecting website quality have been assigned same value which is not true practically, since all factors have different values (effects) depending on various parameters affecting them.

Moreover, interdependencies have been assigned value depending whether it is there or not. To consider the effect of factors and their interdependencies, another matrix, website quality variable characteristic matrix (VCM) is considered.

### 7.6 Website quality variable characteristic matrix (VCM-W)

It is proposed to characterize the website quality system by various factors and their effects through VCM. For this let us consider digraph in Figure 7.2 for defining VCM-W. Consider a matrix $D$ with off-diagonal elements $w_{ij}$ representing interactions between quality factors, i.e. instead of 1 (as in matrix 1).

Consider another matrix $E$ with diagonal elements $W_i$, $i = 1, 2, 3, 4$ where $W_i$ represent quality effect of various factor, i.e. instead of $W$ only (as in matrix 7.2).

Considering matrices $D$ and $E$, the VCM-W is written as $H = [E - D]$.

$$
H = \begin{bmatrix}
W_1 & -w_{1,2} & -w_{1,3} & -w_{1,4} \\
-w_{2,1} & W_2 & -w_{2,3} & -w_{2,4} \\
-w_{3,1} & -w_{3,2} & W_3 & -w_{3,4} \\
-w_{4,1} & -w_{4,2} & -w_{4,3} & W_4
\end{bmatrix}
$$

(7.3)

The matrix provides a powerful tool through its determinant called variable characteristic quality multinomial (VCQM). This is a characteristic of the system and represents the overall quality effect of the system consisting of quality effect of factors and their interactions.

Determinant of matrix equation (7.3), i.e. VCQM carries positive and negative signs with some of its coefficient. Hence, complete information on quality effect will not be
obtained as some will be lost due to addition and subtraction of numerical values of diagonal and off diagonal elements (i.e. \( W_i \) and \( w_{ij} \)). Thus, the determinant of VCM-W, i.e. matrix equation (7.3) does not provide complete information concerning website quality effect. For this, another matrix, website quality variable permanent matrix (VPM-W) is introduced.

**7.7 Website quality variable permanent matrix (VPM-W)**

Overall website quality effect is maximum when the quality effect of individual factors is maximum. Since, total quantitative information is not obtained in VCM-W, VPM-W is defined for the system in general (assuming interactions among all factors) as \( W = E + D \).

\[
W = \begin{bmatrix}
1 & 2 & 3 & 4 \\
W_1 & W_{1,2} & W_{1,3} & W_{1,4} \\
W_{2,1} & W_2 & W_{2,3} & W_{2,4} \\
W_{3,1} & W_{3,2} & W_3 & W_{3,4} \\
W_{4,1} & W_{4,2} & W_{4,3} & W_4
\end{bmatrix}
\]

Where \( E \) and \( D \) have meaning as in matrix equation (7.3).

The permanent of this matrix \( 'W' \) i.e. the \( 'Per W' \), is defined as the qualitative function of the website quality system. The \( 'Per W' \) is a standard matrix function and is generally used in combinatorial mathematics. It is calculated in the same manner as the determinant, but all negative terms obtained after expansion for the calculation of the determinant of the matrix are replaced with positive equivalent terms. This computation results in a monomial every term of which has a significance related to the overall evaluation of the website quality system function and no loss of significant term is observed during this kind of computation. The website quality system function is represented as VPF-W (Variable Permanent Function - Website):

\[
VPF - WQE = Per(W) = \prod_i W_i + \sum_i \sum_j \sum_k \sum_l \left( w_{ij} w_{jk} W_{kl} + w_{ik} w_{jk} w_{jl} \right) W_i W_j + \sum_i \sum_j \sum_k \sum_l \left( w_{ij} w_{jk} \left( w_{kl} W_{il} \right) \right)
\]

\[
\left. + \sum_i \sum_j \sum_k \sum_l \left( w_{ij} w_{jk} w_{kl} W_{il} + w_{ij} w_{jk} w_{ik} w_{jl} \right) \right)
\]

\[
(7.5)
\]
The equation 7.5 is the complete expression in the form of terms arranged in 3 groupings for the quality evaluation of the website system, as it considers the presence of all the attributes and all the possible relative importance between the attributes. The terms are the sets of distinct diagonal elements ($W_i$) and the loops of off-diagonal elements of different sizes. In the permanent 'Per W', various groupings have their own physical significance.

Each term serves as a test for the effectiveness of the relevant group in Per (W).

Equation 7.5 contains terms arranged in 5 groups. The physical significance of various grouping is explained as follows.

