Chapter 1

Introduction

Neutrinos are some of the most elusive Standard Model (SM) particles even though they are the most abundant particles in the universe after photon. They are neutral, interact only through the weak interaction and therefore, do not readily interact with other particles. As a result, neutrinos are particularly difficult to study. Neutrino experiments require extremely massive detectors in order to produce statistically significant results. With so little knowledge about neutrinos, neutrino experiments are integral to understanding weak interaction and are an important probe of new physics.

Neutrino physics has now entered an exciting era in which we are in the process of getting precession measurements of neutrino masses and mixings. It also offers great potential for understanding physics beyond the standard model (BSM). A number of present and future experiments are expected to yield more precise knowledge regarding the many unresolved questions.

In this chapter, we will first briefly summarize the history of neutrinos. Then, after describing the sources of neutrinos, we will talk about the phenomenology of neutrino mixing and neutrino oscillations. We will then mention the present status of the neutrino oscillations and then briefly discuss physics beyond oscillations. Finally, we describe the content of the thesis concisely.
1.1 History of Neutrinos

The history of neutrinos started with the famous letter from Wolfgang Pauli [1]. The experimental data [2, 3] forced Pauli to assume the existence of a new particle, which was later called the neutrino. In 1911 Lise Meitner and Otto Hahn performed an experiment which showed that the energies of electrons emitted during $\beta$-decay had a continuous spectrum, this led to the obvious problem of having the conservation of linear momentum and angular momenta. In his most famous letter written in December 1930, Pauli suggested that in addition to the electrons and protons contained inside the atom, there are also extremely light neutral particles, which he called neutrons. He proposed that these neutrons, which were also emitted during $\beta$-decay and accounted for the missing energy and momentum, had simply not yet been discovered. In 1931, Enrico Fermi built upon Pauli’s ideas of a yet undetected particle, and three years later published a very successful model of $\beta$-decay in which neutrinos were produced. At this time Fermi coined the term "neutrino", which is Italian for "small neutral one", thus giving birth to the neutrino.

In 1956 [4] Clyde Cowan and Frederick Reines discovered the neutrino (actually the antineutrino), by observing the inverse beta decay reaction \( p + \bar{\nu}_e \rightarrow n + e^+ \) occurring from antineutrinos produced in a nuclear reactor. This proved the hypothesis set forth by Pauli many years before. The muon neutrino was discovered in 1962 by Lederman, Schwartz and Steinberger [5] and the tau neutrino was detected in 2000 by the direct observation of the Nu-Tau (DONuT) collaboration [6]. In 1957 Pontecorvo conceptualized the possibility of neutrino oscillations by generalizing the notions related to kaon mixing. As only one flavour of neutrino had been discovered at that time, Pontecorvo’s hypothesis focused on mixing between $\nu$ and $\bar{\nu}$. In 1962, with the knowledge that multiple flavours of neutrinos existed in nature, Maki, Nakagawa and Sakata proposed oscillations between $\nu_e$ and $\nu_\mu$. This framework later extended to tau neutrino.

In 1967, the Homestake experiment, pioneered by Davis and Bahcall uncovered the first indication that supported the neutrino oscillation theory. They sought to measure the rate at which solar neutrinos were captured by chlorine nuclei. They had observed a deficit between the measurement and the prediction, but the source of the discrepancy remained unclear. Many pointed towards an inadequate understanding of the solar model or errors in the neutrino experiments. The deficit phenomenon, however, was not limited to solar neutrino observations. Atmospheric neutrino experiments also reported a
deviation from the approximately 2:1 ratio between muon and electron neutrinos that were produced through the $\pi \to \mu \nu_{\mu}, \mu \to e\nu_e\nu_{\mu}$. IMB [7] experiment, MACRO [8], and Kamiokande collaboration [9] found significant deficits in $\nu_{\mu}$ fluxes. In 1998, Super-Kamiokande [10] explained the shortfall by fitting their results with $\nu_{\mu} \leftrightarrow \nu_{\tau}$ oscillation framework. The debate in the solar neutrino sector ended in 2001 when the Sudbury Neutrino Observatory (SNO) [11] experiment provided conclusive evidence that roughly two-thirds of the solar neutrino flux was related to non-$\nu_e$ flavours. This result supported the notion of neutrino oscillations and reconciled the total flux measurement with the standard solar model (SSM) prediction.

1.2 Sources of Neutrinos

Neutrinos are generated from different sources with a wide range of energies. They can be classified as natural and artificial. In this section we summarize different sources of neutrinos which will play a major role in understanding neutrino oscillations.

**Solar neutrinos**: According to the standard solar model (SSM), the net thermonuclear reaction which takes places inside the core of the Sun is the fusion of four protons and two electrons into a $^4He$ nucleus, two electron neutrinos and an energy release of $Q = 26.73$ MeV, in the form of photons. This process can be written as

$$4p + 2e^- \to ^4He + 2\nu_e + Q$$  \hspace{1cm} (1.1)

With the extremely large number of reactions taking place inside the core of the Sun, it can be seen that a large flux of electron flavour neutrinos is created. The reaction is mainly divided into two processes: a) proton-proton (pp) chain and b) Carbon-Nitrogen-Oxygen(CNO)cycle. The energies of the solar neutrinos are in the hundreds of KeV to a few MeV range. The total solar neutrino flux on the earth is about $5.94 \times 10^{10}\ cm^{-2}s^{-1}$.

