Rewriting Eq.(3.55) as

$$\frac{d\phi_{kk}(t)}{dt} = \frac{nI_0}{4m^2p_0^2} \int d\mathbf{r} d\mathbf{p} G\left(\frac{\mathbf{p}}{\sqrt{2}}\right) g(r)p_x p_y p_y(t) p_x(t). \quad (A.1)$$

By substituting the value of $p_x(t)$ from Eq.(3.50) the above equation reduces to

$$\frac{d\phi_{kk}(t)}{dt} = \frac{nI_0}{4m^2p_0^2} \int d\mathbf{r} d\mathbf{p} G\left(\frac{\mathbf{p}}{\sqrt{2}}\right) g(r)p_x p_y p_y(t)$$

$$\theta(\sigma^2-b^2) \theta(-\hat{r} \cdot \hat{p}) \delta(t-\tau) (-2(\hat{p} \cdot \hat{r})) \frac{x}{r}. \quad (A.2)$$

At $t = 0$ we obtain

$$\frac{d\phi_{kk}(t)}{dt} \bigg|_{t=0} = -\frac{nI_0}{2m^2p_0^2} \int d\mathbf{r} d\mathbf{p} G\left(\frac{\mathbf{p}}{\sqrt{2}}\right) g(r)p_x p_y^2 (p \cdot \hat{r}) \frac{x}{r}$$

$$\theta(\sigma^2-b^2) \theta(-\hat{r} \cdot \hat{p}) \delta(-\tau). \quad (A.3)$$

Using the value of $\tau$ from Eq.(3.52) we note that

$$\delta(-\tau) = \delta \left[ \frac{m}{p} \left( r \cdot \hat{p} + (\sigma^2 - r^2 + (\hat{r} \cdot \hat{p})^2)^{1/2} \right) \right]$$

$$= \delta \left[ -\frac{m}{p} \left( r + \left( \sigma^2 - r^2 + (\hat{r} \cdot \hat{p})^2 \right)^{1/2} \right) \right] \quad (A.4)$$

where, $\mu$ is the angle between $r$ and $p$. By noting that the above delta function has poles at $r^2 = \sigma^2$. So we write

$$\delta(-\tau) = \frac{p}{m} |\mu| \delta(r-\sigma). \quad (A.5)$$

Using Eq.(A.5) in Eq.(A.3) we obtain

$$\frac{d\phi_{kk}(t)}{dt} \bigg|_{t=0} = -\frac{nI_0}{2m^2p_0^2} \int d\mathbf{r} d\mathbf{p} G\left(\frac{\mathbf{p}}{\sqrt{2}}\right) g(r)p_x p_y^2 \frac{x}{r} p \mu$$

$$\times \theta(\sigma^2-b^2) \theta(-\hat{r} \cdot \hat{p}) \frac{p}{m} |\mu| \delta(r-\sigma). \quad (A.6)$$
By using the addition theorem the angular integration in this equation can be done in the following way. We write the function of $\mu$ as

$$g(\mu) = \sum_{\ell=0}^{\infty} \frac{4\pi}{2\ell + 1} g^1(\mu) Y^*_{\ell m}(\theta, \phi) Y_{\ell m}(\theta, \phi)$$  \hspace{1cm} (A.8)

where $\theta$ and $\phi$ are the polar angles and $y_{\ell m}(\theta, \phi)$ are spherical harmonics. The coefficient $g^1(\mu)$ can be obtained from the relation

$$g^1(\mu) = \frac{2\ell + 1}{2} \int_{-1}^{1} d\mu \ P_\ell(\mu) g(\mu)$$  \hspace{1cm} (A.9)

where $P_\ell(\mu)$ is the Legendre polynomial. By using the above procedure and noting that $r = \sigma$ lies on the boundary we have

$$\frac{d\phi}{dt} \bigg|_{t=0} = -\frac{nI_0}{2} \frac{n}{m^2P_0^2} \sigma^2 \pi g(\sigma) \int dp G\left(\frac{p}{\sqrt{2}}\right)$$

$$\chi \int d\theta \sin^2\theta \cos^2\phi \int_{-1}^{1} d\mu \ \mu^2 |\mu| \delta(-\mu) \ du.$$  \hspace{1cm} (A.10)

$\theta(-\mu)$ in above equation restricts the limit of integration from -1 to 0. We finally obtain

$$\frac{d\phi}{dt} \bigg|_{t=0} = -\frac{16n}{5} \frac{g(\sigma)}{m^3P_0^2} \sigma^2 \frac{162 \sqrt{2} P_0^5}{\pi \sqrt{n}} \frac{192 \sqrt{2} P_0^5}{81 \pi} \int_{-1}^{0} \mu^2 |\mu| \ du$$  \hspace{1cm} (A.11)

$$= -\frac{16}{5} \frac{n}{m^3} \frac{g(\sigma)}{v_0^3} \sigma^2 \sqrt{n}$$  \hspace{1cm} (A.12)

which is same as Eq.(3.56).