- The first term (grouping) represents a set of 4 unconnected elements, i.e. $W_1$, $W_2$, $W_3$, and $W_4$.
- The second grouping is absent in the absence of self-loops.
- Each term of the third grouping represents a set of two-element WQE loops (i.e. $w_{ij}w_{ji}$) and is the resultant WQE dependence of characteristics i and j and the WQE measure of the remaining 3 unconnected elements.
- Each term of the fourth grouping represents a set of three-element WQE loops ($w_{ij}w_{jk}w_{ki}$ or its pair $w_{ik}w_{kj}w_{ji}$) and the WQE measure of the remaining 2 unconnected elements.
- The fifth grouping contains two subgroups. The terms of the first sub-group consist of two-element WQE loops (i.e. $w_{ij}w_{ji}$ and $w_{kl}w_{lk}$). The terms of the second sub-group are a product of four-element WQE loops (i.e. $w_{ij}w_{jk}w_{kl}w_{li}$) or its pair (i.e. $w_{il}w_{lk}w_{kj}w_{ji}$).

### 7.8 Quantification of $W_i$ and $w_{ij}$

The website quality system index is a measure of the factors that affect the quality of the constituent subsystems. The website quality system function as defined in equation 7.5 is used for the evaluation of the website quality system index as it contains the presence of all the factors and their relative importance. The numerical value of the website quality system function is called the website quality system index. As the website quality system function mentioned above contains only the positive terms, higher the value of inheritance level ($W_i$) and/or the interdependency
(i.e. \( w_{ij} \)), higher will be the value of the website quality system function. In order to calculate, this index, the detailed information about ‘\( W_i \)’ and ‘\( w_{ij} \)’ is required. The diagonal elements in VPM-WQE (equation 7.5) representing interactions of dimensions of website quality in web environment need to be assigned its importance weight. The quality dimensions identified may not be equally important to achieve website quality in web environment. Hence a suitable scale may be used to assign weights to each dimension. A scale has been suggested in Table 7.2 for the purpose. Similarly each factor identified within each dimension may be assigned weights as suggested in Table 7.3.

**Table 7.2 Inheritance of web quality environment variables**

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Qualitative measure of web quality factor</th>
<th>Assigned value of web quality factor (( W_i ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Exceptionally low</td>
<td>0.1</td>
</tr>
<tr>
<td>2</td>
<td>Very low</td>
<td>0.2</td>
</tr>
<tr>
<td>3</td>
<td>Low</td>
<td>0.3</td>
</tr>
<tr>
<td>4</td>
<td>Below average</td>
<td>0.4</td>
</tr>
<tr>
<td>5</td>
<td>Average</td>
<td>0.5</td>
</tr>
<tr>
<td>6</td>
<td>Above average</td>
<td>0.6</td>
</tr>
<tr>
<td>7</td>
<td>High</td>
<td>0.7</td>
</tr>
<tr>
<td>8</td>
<td>Very high</td>
<td>0.8</td>
</tr>
<tr>
<td>9</td>
<td>Exceptionally high</td>
<td>0.9</td>
</tr>
</tbody>
</table>

**Table 7.3 Interdependence of web quality environment variables**

<table>
<thead>
<tr>
<th>S. No</th>
<th>Qualitative measure of interdependencies</th>
<th>Assigned value of interdependency (( w_{ij} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Very strong</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>Strong</td>
<td>0.4</td>
</tr>
<tr>
<td>3</td>
<td>Medium</td>
<td>0.3</td>
</tr>
<tr>
<td>4</td>
<td>Weak</td>
<td>0.2</td>
</tr>
<tr>
<td>5</td>
<td>Very weak</td>
<td>0.1</td>
</tr>
</tbody>
</table>

It may be mentioned that one may choose any scale, e.g., 0 to 1, 0 to 5, 1 to 5, 0 to 10, 1 to 10, 1 to 11, 0 to 50, 0 to 100, 1 to 100, 1 to 110, 0 to 1000, 1 to 1000, or any other scale for ‘\( W_i \)’ and ‘\( w_{ij} \)’. But the final ranking will not change, as these are relative values. It is, however, desirable to choose a lower scale for ‘\( W_i \)’ and ‘\( w_{ij} \)’ to obtain a manageable value of website quality index. It may be further mentioned that the scales adapted for ‘\( W_i \)’ and ‘\( w_{ij} \)’ can be independent of each other. Whenever the website quality index is calculated, only the diagonal elements will change, i.e., (\( W_i \)), and the off-diagonal elements (\( w_{ij} \)) remain the same.
The website quality index for each type of available website system may be evaluated by using equation 7.5 by substituting the values of $W_i$ and $w_{ij}$. The website quality systems may be arranged in the ascending order or descending order of the evaluated website quality index. The website system which has highest quality index is considered to be the best choice as per the qualitative standards.

**7.9 Website Quality Index**

The quantification of website quality factors is very difficult to calculate under actual interactive conditions as the overall website quality is a function of inheritance of quality factors and their interdependencies. All possible combination of factors in equation 7.5 represents different states of website quality. Variable permanent function (equation 7.5), which thus represents structural complexity, effect of characteristics and their interdependence, is a useful tool for developing an index for website quality. This is the permanent of VPM-W, which is given by equation 7.5. The numerical values of various factors and dependencies required for W can be determined using the procedure already explained. The features of website quality index are highlighted below:

This index is a single numerical value and a means to evaluate the content of factors needed for an environment in an organization conducive to website quality.