**Atmospheric neutrinos**: When primary cosmic rays interact with nuclei in the atmosphere of the Earth, secondary particles, mostly pions and some kaons, are produced in hadronic showers. Atmospheric neutrinos are produced from the decay of those secondary particles, dominantly by the following decay chains of pions:

$$\pi^+ \to \mu^+ + \nu_{\mu}; \quad \pi^- \to \mu^- + \bar{\nu}_{\mu}$$  \hspace{1cm} (1.2)
At high energies, kaons also contribute to the production of muons and neutrinos. The muons then decay before reaching the Earth’s surface and give rise to electrons, electron neutrinos and muon neutrinos through the following processes

$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu; \quad \mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$$ (1.3)

Hence the ratio of muon and electron type neutrino is 2:1. Energy range of the atmospheric neutrinos is from a few hundred of MeV to $10^8$ GeV. Atmospheric neutrino flux falls steeply as $E^{-2.7}$ for energies above 1 GeV and the flux becomes undetectably small after about a 100 TeV (See [12] for a detail review). Atmospheric neutrino flux peaks at zenith angle $\approx 90^\circ$, i.e near the horizon, due to the larger length of atmosphere available in this direction. Atmospheric neutrinos have long energy and oscillation length, hence atmospheric neutrinos will be very useful to study neutrino oscillations as well as new physics.

**Relic neutrinos**: Relic neutrinos are one of the most important products of the Big Bang. In the early Universe, neutrinos were in thermal equilibrium through weak interaction with the other particles. As the Universe expanded and cooled, the rates of weak interaction processes decreased and neutrinos decoupled when these rates became smaller than the expansion rate. The weak interaction cross-section of the Relic neutrino with matter is very small due to extremely small temperatures. Therefore, the direct detection of Relic neutrinos is an extremely difficult task with present experimental techniques.

**Astrophysical source of neutrinos**: Very high energy neutrinos are produced by astrophysical sources like Gamma-Ray Burst (GRB), Active Galactic Nuclei (AGN). Supernova Remnants (SNRs), AGNs, GRBs and other astrophysical sources can accelerate protons to such energies (and above) by the Fermi acceleration mechanism. The interactions of these protons with soft photons or matter from the source can give Ultra high energy (UHE) neutrinos through the following process: $p\gamma, pp \rightarrow \pi^\pm X, \pi^\pm \rightarrow \mu^\pm \nu_\mu(\bar{\nu}_\mu), \mu^\pm \rightarrow e^\pm \bar{\nu}_e(\nu_e)$ with a flux ratio of $\phi_{\nu_e} : \phi_{\nu_\mu} : \phi_{\nu_\tau} = 1 : 2 : 0$. Recently, the IceCube detector [13] has observed a total of 37 neutrino events with deposited energies ranging from 30 TeV to 2 PeV. The 2 PeV event is the highest-energy neutrino event ever observed.

**Accelerator based neutrinos**: Accelerator based neutrino beam are mainly composed of muon neutrinos. They are produced by the decay of charged pions and kaons. Initially a beam of high energy protons is directed to a thick nuclear target, producing
secondaries such as positively charged pions and kaons which are collected and focused using a magnetic horn. These mesons then enter an evacuated decay tunnel where they decay to produce muons and neutrinos. The muons and the remaining mesons are absorbed in a beam dump at the end of the tunnel, producing a neutrino beam. Semi-leptonic decays of charmed particles have also been used to produce neutrinos where a high-energy proton beam is stopped in a thick target to generate heavy hadrons. The charmed heavy hadrons decay promptly emitting equal fluxes of high-energy electron and muon neutrinos.

**Reactor neutrinos**: Nuclear reactors are the major sources of artificially produced neutrinos. Power generation in nuclear reactors take place through the fission of neutron-rich isotopes like $^{235}\text{U}$, $^{238}\text{U}$, $^{239}\text{Pu}$. Electron antineutrinos are produced by the chain of $\beta$-decays of the fission products in the energy range of 0.1 to 10 MeV.

### 1.3 Neutrino Oscillations

It has been seen that neutrinos change flavour during their propagation. To explain this it is essential that at least two of the neutrinos have masses, which are different from each other and the flavour eigenstates are different from the mass eigenstates. In this section, the standard formalism and expressions for neutrino oscillation and survival probabilities are described in detail.

Neutrino oscillation arises from a mixture between the flavour and mass eigenstates of neutrinos. The neutrino flavour eigenstate $|\nu_\alpha\rangle (\alpha = e, \mu, \tau)$ can be written as a superposition of mass eigenstates $|\nu_j\rangle (j=1,2,3)$.

$$|\nu_\alpha\rangle = \sum_{j=1,2,3} U_{\alpha j}^* |\nu_j\rangle \quad (1.4)$$

where $U_{\alpha j}$ is a $3\times3$ unitary mixing matrix, known as the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [14, 15].

**Parametrization of U:**

In general for an $N\times N$ unitary matrix, there are $\frac{N(N-1)}{2}$ mixing angles and $\frac{N(N+1)}{2}$ phases. But $2(N-1)$ phases can be absorbed by the redefinition of the fields. Hence, in case of $N$ flavours the leptonic mixing matrix $U_{\alpha j}$ depends on $\frac{(N-1)(N-2)}{2}$ Dirac CP-violating phases ($\delta_{CP}$). If the neutrinos are Majorana particles, there are $(N-1)$ additional Majorana phases.
Standard Derivation of the Oscillation Probability:

In the standard theory of neutrino oscillations, the mass states $|\nu_j\rangle$ are eigenstates of the Hamiltonian,

$$H |\nu_j\rangle = E_j |\nu_j\rangle$$

with energy eigenvalues $E_j = \sqrt{p^2 + m_j^2}$. By solving the Schrodinger equation, the time evolution of neutrino mass eigenstates can be written as

$$|\nu_j(t)\rangle = e^{-iE_j t} |\nu_j\rangle$$

Thus the time evolution of a flavour state can be written as

$$|\nu_{\alpha}(t)\rangle = \sum_j U_{\alpha j}^* e^{-iE_j t} |\nu_j\rangle$$

At $t = 0$, the flavour state can be written as $|\nu_{\alpha}(t = 0)\rangle = |\nu_{\alpha}\rangle$. Using the unitary relation $U^\dagger U = 1$, the mass eigenstates can be written in terms of flavour eigenstates as

$$|\nu_j\rangle = \sum_{\alpha} U_{\alpha j} |\nu_{\alpha}\rangle$$

The amplitude of transition from a flavour state $\nu_{\alpha}$ to $\nu_{\beta}$ can then be written as

$$S_{\nu_{\alpha} \rightarrow \nu_{\beta}}(t) = \langle \nu_{\beta} | \nu_{\alpha}(t) \rangle = \sum_j U_{\alpha j}^* U_{\beta j} e^{-iE_j t}$$

The oscillation probability from a flavour state $\nu_{\alpha}$ to $\nu_{\beta}$ can be written as

$$P_{\nu_{\alpha} \rightarrow \nu_{\beta}}(t) = |S_{\nu_{\alpha} \rightarrow \nu_{\beta}}(t)|^2 = \sum_{j,k} U_{\alpha j}^* U_{\beta j} U_{\alpha k} U_{\beta k}^* e^{-i(E_j - E_k) t}$$

The quartic products in eqn 1.10 are free from the Majorana phases, hence Majorana phases cannot be measured in neutrino oscillation experiment. Neutrino oscillations have two different channels, if the flavour $\nu_{\alpha} \neq \nu_{\beta}$, then it is called the oscillation probability and if $\nu_{\alpha} = \nu_{\beta}$ then it is known as the survival probability.
1.3.1 Neutrino Oscillations in vacuum

We will first describe neutrino oscillations in vacuum. In vacuum, neutrino mass eigenstates evolve independently. At first, we will derive the oscillation probability formula for the two neutrino flavour case. Then we will go on to the three flavour case.

**Two flavour oscillations:**

In the two flavour oscillation case, we consider two flavour eigenstates of neutrinos as $\nu_\alpha$ and $\nu_\beta$. They are the superposition of two mass eigenstates $\nu_1$ and $\nu_2$ of masses $m_1$ and $m_2$ respectively. The mixing matrix in the two flavour case is

$$U = \begin{pmatrix} 
    \cos\theta & \sin\theta \\
    -\sin\theta & \cos\theta 
\end{pmatrix} \quad (1.11)$$

where $\theta$ is the mixing angle. In the two flavour case, there is only one mixing angle, $\theta$, and there is no CP phase.

From equation 1.10, it is straightforward to derive the expression for the probability of oscillation from $\nu_\alpha$ to $\nu_\beta$

$$P(\nu_\alpha \to \nu_\beta) = \sin^2\frac{2\theta}{sin\theta}\sin^2\left(\frac{\Delta m^2 L}{4E}\right) \quad (1.12)$$

In the case $\alpha = \beta$, the survival probability can be obtained from the unitarity relation of the probability

$$P(\nu_\alpha \to \nu_\alpha) = 1 - \sin^2\frac{2\theta}{sin\theta}\sin^2\left(\frac{\Delta m^2 L}{4E}\right) \quad (1.13)$$

If we use natural units then the survival probability can be written as

$$P(\nu_\alpha \to \nu_\alpha) = 1 - \sin^2\frac{2\theta}{sin\theta}\sin^2\left(1.27\frac{\Delta m^2[eV^2]L[km]}{E[GeV]}\right) \quad (1.14)$$

The oscillation length is $L^{osc} = \frac{4\pi E}{\Delta m^2}$. From equations 1.12 and 1.14, we see that the oscillation and survival probabilities depend on the mixing angle $\theta$, the mass squared difference
\( \Delta m^2 \), oscillation distance \( L \) and energy \( E \) of the neutrino.

**Three flavour oscillations:**

In case of 3 flavour oscillation, there are 3 flavour eigenstates and 3 mass eigenstates. There will be 3 mixing angles and one Dirac CP phase. If the neutrinos are Majorana, there will be extra 2 phases. However, the rephasing invariants in eqn 1.10 (quartic products) are independent of the Majorana phases, hence it follows that they have no effect on the neutrino oscillations. The mixing matrix can be written using the PMNS parametrization in terms of \( \theta_{12}, \theta_{13}, \theta_{23} \) and phase \( \delta \)

\[
U = R^{23} R^{13} R^{12} = \begin{pmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{pmatrix}
\begin{pmatrix}
c_{13} & 0 & s_{13}e^{-i\delta} \\
0 & 1 & 0 \\
-s_{13}e^{-i\delta} & 0 & c_{13}
\end{pmatrix}
\begin{pmatrix}
c_{12} & s_{12} & 0 \\
-s_{12} & c_{12} & 0 \\
0 & 0 & 1
\end{pmatrix}
\]  \tag{1.15}

where \( c_{ij} = \cos \theta_{ij} \), \( s_{ij} = \sin \theta_{ij} \). Now using the mixing matrix and using equation 1.10, we can write the oscillation probability

\[
P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{j>k} \text{Re}[U_{\alpha j}^* U_{\beta j} U_{\alpha k} U_{\beta k}^*] \sin^2 \left( \frac{\Delta m_{jk}^2 L}{4E} \right) + 2 \sum_{j>k} \text{Im}[U_{\alpha j}^* U_{\beta j} U_{\alpha k} U_{\beta k}^*] \sin^2 \left( \frac{\Delta m_{jk}^2 L}{2E} \right)
\]  \tag{1.17}

Imaginary part of the probability expression contains information about the CP violating phase. For CP-transformed reaction \( \bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta \), the transition probability will be the same except that the sign of the imaginary term is reversed.
The probability expressions are not as simple as in the case of two flavour case, but one can use approximations to get ideas about the probability. One of the most important approximation is one mass squared dominance approximation (OMSD), where $\Delta m_{21}^2$ is taken as zero. The OMSD approximation is good enough for a reasonably large range of energies and baselines, and it has the advantage of being exact in the parameter $\theta_{13}$. The expressions of the probabilities $P_{\mu\mu}$ and $P_{\mu\tau}$ in vacuum are given by the following expressions obtained in the OMSD approximation [16–20],

\[
P_{\mu\mu}^\nu = 1 - \sin^2 2\theta_{23} \sin^2 \left[ 1.27 \Delta m_{31}^2 \frac{L}{E} \right] + 4 \sin^2 \theta_{13} \sin^2 \theta_{23} \cos 2\theta_{23} \sin^2 \left[ 1.27 \Delta m_{31}^2 \frac{L}{E} \right]
\]

\[
P_{\mu\tau}^\nu = \sin^2 \theta_{23} \sin^2 2\theta_{13} \sin^2 \left[ 1.27 \Delta m_{31}^2 \frac{L}{E} \right]
\]

Atmospheric neutrinos cover large distances and wide range of energies, additionally recent large value of $\theta_{13}$ will be good enough to use OMSD approximation. Hence, the expressions of $P_{\mu\mu}$ and $P_{\mu\tau}$ using OMSD approximation will be relevant in this thesis.