A higher value of index is an indicator of more conducive environment to website quality.

The value may be used for self-analysis of website of an organization and by this procedure the permanent value can be increased by varying (increasing in particular) the quality factors identified.

**7.10 Methodology**

The graph theoretic approach evaluates the website quality in terms of a single numerical index. This takes into consideration the inheritance effect of factors and their interdependencies. The various steps in the proposed approach are presented here, which will help in evaluation process of the website quality.

1. Identify various factors that affect website quality. Different websites may have a different set of factors affecting website quality depending on the type of website (e-commerce or social networking or blog etc).
2. Broadly group these factors (four dimensions are framed in this study as shown in figure 7.1). For the application of this methodology the factors are written in composite form to avoid mathematical complexity in the further analysis.

3. Logically develop a digraph between the factors depending on their interdependencies (similar to figure 7.2). The nodes in the digraph represent factors while edges represent interaction among factors.

4. Identify the factors affecting website quality from factors identified in step (1).

5. Identify the sub-factors affecting factors identified in step (4).

6. Develop a factor digraph considering inheritances and interactions among one of the group of factors. The nodes in the digraph represent factors while edges represent interaction among factors.

7. Develop factor matrix with diagonal elements representing inheritances and the off diagonal elements representing interactions among them (equation 7.4).

8. At the sub-system level use Tables 7.2 and 7.3. This will provide numerical values for inheritance of attributes and their interactions with the help of experts.

9. Find the value of permanent function for factor (equation 7.5).

10. Repeat steps (6)-(9) for each factor.

11. Similarly, repeat steps (6)-(9) for each sub-factor.

12. Develop quality factor digraph and quality matrix at system level as explained in steps (3) and (7).

13. At system level, the permanent value of each factor (obtained in step 9) provides inheritance of website quality dimension (i.e. diagonal elements in equation 7.4). The quantitative value of interactions among dimensions (i.e. off diagonal elements in equation 7.4) is obtained from Table 7.3 through proper interpretation by experts. This will form quality matrix at system level similar to equation (4).

14. Find the value of permanent function for the system (equation 7.5). This is the value of the website quality index.

The quality of a website can thus be evaluated based on the above discussed methodology.

**7.11 Example**

To demonstrate the proposed methodology, website of an organization is taken as an example. It is proposed to find the value of website quality index. For determining the index we require numerical values of all quality factors and their interdependencies, i.e. all values in website quality variable permanent matrix.
(equation 7.4). The value of diagonal elements in the VPM-W, i.e. the value of quality factors $W_1; W_2; W_3; W_4$ are evaluated by applying graph theoretic methodology.

Step by step methodology discussed in previous section is used to evaluate website quality index in this example.

Step 1: The various factors affecting website quality of an organization are identified in Figure 7.1.

Step 2: The factors and sub-factors affecting the website quality as discussed earlier in chapter 4 and are listed in Table 7.4.

Table 7.4: Degree of influence of quality attributes $j$ on $i$

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Dimension/Factors</th>
<th>Factors/Sub-Factors</th>
<th>Degree of influence of web quality dimension / factor $j$ on $i$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>very strong ($w_{ij} = .5$) Strong ($w_{ij} = .4$) Medium ($w_{ij} = .3$) Weak ($w_{ij} = .2$) very weak ($w_{ij} = .1$)</td>
</tr>
<tr>
<td>Level 1: Dimensions of web quality model</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Quality Attributes</td>
<td>-Efficiency -Functionality -Maintainability -Portability -Reliability -Usability</td>
<td>2, 3, 4</td>
</tr>
<tr>
<td>2</td>
<td>Website Features</td>
<td>-Content -Navigation -Presentation</td>
<td>1, 3, 4</td>
</tr>
<tr>
<td>3</td>
<td>Lifecycle Processes</td>
<td>-Development -Maintenance -Service</td>
<td>2, 4 1</td>
</tr>
<tr>
<td>4</td>
<td>Cognition Factors</td>
<td>-Assurance -Proximity</td>
<td>1, 2, 3</td>
</tr>
<tr>
<td>Level 2: Factors of Quality Attributes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Efficiency</td>
<td>-Download time -Timeliness</td>
<td>5 2, 6 3 4</td>
</tr>
<tr>
<td>2</td>
<td>Functionality</td>
<td>-Accuracy -Interoperability -Security -Suitability</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>Maintainability</td>
<td>-Manageability -Reusability -Stability</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>Portability</td>
<td>-Adaptability -Installability</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>Reliability</td>
<td>-Availability -Fault tolerance -Maturity -Recoverability</td>
<td>1 2 3, 4, 6</td>
</tr>
<tr>
<td></td>
<td>Usability</td>
<td>2</td>
<td></td>
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<td>Understandability</td>
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<tr>
<td></td>
<td>User friendliness</td>
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**Level 2: Factors of Website Features**