### 1.3.2 Neutrino Oscillations in matter

When neutrinos travel through a dense medium, their propagation can be significantly modified by the coherent forward scattering from particles they encounter along the way. Potential due to the scattering on matter modifies the mixing of the neutrinos. As a result, the oscillation probability differs from the oscillation in vacuum. The flavour-changing mechanism in matter was formulated by Mikhaev, Smirnov and Wolfenstein (MSW) [21], who first pointed out that there is an interplay between flavour-non-changing neutrino-matter interactions and neutrino mass and mixing. The MSW effect stems from the fact that electron neutrinos (and antineutrinos) have different interactions with matter compared to neutrinos of other flavours. In particular, $\nu_e$ can have both charged current and neutral current elastic scattering with electrons, while $\nu_\mu$ or $\nu_\tau$ have only neutral current interactions with electron. When the neutrino traverses the Earth, the oscillation probability is calculated taking into account Earth’s matter potential due to the forward scattering amplitude of charged current $\nu_e$ interactions with electrons. Neutral current interactions are neglected here because they lead to flavour-independent terms which are irrelevant.
for the oscillation probabilities.

The effective CC Hamiltonian for an electron neutrino propagating through matter(electrons) can be written as

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \bar{\nu}_e \gamma^\mu (1 - \gamma^5) \nu_e \bar{e} \gamma^\mu (1 - \gamma^5) e$$  \hspace{1cm} (1.20)

Using Fierz transformation, we can write

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \bar{\nu}_e \gamma^\mu (1 - \gamma^5) \nu_e \bar{e} \gamma^\mu (1 - \gamma^5) e$$  \hspace{1cm} (1.21)

If we consider the effective Hamiltonian over the electron background and integrate over the electron momentum, the Hamiltonian can then be written as

$$H_{\text{eff}} = V_{cc} \bar{\nu}_e \gamma^0 \nu_e$$  \hspace{1cm} (1.22)

where the charged current potential

$$V_{cc} = \sqrt{2} G_F N_e$$  \hspace{1cm} (1.23)

Here $N_e$ is the electron density of the medium.

Similarly, the neutral current potential of neutrinos propagating in a medium can be calculated. In an electrically neutral medium, the total number of protons and electrons is the same, hence they will cancel the overall potential. The potential will arise only from the interaction with neutrons. Hence, the net effective potential will be

$$V_{NC} = \frac{G_F N_n}{\sqrt{2}}$$  \hspace{1cm} (1.24)

It is important to note that the neutral current potential is flavour independent, hence it will not have any effect on neutrino oscillations.

It is useful to write the matter potential in terms of the matter density $\rho$ and the electron-density.
fraction in the nucleon, $Y_e$ as

$$V_{cc} = 7.56 \times 10^{-14} \left( \frac{\rho}{g/cm^3} \right) Y_e eV$$

(1.25)

For earth matter $Y_e \approx 0.5$. For anti-neutrinos the matter potential will be negative, this is mainly due to the anti-commutation relation of the creation and annihilation operator. The number operator for neutrino is positive and for anti-neutrino it is negative, so $V_{cc}$ will be $-V_{cc}$ for anti-neutrinos.

**Two flavour oscillations in matter:**

The matter effect on neutrino oscillation probability can be better understood if we start with the two flavour case. Let’s take two flavour eigenstates $\nu_e$ and $\nu_\mu$ and two mass eigenstates $\nu_1$ and $\nu_2$. The time evolution equation in flavour state can be written as

$$i \frac{d}{dt} \begin{pmatrix} \nu_e(t) \\ \nu_\mu(t) \end{pmatrix} = H_f \begin{pmatrix} \nu_e(t) \\ \nu_\mu(t) \end{pmatrix}$$

(1.26)

The effective Hamiltonian in flavour basis is

$$H_f = U \begin{pmatrix} -\Delta & 0 \\ 0 & \Delta \end{pmatrix} U^\dagger + \begin{pmatrix} V_e & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} -\Delta \cos 2\theta + V_e & \Delta \sin 2\theta \\ \Delta \sin 2\theta & \Delta \cos 2\theta - V_e \end{pmatrix}$$

(1.27)

where $\Delta = \frac{\Delta m^2}{4E}$ and $V_e = \sqrt{2}G_F N_e$. We will diagonalize the $H_f$ by the orthogonal transformation

$$H_m = U_m^\dagger H_f U_m$$

(1.28)

where the unitary matrix

$$U_m = \begin{pmatrix} \cos \theta_m & \sin \theta_m \\ -\sin \theta_m & \cos \theta_m \end{pmatrix}$$

(1.29)

is the effective mixing matrix in matter. After diagonalization, the mass squared differ-
ence and mixing angle in matter are

\[ \Delta m_m^2 = \sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2} \]  

(1.30)

and the mixing angle in presence of matter is

\[ \tan 2\theta_m = \frac{\tan 2\theta}{1 - \frac{A}{\Delta m^2 \cos 2\theta}} \]  

(1.31)

where \( A = 2EV_c \). The oscillation probability will be the same but with modified mixing parameters

\[ P(\nu_\alpha \rightarrow \nu_\beta) = \sin^2 2\theta_m \sin^2 \left(\frac{\Delta m^2_L}{4E}\right) \]  

(1.32)

Long baselines or high matter densities are required to observe significant matter effects. Under the resonant condition \( \Delta m^2 \cos 2\theta = A \), the oscillation will be significantly enhanced. The mixing angle in matter will be \( \theta_m = \frac{\pi}{4} \) or maximal irrespective of the vacuum mixing angle.