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<tr>
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<th>Content</th>
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<tbody>
<tr>
<td>1</td>
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<tr>
<td></td>
<td>Search Capability</td>
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<tr>
<td></td>
<td>Multi Language Support</td>
<td></td>
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<tr>
<td></td>
<td>Completeness</td>
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<tr>
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<td>Trustworthy</td>
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<tr>
<td></td>
<td>Comprehensibility</td>
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<thead>
<tr>
<th></th>
<th>Navigation</th>
<th>1, 3</th>
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<tbody>
<tr>
<td>2</td>
<td>Menus</td>
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<tr>
<td></td>
<td>Consistency</td>
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<tr>
<td></td>
<td>Quick Access Pages</td>
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<td></td>
<td>Site Map</td>
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<table>
<thead>
<tr>
<th></th>
<th>Presentation</th>
<th>1 2</th>
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<tbody>
<tr>
<td>3</td>
<td>Browser related Deficiencies</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Graphics Clarity</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Multimedia</td>
<td></td>
</tr>
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<td></td>
<td>Web Page Design</td>
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**Level 2: Factors of Lifecycle Processes**

<table>
<thead>
<tr>
<th></th>
<th>Development</th>
<th>3</th>
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<tbody>
<tr>
<td>1</td>
<td>Object Oriented Programming</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Web Page control Structure</td>
<td></td>
</tr>
</tbody>
</table>

<table>
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<tr>
<th></th>
<th>Maintenance</th>
<th>1 1</th>
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</thead>
<tbody>
<tr>
<td>2</td>
<td>Code Reusability</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Interlinking Effort</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Variability</td>
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</tr>
</tbody>
</table>

<table>
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<tr>
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<tr>
<td>3</td>
<td>Collaboration</td>
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<td>Communication</td>
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<td>Responsiveness</td>
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<tr>
<td></td>
<td>Support</td>
<td></td>
</tr>
</tbody>
</table>

**Level 2: Factors of Cognition Factors**

<table>
<thead>
<tr>
<th></th>
<th>Perception</th>
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</thead>
<tbody>
<tr>
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<td>Empathy</td>
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<tr>
<td></td>
<td>Intimacy</td>
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</table>

<table>
<thead>
<tr>
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<th>Reasoning</th>
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<td></td>
<td>Proximity</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Uniqueness</td>
<td></td>
</tr>
</tbody>
</table>

Step 3: The dependencies of factors at subsystem level are visualized through digraphs shown in Figures 7.3-7.20. Subscript denotes the subsystem while
superscript denotes the factors affecting the subsystem. The subscript starting with `s' represents subsystem and subscript starting with `ss' represents sub-sub-system.

As explained the nodes in the digraph represent attributes identified in each factor. The interaction among attributes is represented by edges.

Figure 7.3 Digraph of factors of website quality attributes

Figure 7.4 Digraph of factors of website Features

Figure 7.5 Digraph of factors of Lifecycle Processes

Figure 7.6 Digraph of factors of Cognition Factors

Figure 7.7 Digraph of factors of Efficiency
Figure 7.8 Digraph of factors of Functionality

Figure 7.9 Digraph of factors of Maintainability

Figure 7.10 Digraph of factors of Portability

Figure 7.11 Digraph of factors of Reliability

Figure 7.12 Digraph of factors of Usability
Figure 7.13 Digraph of factors of Content

Figure 7.14 Digraph of factors of Navigation

Figure 7.15 Digraph of factors of Presentation

Figure 7.16 Digraph of factors of Development

Figure 7.17 Digraph of factors of Maintenance
Step 4: Variable permanent matrix for digraph for each subsystem and sub subsystem is written. At sub subsystem level, variable permanent matrix for digraph for sub subsystem 1 (Figure 7.7) in general form is considered. Similar to equation 7.4, VPM $W_{ss11}$ is given by

$$
VPM-W_{ss11} = \begin{bmatrix}
W_1 & w_{1,2} \\
W_{2,1} & W_2
\end{bmatrix}
$$

(7.6)

Step 5: At the sub subsystem level Tables 7.2 and 7.3 are used to determine numerical values for inheritance of attributes and their interactions. For sub subsystem 1 (i.e. Efficiency), the inheritance values taken from Table 7.2 are $W_1=0.8$ and $W_2=0.8$ and interaction values taken from Table 7.3 are $w_{1,2}=0.2$ and $w_{2,1}=0.4$. The VPM-$W_{ss11}$ shall be
Similarly, the variable permanent matrices for different subsystems (based on their digraphs) are written through equations (7.8)-(7.20).

\[
\begin{align*}
VPM-W_{m1} &= \begin{bmatrix}
1 & 2 & \text{Factor} \\
.8 & .2 & 1 \\
.4 & .8 & 2 \\
\end{bmatrix} \\
VPM-W_{m2} &= \begin{bmatrix}
1 & 2 & 3 & 4 & \text{Factor} \\
.6 & 0 & 0 & .2 & 1 \\
0 & .7 & .4 & .4 & 2 \\
0 & .2 & .4 & .2 & 3 \\
0 & .1 & 0 & .8 & 4 \\
\end{bmatrix} \\
VPM-W_{m3} &= \begin{bmatrix}
1 & 2 & 3 & \text{Factor} \\
.7 & .2 & .3 & 1 \\
.4 & .7 & .1 & 2 \\
.3 & 0 & .5 & 3 \\
\end{bmatrix} \\
VPM-W_{m4} &= \begin{bmatrix}
1 & 2 & \text{Factor} \\
.8 & .3 & 1 \\
.2 & .7 & 2 \\
\end{bmatrix} \\
VPM-W_{m5} &= \begin{bmatrix}
1 & 2 & 3 & 4 & \text{Factor} \\
.8 & 0 & .5 & 0 & 1 \\
.3 & .7 & .3 & .2 & 2 \\
.2 & 0 & .7 & 0 & 3 \\
.3 & .2 & .4 & .5 & 4 \\
\end{bmatrix} \\
VPM-W_{m6} &= \begin{bmatrix}
1 & 2 & 3 & 4 & 5 & 6 & 7 & \text{Factor} \\
.2 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
.3 & .5 & 0 & 0 & .4 & .4 & .3 & 2 \\
0 & 0 & .4 & .2 & 0 & 0 & .2 & 3 \\
0 & 0 & 0 & .5 & .4 & .4 & .1 & 4 \\
0 & 0 & 0 & .8 & .3 & 0 & 5 \\
0 & .3 & .1 & 0 & .3 & .8 & .3 & 6 \\
0 & 0 & .2 & .5 & .4 & .4 & .6 & 7 \\
\end{bmatrix}
\end{align*}
\]
\[
\text{VPM-}W_{a21} = \begin{bmatrix}
.7 & 0 & .2 & .1 & 0 & .2 \\
.2 & .8 & .2 & 0 & .3 & .5 \\
0 & .3 & .8 & .1 & 0 & .4 \\
.1 & .2 & .2 & .5 & .2 & .3 \\
.2 & 0 & 0 & 0 & .7 & .2 \\
.2 & .2 & .3 & 0 & 0 & .9 \\
\end{bmatrix}
\]

(7.13)

\[
\text{VPM-}W_{a22} = \begin{bmatrix}
.7 & .3 & .3 \\
0 & .7 & .2 \\
0 & 0 & .7 \\
\end{bmatrix}
\]

(7.14)

\[
\text{VPM-}W_{a23} = \begin{bmatrix}
.9 & .2 & .4 & .5 \\
0 & .7 & .3 & .2 \\
0 & 0 & .7 & .2 \\
0 & 0 & 0 & .9 \\
\end{bmatrix}
\]

(7.15)

\[
\text{VPM-}W_{a31} = \begin{bmatrix}
.6 & .3 \\
.2 & .7 \\
\end{bmatrix}
\]

(7.16)

\[
\text{VPM-}W_{a32} = \begin{bmatrix}
.7 & .2 & .2 \\
0 & .8 & .2 \\
.3 & 0 & .5 \\
\end{bmatrix}
\]

(7.17)

\[
\text{VPM-}W_{a33} = \begin{bmatrix}
.5 & 0 & .2 & .3 \\
.2 & .7 & .4 & .4 \\
.1 & .1 & .6 & .3 \\
.3 & .1 & .2 & .8 \\
\end{bmatrix}
\]

(7.18)
Step 6: The permanent of matrix 7.7 i.e. \( \text{Per } W_{ss11} \), is calculated by evaluating the matrix on the lines of equation 7.5. The value of permanent function which leads to the inheritance of quality factor \( W_{ss11} \) may be written as \( \text{Per } W_{ss11} = 0.72 \).