**Three flavour oscillations in matter:**

In the case of 3 neutrino flavours, the effective matter Hamiltonian in flavour basis is given by

\[ H_f = U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta_{21} & 0 \\ 0 & 0 & \Delta_{31} \end{pmatrix} U^\dagger + \begin{pmatrix} V_c & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \]  

(1.33)

In order to compute the effective masses and mixing angles in matter, the above Hamiltonian is diagonalized. The normalized eigenvectors of \( H_f \) form the modified matter mixing matrix. Hence, the matter and vacuum parameters are related by comparison. This technique has been implemented exactly by a numerical solution of the evolution equation in matter.

Here we give the analytic expressions for the probabilities in the OMSD approximation, which amounts to neglecting the smaller mass-squared difference \( \Delta m^2_{21} \) in comparison to \( \Delta m^2_{31} \), as we have already discussed. We have checked that this approximation
works well at energies and length scales relevant here. However, all the plots we give are obtained by numerically solving the full three flavour neutrino propagation equation using Prem Reference Earth Model (PREM) [22] density profile for the earth. Making this approximation, the mass squared difference $(\Delta m^2_{31})^m$ and mixing angle $\sin^2 \theta^m_{13}$ in matter can be related to their vacuum values by

$$ (\Delta m^2_{31})^m = \sqrt{(\Delta m^2_{31} \cos 2\theta_{13} - A)^2 + (\Delta m^2_{31} \sin 2\theta_{13})^2} $$

(1.34)

$$ \sin 2\theta^m_{13} = \frac{\Delta m^2_{31} \sin 2\theta_{13}}{\sqrt{(\Delta m^2_{31} \cos 2\theta_{13} - A)^2 + (\Delta m^2_{31} \sin 2\theta_{13})^2}} $$

(1.35)

Using the above substitutions, the OMSD probabilities $P_{\mu\mu}$ and $P_{\mu e}$ in matter are given by

$$ P^m_{\mu\mu} = 1 - \cos^2 \theta^m_{13} \sin^2 2\theta_{23} \sin^2 \left[ 1.27 \left( \frac{\Delta m^2_{31} + A + (\Delta m^2_{31})^m}{2} \right) \frac{L}{E} \right] $$

$$ - \sin^2 \theta^m_{13} \sin^2 2\theta_{23} \sin^2 \left[ 1.27 \left( \frac{\Delta m^2_{31} + A - (\Delta m^2_{31})^m}{2} \right) \frac{L}{E} \right] $$

$$ - \sin^4 \theta_{23} \sin^2 2\theta^m_{13} \sin^2 \left[ 1.27 (\Delta m^2_{31})^m \frac{L}{E} \right] $$

(1.36)

$$ P^m_{\mu e} = \sin^2 \theta_{23} \sin^2 2\theta^m_{13} \sin^2 \left[ 1.27 (\Delta m^2_{31})^m \frac{L}{E} \right] $$

(1.37)

The probability for the time-reversed transition $P^m_{e\mu}$ is same as $P^m_{\mu e}$ with the replacement $\delta_{cp} \rightarrow -\delta_{cp}$. The OMSD analytical expressions are insensitive to $\delta_{cp}$, hence $P^m_{\mu e} = P^m_{e\mu}$. For an inverted neutrino mass hierarchy, the corresponding probabilities are obtained by reversing the sign of $\Delta m^2_{31}$. The antineutrino probabilities can be written down by making the replacement $A \rightarrow -A$ in the above equations.

In Fig 1.1 and 1.2, we plot the 3-flavour muon neutrino(left panel), anti-neutrino (right panel) survival and oscillation probabilities in vacuum(upper panel) and in matter(lower panel) as a function of energy for both normal and inverted hierarchies for a baseline of 6000 km. The oscillation parameters used as mentioned in Table 1.1. In case of inverted hierarchy, we have only changed the sign of $\Delta m^2_{31}$ as negative, other oscillation parameters are taken same as the normal hierarchy oscillation parameters.
Figure 1.1: Muon neutrino (left) and anti-neutrino(right) survival probability as a function of energy for 6000 Km. Lower and upper panel shows for vacuum and matter respectively. Black solid and magenta dotted curve are for $\Delta m^2_{31} > 0$ and $\Delta m^2_{31} < 0$ respectively. True value of $\delta_{\nu}$, taken as zero.
Figure 1.2: Muon neutrino (left) and anti-neutrino (right) oscillation probability as a function of energy for 6000 Km. Lower and upper panel shows for vacuum and matter respectively. Black solid and magenta dotted curve are for $\Delta m^2_{31} > 0$ and $\Delta m^2_{31} < 0$ respectively. True value of $\delta_{cp}$ taken as zero.
We point out some of the interesting qualitative features demonstrated by the probabilities in Fig 1.1 and 1.2

a) It is clear from the figures that the survival probabilities for neutrinos and anti-neutrinos are same for vacuum. This is due to the fact that, there is no matter interactions, which are different for neutrinos and anti-neutrinos.

b) In case of matter effects, the differences between the probability values for normal and inverted hierarchy in all the channels are maximized in the energy range 4-8 GeV.

c) It shows that $P_{\mu\mu}$ in matter for neutrinos with normal hierarchy deviates from the vacuum oscillations and for anti-neutrinos it is $P_{\bar{\mu}\bar{\mu}}$ that exhibits this deviation for inverted hierarchy.

d) Matter effects in $P_{\mu e}$ in case of a NH manifest themselves by a rise over the corresponding value for an IH. In the case of $P_{\bar{\mu}e}$, IH rises over the NH.

1.4 Present status of neutrino parameters

In the current Standard Model (SM) framework, neutrinos are regarded as massless, uncharged leptons that interact with matter only via the weak force. However, there is strong evidence which suggests that neutrinos undergo transformations between flavours, a quantum mechanical phenomenon known as "neutrino oscillations", which require neutrinos to be massive. Recently large number of neutrino experiments started data taking and reconfirming the neutrino oscillations as well as precision measurement of the oscillation parameters.