Step 7: Similarly the values of permanent functions of different sub subsystems are evaluated from the variable permanent matrices in equations (7.8)-(7.20) and are written as under:

- \( \text{Per } W_{ss12} = 0.1872 \)
- \( \text{Per } W_{ss13} = 0.354 \)
- \( \text{Per } W_{ss14} = 0.62 \)
- \( \text{Per } W_{ss15} = 0.2574 \)
- \( \text{Per } W_{ss16} = 0.0275332 \)
- \( \text{Per } W_{ss21} = 0.29109 \)
- \( \text{Per } W_{ss22} = 0.343 \)
- \( \text{Per } W_{ss23} = 0.3969 \)
- \( \text{Per } W_{ss31} = 0.48 \)
- \( \text{Per } W_{ss32} = 0.34 \)
- \( \text{Per } W_{ss33} = 0.31 \)
- \( \text{Per } W_{ss41} = 0.86 \)
- \( \text{Per } W_{ss42} = 0.352 \)

Step 8: Digraph for website quality attributes at sub system level 1 is shown in Figure 7.3. Similarly digraph for website features at sub system level 1 is shown in Figure 7.4, digraph for lifecycle processes at sub system level 1 is shown in Figure 7.5 and digraph for cognition factors at sub system level 1 is shown in Figure 7.6.
variable permanent matrix for this example at sub system level 1 is written in symbolic form as:

\[
\begin{align*}
\text{VPM-}\mathbf{W}_{s1} &= \begin{bmatrix}
1 & 2 & 3 & 4 & 5 & 6 \\
0.72 & 0 & 0 & 0 & .4 & 0 \\
.3 & 0.1872 & .4 & .4 & .3 & .5 \\
.2 & 0 & 0.354 & 0 & .2 & 0 \\
.1 & 0 & 0 & 0.62 & .2 & 0 \\
.4 & 0 & 0 & 0 & 0.2574 & 0 \\
.3 & 0 & 0 & 0 & .2 & 0.0275332
\end{bmatrix} \quad (7.21)
\end{align*}
\]

\[
\begin{align*}
\text{VPM-}\mathbf{W}_{s2} &= \begin{bmatrix}
1 & 2 & 3 \\
0.29109 & .2 & .4 \\
0 & 0.343 & .3 \\
.2 & .2 & 0.3969
\end{bmatrix} \quad (7.22)
\end{align*}
\]

\[
\begin{align*}
\text{VPM-}\mathbf{W}_{s3} &= \begin{bmatrix}
1 & 2 & 3 \\
0.48 & .2 & .3 \\
.3 & 0.34 & .2 \\
.2 & .3 & 0.31
\end{bmatrix} \quad (7.23)
\end{align*}
\]

\[
\begin{align*}
\text{VPM-}\mathbf{W}_{s4} &= \begin{bmatrix}
1 & 2 \\
.86 & .2 \\
.5 & .352
\end{bmatrix} \quad (7.24)
\end{align*}
\]

As explained, the values of diagonal elements are to be taken from step 6 and 7 and the values of off-diagonal elements are taken from Table 7.3. The values of permanent functions of different sub systems are evaluated from the variable permanent matrices in equations (7.21)-(7.24) and are written as under:

\[
\begin{align*}
\text{Per } \mathbf{W}_{s1} &= 0.000390651 \\
\text{Per } \mathbf{W}_{s2} &= 0.0965334 \\
\text{Per } \mathbf{W}_{s3} &= 0.153392 \\
\text{Per } \mathbf{W}_{s4} &= 0.40272
\end{align*}
\]
Step 9: To obtain variable permanent matrix—website quality for this example, values are substituted as per step 8.

\[
VPM \ W = \begin{bmatrix}
1 & 2 & 3 & 4 \\
0.000390651 & .3 & .2 & .2 \\
.2 & 0.0965334 & .3 & .2 \\
.2 & .3 & 0.153392 & .2 \\
.2 & .3 & .3 & 0.40272
\end{bmatrix}
\]

\[
(7.25)
\]

Step 10: Value of permanent function for the system is evaluated as per equation 7.5. The value of permanent of above matrix (equation 7.25) is 0.0523839, which indicates website quality index for the case considered. By carrying out similar analysis website quality index for different web environment can be obtained. As suggested, this will help towards analysis and improve of concerned website. The quality of websites may thus be compared and rated for a particular period of time.

For the scope of further improvement, it is suggested to find hypothetical best and hypothetical worst value of website quality index. Website quality index is at its best when the inheritance of all its factors is at its best. Since, inheritance of factors is evaluated considering sub factors and applying graph theoretic approach at the subsystem level, it is evident that website quality index is at its best when inheritance of sub factors is at its best. Since, Table 7.2 is used at subsystem level, maximum value of Per W\textsubscript{1} is obtained when inheritance of all the sub factors is maximum, i.e. value taken from Table 7.2 is 0.9. Thus, equation (7.7) may be rewritten for the maximum value of Per W\textsubscript{ss11} as

\[
VPM- W_{ss1} (max) = \begin{bmatrix}
1 & 2 \\
.9 & .2 \\
.4 & .9
\end{bmatrix}
\]

\[
(7.26)
\]

The value of the permanent of the above function is 0.89, i.e. Per W\textsubscript{ss11}(max) = 0.89. Similarly website quality index is at its worst when the inheritance of all its factors and sub factors is at its worst. This is the case when inheritance of the entire sub
factor is minimum i.e. value taken from Table 7.2 is 0.1. Thus, equation (7.7) may be rewritten for the minimum value of \( \text{Per } W_{ss11} \) as

\[
\begin{bmatrix}
1 & 2 \\
.1 & .2 \\
.4 & .1 \\
\end{bmatrix}
\begin{bmatrix}
\text{Factor} \\
1 \\
2 \\
\end{bmatrix}
\]

(7.27)

The value of the permanent of the above function is 0.09, i.e. \( \text{Per } W_{ss11}(\text{min}) = 0.09 \). Similarly maximum and minimum values for each subsystem are evaluated and different values of permanent of subsystem matrices are summarized in Table 7.5.