The neutrino mixing matrix, or the PMNS matrix can be parametrized in terms of three mixing angles $\theta_{12}$, $\theta_{13}$, $\theta_{23}$, and a CP violating phase $\delta_{\text{cp}}$. Neutrino oscillations are also governed by two mass squared differences $\Delta m_{21}^2$ and $\Delta m_{31}^2$. Solar neutrino oscillation parameters, $\theta_{12}$ and $\Delta m_{21}^2$ have been measured from the combined analysis of the KamLAND and solar neutrino data. Super-Kamiokande (SK) atmospheric, T2K [23] and MINOS [24] and other atmospheric neutrino experiments have measured the value of
After decades of speculation on whether or not $\theta_{13}$ is zero, data from these experiments have revealed that the value of $\theta_{13}$ is non-zero. The short baseline reactor neutrino experiments, Daya Bay [25], RENO [26] and Double Chooz [27] have excluded $\theta_{13} = 0$ at $5.2\sigma$, $4.9\sigma$ and $3.1\sigma$ respectively. Recent [28] global analysis best fit value of the oscillation parameters are given in Table 1.1

<table>
<thead>
<tr>
<th>Oscillation Parameter</th>
<th>Best-fit value</th>
<th>3$\sigma$ range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sin^2 \theta_{12}/10^{-1}$</td>
<td>3.23</td>
<td>2.78 - 3.75</td>
</tr>
<tr>
<td>$\sin^2 \theta_{23}/10^{-1}$</td>
<td>5.67</td>
<td>3.92 - 6.43</td>
</tr>
<tr>
<td>(NH)</td>
<td>5.73</td>
<td>4.03 - 6.40</td>
</tr>
<tr>
<td>(IH)</td>
<td>2.34</td>
<td>1.77 - 2.94</td>
</tr>
<tr>
<td>$\sin^2 \theta_{13}/10^{-1}$</td>
<td>2.40</td>
<td>1.83 - 2.97</td>
</tr>
<tr>
<td>(NH)</td>
<td>7.60</td>
<td>7.11 - 8.18</td>
</tr>
<tr>
<td>(IH)</td>
<td>2.48</td>
<td>2.30 - 2.65</td>
</tr>
<tr>
<td>$</td>
<td>\Delta m^2_{31}</td>
<td>[10^{-5} \text{ eV}^2]$</td>
</tr>
<tr>
<td>(NH)</td>
<td>2.48</td>
<td>2.30 - 2.65</td>
</tr>
<tr>
<td>(IH)</td>
<td>2.38</td>
<td>2.30 - 2.54</td>
</tr>
<tr>
<td>$\delta/\pi$ (NH)</td>
<td>1.34</td>
<td>0.0 - 2.0</td>
</tr>
<tr>
<td>(IH)</td>
<td>1.48</td>
<td>0.0 - 2.0</td>
</tr>
</tbody>
</table>

Table 1.1: Recent best-fit and 3$\sigma$ range of the oscillation parameters.

1.4.1 Open challenges in oscillations physics

Large value of $\theta_{13}$ has expedited the possibility of resolving the two most important questions, namely, neutrino mass hierarchy or the sign of $\Delta m^2_{31}$, and, measuring the CP violation in the neutrino sector. Apart from these two there is also an important question, known as the Octant degeneracy of $\theta_{23}$.

Neutrino mass hierarchy: Neutrino has three mass eigenstates, $m_1, m_2, m_3$. There are two possibilities, i) $m_3 > m_2 > m_1$, called as Normal hierarchy, or ii) $m_2 > m_1 > m_3$, namely, Inverted hierarchy as shown in Fig 1.3. Oscillation experiments have measured only the absolute value of $\Delta m^2_{31}$, but hierarchy of the mass-spectrum is still unknown.
Large value of non zero $\theta_{13}$ will enhance the oscillation when neutrinos will pass through matter, hence the sign of the mass squared difference can be known from the matter effect. Atmospheric neutrinos travel maximum distance through earth matter. Neutrinos and anti-neutrinos interact differently with earth matter. Non zero value of $\theta_{13}$ will help to resolve mass hierarchy if any atmospheric neutrino experiment can detect neutrino and anti-neutrino differently.

**CP violation:** $\delta_{CP}$ appears in the PMNS matrix, if the PMNS is real then there is no CP violation, in other words if $\delta_{CP}$ is zero or $\pi$ then CP is conserved. CP violation has been observed in the quark sector, hence there can be a complex phase analogous to the CKM phase. Since $\delta_{CP}$ occurs with the mixing angle $\theta_{13}$ in the PMNS matrix, the recent measurement of non zero and moderately large value of $\theta_{13}$ by reactor and accelerator experiment is expected to be conducive for the measurement of $\delta_{CP}$.
Octant degeneracy: Another unknown parameter in neutrino oscillation physics is the exact value of $\theta_{23}$. Atmospheric neutrino experiments, like SK has measured $\sin^2 2\theta_{23}$, however the octant in which this mixing angle lies is not yet decisively determined by the data. Octant degeneracy in neutrino oscillation means that, it is not known clearly whether the angle is $\theta_{23}$ or $\frac{\pi}{2} - \theta_{23}$.

1.5 Beyond neutrino oscillations

Recently, mass eigenstate mixing induced neutrino oscillations has become the standard theory accounting for the solar, atmospheric and long baseline neutrino experiments. However, there are other alternative theories which can also induce neutrino oscillations or can cause effects similar to some of the experimental observations. Mass eigenstate mixing induced neutrino oscillations are basically due to the fact that different mass eigenstates have different energies for the same momentum. Thus, in principle, any theories which can modify the dispersion relation to meet this condition can induce neutrino oscillations, such as violation of Lorentz invariance (LIV) and CPT(CPTV), non standard interactions etc.

1.5.1 Lorentz Invariance Violation / CPT Violation

Lorentz invariance is intimately related to CPT symmetry as stated in the CPT theorem [29–31]. For a local field theory, the Lorentz invariance leads to CPT symmetry, while breaking CPT symmetry naturally causes Lorentz invariance violation, which is proven in [32].

The Standard Model (SM) and supersymmetric (SUSY) models are designed to incorporate CPT invariance. However, these models do not include gravity. The Standard Model Extension (SME) [33, 34] is an effective field theory that incorporates gravity with the SM, by way of introducing CPT-even and CPT-odd terms. The fundamental scale of the SME is the Planck scale $m_p \approx 10^{19} GeV$, which is about 17 orders of magnitude larger than electroweak scale $m_w$ associated with SM. By treating the SM as a low energy effective theory of underlining fundamental physics, all the possible Lorentz symmetry breaking terms are added into the SM Lagrangian using the SM fields and gravitational
fields. To generate neutrino oscillation effects, it turns out that the minimal renormalizable version of SME [35] is sufficient.

The possible origin of CPT violation in the neutrino sector has been studied in the context of extra dimensions [36, 37], non-factorizable geometry [38], and non-local causal Lorentz invariant theories [39]. There are other alternative studies, which tries to put bounds on CPT sector in many different contexts (See [40–42] for a detailed review). The original formalism to analyze CPT violating effects on neutrino oscillations has been proposed in [43]. The CPT violating term contributes to the standard Hamiltonian, which will alter the effective neutrino masses. This change in neutrino mass would affect the neutrino oscillation wavelengths. CPT violation in neutrino oscillations would manifest itself in the observation $P(\nu_\alpha \to \nu_\beta) \neq P(\bar{\nu}_\beta \to \bar{\nu}_\alpha)$. However, when neutrinos propagate through matter, the matter effects give rise to "fake" CP and CPT violation even if the vacuum Hamiltonian is CPT conserving. These fake effects need to be accounted for while searching for CPT violation.

CPT violation may also occur if particle and anti-particle masses are different. Such violation, however, also breaks the locality assumption of quantum field theories [32]. Many authors have studied how best to parametrize and/or use neutrino oscillations and neutrino interactions to perform tests of CPT and Lorentz symmetry breaking in different contexts, ranging from neutrino factories and telescopes to long baseline, atmospheric, solar and reactor experiments, including those looking for supernova neutrinos (See [43–68] for a detail review).

1.5.2 Non standard neutrino interactions

As discussed in section 1.3, standard matter interactions affect the survival and oscillation probability of the electron neutrino, but not of the muon and tau neutrinos, since muon and tau particle are absent in normal matter. There can be, however, non-standard matter interactions (NSI) that influence the survival and oscillation probabilities of second and third generation neutrinos. These are interactions of neutrinos with matter (up and down quarks and leptons). One way in which non-standard interactions can arise is through the violation of the unitarity of the lepton mixing matrix in certain see-saw models [69]. Sizable NSI effects can be introduced via new scalar exchange bosons such as Higgs bosons.
or scalar leptoquarks, some supersymmetric models may also give rise to NSI [70].

In future neutrino experiments, ‘new physics’, beyond the SM may appear in the form of unknown couplings involving neutrinos, which are usually referred to as non-standard neutrino interactions (NSI). Compared with standard neutrino oscillations, NSIs could contribute to the oscillation probabilities and neutrino event rates as sub-leading effects, and may bring in very distinctive phenomena. Running and future neutrino experiments will provide us with more precision measurements on neutrino flavour transitions, and therefore, the window of searching for NSIs is open.

In principle, NSIs could exist in the neutrino production, propagation, and detection processes. In general, the NSI can impact the neutrino oscillation signals via two kinds of interactions: (a) charged current (CC) interactions (b) neutral current (NC) interactions. However, CC interactions affect processes only at the source or the detector and these are clearly discernible at near detectors (via the zero distance effect). On the other hand, the NC interactions affect the propagation of neutrinos. The main motivation to study NSIs is that if they exist we ought to know their effects on physics.

In the literature, there exist several theoretical and phenomenological studies of NSIs for atmospheric, accelerator, reactor, solar and supernova neutrinos (See for details [71–76]). In addition, some experimental collaborations have obtained bounds on NSIs [77, 78].

Other than these two phenomena, there are various questions, which are yet unresolved, few of them are

**Existence of sterile neutrinos:**

The hypothesis of sterile neutrinos are based on recent anomalies observed in neutrino experiments. Decades of experimentation have produced a vast number of results in neutrino physics and astrophysics, some of which are in perfect agreement with only three active neutrinos, while a small subset calls for physics beyond the standard model.

The first, and individually still most significant, piece pointing towards new physics is the LSND [79] result, where electron antineutrinos were observed in a pure muon antineutrino beam. The most straightforward interpretation of the LSND result is antineu-
trino oscillation with a mass squared difference, $\Delta m^2$, of about 1 eV$^2$. Given the other neutrino oscillation parameters, the LSND $\Delta m^2$ requires a fourth neutrino.

A new anomaly supporting the sterile neutrino hypothesis emerges from the recent re-evaluations of reactor antineutrino fluxes [80], which find a 3% increased flux of antineutrinos relative to the previous calculations. As a result, more than 30 years of data from reactor neutrino experiments, which formerly agreed well with the flux prediction, have become the observation of an apparent 6% deficit of electron antineutrinos. This is known as the reactor antineutrino anomaly and is compatible with sterile neutrinos having $\Delta m^2_{\text{sterile}} > 1\text{eV}^2$.

Another hint consistent with sterile neutrinos comes from the source calibrations performed for radio-chemical solar neutrino experiments based on gallium [81, 82]. Both the source strength and reaction cross section are known with some precision and a 5-20% deficit of the measured to expected count rate was observed. Again, this result would find a natural explanation by a sterile neutrino $\Delta m^2_{\text{sterile}} > 1\text{eV}^2$.