**Table 7.5 Values for maximum/minimum website quality index**

<table>
<thead>
<tr>
<th>System/Subsystem</th>
<th>Current value</th>
<th>Maximum value</th>
<th>Minimum value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{Per } W_{ss11} )</td>
<td>0.72</td>
<td>0.89</td>
<td>0.09</td>
</tr>
<tr>
<td>( \text{Per } W_{ss12} )</td>
<td>0.1872</td>
<td>0.7605</td>
<td>0.0021</td>
</tr>
<tr>
<td>( \text{Per } W_{ss13} )</td>
<td>0.354</td>
<td>0.888</td>
<td>0.024</td>
</tr>
<tr>
<td>( \text{Per } W_{ss14} )</td>
<td>0.62</td>
<td>0.87</td>
<td>0.07</td>
</tr>
<tr>
<td>( \text{Per } W_{ss15} )</td>
<td>0.2574</td>
<td>0.7735</td>
<td>0.0055</td>
</tr>
<tr>
<td>( \text{Per } W_{ss16} )</td>
<td>0.0275332</td>
<td>0.921937</td>
<td>0.0003366</td>
</tr>
<tr>
<td>( \text{Per } W_{ss21} )</td>
<td>0.29109</td>
<td>0.912775</td>
<td>0.003087</td>
</tr>
<tr>
<td>( \text{Per } W_{ss22} )</td>
<td>0.343</td>
<td>0.729</td>
<td>0.001</td>
</tr>
<tr>
<td>( \text{Per } W_{ss23} )</td>
<td>0.3969</td>
<td>0.6561</td>
<td>0.0001</td>
</tr>
<tr>
<td>( \text{Per } W_{ss31} )</td>
<td>0.48</td>
<td>0.87</td>
<td>0.07</td>
</tr>
<tr>
<td>( \text{Per } W_{ss32} )</td>
<td>0.34</td>
<td>0.795</td>
<td>0.019</td>
</tr>
<tr>
<td>( \text{Per } W_{ss33} )</td>
<td>0.31</td>
<td>0.9176</td>
<td>0.0184</td>
</tr>
<tr>
<td>( \text{Per } W_{ss41} )</td>
<td>0.86</td>
<td>1.194</td>
<td>0.138</td>
</tr>
<tr>
<td>( \text{Per } W_{ss42} )</td>
<td>0.352</td>
<td>0.81</td>
<td>0.01</td>
</tr>
<tr>
<td>( \text{Per } W_{s1} )</td>
<td>0.000390651</td>
<td>0.459559</td>
<td>1.90592e-10</td>
</tr>
<tr>
<td>( \text{Per } W_{s2} )</td>
<td>0.0965334</td>
<td>0.561664</td>
<td>0.0122652</td>
</tr>
<tr>
<td>( \text{Per } W_{s3} )</td>
<td>0.153392</td>
<td>0.824614</td>
<td>0.0414685</td>
</tr>
<tr>
<td>( \text{Per } W_{s4} )</td>
<td>0.40272</td>
<td>1.06714</td>
<td>0.10138</td>
</tr>
<tr>
<td>( \text{Per } W )</td>
<td>0.0523839</td>
<td>0.513182</td>
<td>0.0334043</td>
</tr>
</tbody>
</table>
Maximum value of website quality index at system level is evaluated by considering maximum values of all subsystems and minimum value of website quality index at system level is evaluated by considering minimum values of all subsystems. The value of per W indicates the value of website quality index. Thus, the maximum and minimum value of website quality index indicates the range within which it can vary. Experts can use this range to decide a threshold value for a given set of similar websites. Monitoring at regular interval may be carried out by third party to assess the quality of web services being provided by an organization. Moreover, the values may be used for self-assessment of website for further improvement.