However, cosmological data, observations of the cosmic microwave background and large scale structure favor the existence of a fourth light degree-of-freedom which could be a sterile neutrino. Recent Planck results [83] show that effective number of neutrinos is, $N_{\text{eff}} = 3.30 \pm 0.27$ and an upper limit of 0.23 eV for the summed neutrino mass.

**Dirac or Majorana nature:**

If massive neutrinos are Dirac particles, they must be distinguishable from their antiparticles because of lepton number conservation. On the other hand, Majorana neutrino is its own antiparticle which can participate in lepton-number-violating processes. Only charge-neutral fermions can be Majorana and hence neutrinos stand out to be the only probable candidates within the Standard Model.

Establishing whether neutrinos are Dirac fermions possessing distinct antiparticles, or are Majorana fermions, i.e. spin 1/2 particles that are identical with their antiparticles, is of fundamental importance for understanding the underlying symmetries of particle interactions and the origin of neutrino masses. The neutrinos with definite mass will be Dirac fermions if particle interactions conserve some additive lepton number, e.g. the total lepton charge $L = L_e + L_\mu + L_\tau$. If no lepton charge is conserved, the neutrinos will be Majorana fermions (see, e.g. [84]).

The massive neutrinos are predicted to be of Majorana nature by the see-saw mechanism of neutrino mass generation [85], which also provides an attractive explanation of
the smallness of neutrino masses and, through the leptogenesis theory [86], of the observed baryon asymmetry of the Universe. If the neutrinos are Majorana in nature, then Majorana phases are same for all the massive neutrinos, which violates lepton number. Hence, they do not have any effect on lepton number conserving process like the neutrino oscillations.

**Absolute scale of neutrino mass:**

Significant constraints on the absolute scale of neutrino mass can be obtained through $\beta$-decay, neutrinoless double beta decay experiments and also from cosmology. The experimental searches for neutrinoless double beta decay have a long history (see, e.g. [87, 88]). The best sensitivity was achieved in the Heidelberg–Moscow $^{76}$Ge experiment [89]: Recent study [90] shows the strongest mass upper limit of $m_{\beta\beta}^{0\nu}$ ranges from 0.115 to 0.339 eV depending on different choices of nuclear matrix elements. While the bound from cosmology [83] is 0.23 eV.

**Mechanism of neutrino mass generation:**

There are different mass models which try to explain small neutrino masses. Important models are Left-Right symmetric models, models with spontaneous B-L violation, radiative mass models, Grand unified models (SU(5),SO(10)) and supersymmetric models. One of the most important mechanism of neutrino mass generation is the see-saw mechanism [85]. The see-saw mechanism is briefly described below—

**see-saw mechanism:**

The principle of the seesaw mechanism can be understood by looking at the neutrino mass matrix. One has to assume that besides the usual left handed (LH) neutrinos $\nu_L$, there are right handed (RH) neutrinos $\nu_R$. Therefore one can construct a Dirac mass term for neutrinos,

$$L^D_{\text{mass}} = m_D \bar{\nu}_R \nu_L + h.c. = \frac{1}{2} (m_D \bar{\nu}_R \nu_L + m_D \bar{\nu}_L \nu_R) + h.c. \quad (1.38)$$

since neutrinos have zero electric charge, in general also Majorana mass terms are possible,

$$L^L_{\text{mass}} = \frac{1}{2} m_L \bar{\nu}_L \nu_L + h.c. \quad (1.39)$$
Now one can introduce a mass matrix $M$, such that

$$
\mathcal{L}_{\text{mass}} = \mathcal{L}_{\text{mass}}^D + \mathcal{L}_{\text{mass}}^L + \mathcal{L}_{\text{mass}}^R
$$

(1.41)

where

$$
M = \begin{pmatrix} m_L & m_D \\ m_D^T & m_R \end{pmatrix}
$$

(1.42)

in the most general case. In the seesaw scenario the RH neutrino fields $\nu_R = N_R$ are assumed to be fields with a heavy mass, whereas $m_D$ is of the electroweak scale. Therefore $m_D << m_R$. Since $\nu_L$ possesses non zero isospin and hypercharge, LH Majorana is forbidden by the SM, so $m_L = 0$. Hence the mass eigenstates will be

$$
m_1 \approx \frac{m_D^2}{m_R}
$$

(1.43)

$$
m_2 \approx m_R
$$

(1.44)

As a consequence, one has a neutrino at a mass scale $\lambda_N = m_R$ of new physics and a very light neutrino, the mass of which is suppressed by $\frac{m_D}{\lambda_N}$.

Whether these non-standard effects can explain neutrino experimental data is certainly worth investigating. On the other hand, these alternatives are new physics beyond the Standard Model and testing them is theoretically important. Atmospheric neutrinos cover a wide range of energies, path lengths and matter densities, and will prove to be a powerful tool to explore new physics.
1.6 An overview of the thesis

Large number of experiments have been designed to study these unresolved questions, many are in proposal state, which have potential to resolve these interesting questions. Motivated by these aspects, the thesis mainly focuses on the simulation study, oscillation and new physics study at the future neutrino experiment ICAL detector at INO. The thesis is organized in the following way:

Chapter 2 describes the ICAL detector. In this chapter, we have described the ICAL detector geometry, working principle of the active detector elements, which are Resistive Plate chambers (RPC) and electronic systems of ICAL.

In Chapter 3, we have discussed the simulation of muons at the ICAL detector, which is very crucial to study neutrino oscillation physics.

In Chapter 4, we have studied the possibility of determining the octant of $\theta_{23}$ at the ICAL detector in conjunction with long baseline experiments T2K and NO$\nu$A, in the light of the non-zero value of $\theta_{13}$ measured by reactor experiments.

Chapter 5 contains the sensitivity of the ICAL detector to Lorentz and CPT violation. We have also shown that strong constraints on CPT violating parameters can be achieved.

In chapter 6, we have shown the effects of non-standard interactions on the oscillation pattern of atmospheric neutrinos at the ICAL detector.

Chapter 7 is a summary of the thesis and its main conclusion.