7.12 Concluding Remarks
The chapter endeavours to quantify the overall effect of website quality factors in developed web quality model through systematic approach. The website quality factors not only help an organization to achieve intangible objectives - better quality, customer satisfaction, goodwill, and responsiveness through continuous improvement but also have long lasting effect on tangible objectives - profitability through productivity.
The current value of website quality index for the considered example is 0.0523839 which is far below than the maximum value of 0.513182 and more nearer to minimum value of 0.0334043. This value in itself speaks of the scope of improvement in the website.
The developed procedure may be useful for self-analysis and comparison among various websites. The key attributes affecting website quality is very difficult to compile as it varies with type of website and its intended users. The web quality model considers general factors, which may vary depending on type of website, users of website and website management team. There is a scope of research in factor specific website. The practical implications of this process is to provide a useful methodology for website managers to assess key quality attributes affecting overall website quality and improve upon therein. Procedure for stepwise application of methodology is given with example that may help a website manager to implement it. The chapter attempts to quantify the web quality factors through systematic approach and is of value to website managers to improve upon their website environment.
Since, the graph theoretic approach as adopted can incorporate more number of factors during the modeling and matrix representation of the website quality systems;
it becomes easier to upgrade such systems with modifications in the attribute constraints represented in matrix form.

The result shown in Table 7.5 may further be evaluated to provide overview of shortcomings in the system or subsystem. For this purpose, quality opportunity loss and quality opportunity gain was calculated as shown in Table 7.6. Quality opportunity loss shows the percentage by which we lag to reach the maximum value of opportunity presented to us. Similarly, quality opportunity gain shows the percentage gained with respect to minimum value.

Table 7.6 Values for Quality Opportunity Loss and Gain

<table>
<thead>
<tr>
<th>System/Subsystem</th>
<th>Quality Opportunity Loss</th>
<th>Quality Opportunity Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Per W_{ss11}</td>
<td>0.2125</td>
<td>0.7875</td>
</tr>
<tr>
<td>Per W_{ss12}</td>
<td>0.75593</td>
<td>0.24407</td>
</tr>
<tr>
<td>Per W_{ss13}</td>
<td>0.61806</td>
<td>0.38194</td>
</tr>
<tr>
<td>Per W_{ss14}</td>
<td>0.3125</td>
<td>0.6875</td>
</tr>
<tr>
<td>Per W_{ss15}</td>
<td>0.67201</td>
<td>0.32799</td>
</tr>
<tr>
<td>Per W_{ss16}</td>
<td>0.97049</td>
<td>0.02951</td>
</tr>
<tr>
<td>Per W_{ss21}</td>
<td>0.6834</td>
<td>0.3166</td>
</tr>
<tr>
<td>Per W_{ss22}</td>
<td>0.53022</td>
<td>0.46978</td>
</tr>
<tr>
<td>Per W_{ss23}</td>
<td>0.39512</td>
<td>0.60488</td>
</tr>
<tr>
<td>Per W_{ss31}</td>
<td>0.4875</td>
<td>0.5125</td>
</tr>
<tr>
<td>Per W_{ss32}</td>
<td>0.58634</td>
<td>0.41366</td>
</tr>
<tr>
<td>Per W_{ss33}</td>
<td>0.67571</td>
<td>0.32429</td>
</tr>
<tr>
<td>Per W_{ss41}</td>
<td>0.31629</td>
<td>0.68371</td>
</tr>
<tr>
<td>Per W_{ss42}</td>
<td>0.5725</td>
<td>0.4275</td>
</tr>
<tr>
<td>Per W_s1</td>
<td>0.99915</td>
<td>0.00085</td>
</tr>
<tr>
<td>Per W_s2</td>
<td>0.84662</td>
<td>0.15338</td>
</tr>
<tr>
<td>Per W_s3</td>
<td>0.85708</td>
<td>0.14292</td>
</tr>
<tr>
<td>Per W_s4</td>
<td>0.68798</td>
<td>0.31202</td>
</tr>
<tr>
<td>Per W</td>
<td>0.96044</td>
<td>0.03956</td>
</tr>
</tbody>
</table>

Quality Opportunity Loss = \( \frac{(\text{Maximum Value - Current Value})}{(\text{Maximum Value - Minimum Value})} \)
Quality Opportunity Gain = \frac{(Current Value - Minimum Value)}{(Maximum Value - Minimum Value)}

The maximum value for quality opportunity gain is 78.75 for $W_{ss11}$ (Efficiency). This maximum value actually depicts the percentage by which it has already treaded the path and need less attention. On the other hand system or subsystem having low value of quality opportunity gain needs more attention to the amount of percentage shown in the quality opportunity loss column. The overall value of quality opportunity loss for website quality index stands at 96.04 which being very low, requires urgent attention towards improvement in the quality of the website.

The proposed structural approach based on digraph and matrix method for the evaluation of website quality has the following features:

- It identifies factors pertaining to website quality.
- It permits modeling of dependence among factors.
- Application of graph theoretic approach makes it convenient for visual analysis and computer processing.
- The presence of web quality factors in success of an online venture is indicated by a single numerical index.
- It permits self-analysis and comparison of organizations.
- Cause and effect analysis is useful in improving the website quality.
- Systematic methodology for conversion of qualitative factors to quantitative values and mathematical modeling gives an edge to the proposed technique over conventional methods.