MATHEMATICAL MODELING IN TERMINAL BALLISTICS AND ITS APPLICATION IN DEFENCE AND MEDICAL SCIENCES

ABSTRACT
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Terminal ballistics is the study of terminal aspect of weapon impact on the target. The study aims at interaction between the weapon and the target and the assessment of the damage involved in either with the target or to the weapon itself. In later case it is referred as weapon failure or a target with better protection. In either case study involves the dynamic response of the weapon on the target which may be moving or stationary.

Ballistics comprises of many aspects; Kinetic Energy (KE) projectile, hollow charge (HEAT) (High Explosive Anti Tank), HESH (High Explosive Squash Head) projectile, mines, small arms (Rifle/Pistol/Machine Gun/Hand released Munitions), and Wound Ballistics. Modeling and Simulation helps in saving lots of money in knowing the outcome of the trial before going for the final production and introducing into the services (Army/Navy/Air force). Earlier, the domain knowledge was restricted to only services, since the use of explosives were restricted only during the war scenario. In the modern society and due to terrorism, where the weapons have in the hands of antisocial elements, it has become equally essential to know the knowledge of the destructive power of the weapon for both protective and offensive purposes for the counter reactive on it. In view of its importance, study pertaining to the terminal ballistics has been initiated in the present dissertation.

In order to understand the damage caused by a particular weapon on the target either full fledged experimental set up to be there in a lab or at a field site. If not at least to have a strong mathematical and simulation based background for understanding the damage caused to the target. The former is quite expensive and at times it would not be possible to estimate parameters which cause damage to the target. In view of its limitations and expenses, it would be always ideal to model and simulate the flow parameters which are responsible for the material failure or
penetration aspect in the terminal ballistics studies. Hence, studies on impact dynamics have been chosen in the present dissertation. The dissertation has over all 10 chapters of which first four are introductory and also focus on literature survey.

**Chapter 1** deals with brief introduction to interior ballistics of guns, exterior ballistics of projectile, and on terminal ballistics.

**Chapter 2** deals with introduction to hollow charge (High Explosive Anti Tank weapon), brief about Munroe effect, estimation of jet and slug velocity and their mass. Limitation of the basic model on hollow charge and recent developments in hollow charge has been discussed in this chapter.

In **Chapter 3**, Kinetic Energy projectile has been discussed. Basic concepts behind estimation of impact (penetration) velocity from Bernoulli’s principle, and depth of penetration have been explained. Models which account for empirical relations with flow variables (velocity, depth of penetration, stress and so forth) and which relates to material parameters of projectile and target have also been discussed in this chapter.

Wound ballistics is the study of wounding capability of a bullet inside human body. **Chapter 4** discuss about wound ballistics and details about how a weapon (bullet) makes injury in different parts of the body. State of the art of the mathematical and experimental methods which exists in wound ballistics has also been discussed in this chapter.

Estimation of jet velocity of hollow charge from the concept of hydrodynamic jet theory has been taken up in **Chapter five**. One of the basic motivations behind the studies is, in the present chapter, there are no specific formula which could be used for the estimation of jet velocity for various combinations of metallic liner and explosives. The analysis has been carried out
with the presumption that, under high explosions, the temperature and pressure built up inside the casing of hollow charge are so high that, metallic liner melts like liquid and forms jet and slug. Hydrodynamic jet theory has been applied for the estimation of jet velocity and pressure. The results have been computed for different explosives [TNT, RDX, HMX, PETN], and for different metallic liners [copper, steel, lead]. Result indicates that velocity reaches to its maximums at a $1/10^{th}$ of the standoff distance before impacting on to target. The results on pressure indicates that, the pressure generated at the time of detonations are quite high [1200 to 1400 MPa] for a metallic liner to melt like a liquid. This observation supports the presumption that, penetration mechanism can be modeled by using hydrodynamic jet theory.

Estimation of temperature and other flow parameters distribution within the hollow charge projectile has been modeled. In Chapter 6, mathematical model has been developed based on conservation of mass, momentum and energy principle. Flow parameters such as velocity, pressure and temperature have been computed for various combinations of metallic liner and target densities. The model has been computed for different explosives also. The results exhibit that temperature increases with the increase in density of metallic liner and explosives. Birkhoff theory which propounds hydrodynamic jet theory has been re-visited for computing relation for jet velocity of various combination of collapsing metallic liner angles and explosive and material liner densities. The results indicate that, the jet velocity which is depending on collapsing angle (of metallic liner) found to depend on metallic liner and explosive density and also on detonation properties. The model also found to be in general agreement with physics of flows and deformation and with other published works.
Mathematical modeling of hollow charge jet penetrating into different target materials has been studied in **Chapter seven**. Using work done principle and Bernoulli’s principle at the interface of target and projectile, flow parameters such as impact velocity and depth of penetration have been computed for different combination of metallic liner and target densities. Momentum and energy per unit distance which are one of the important parameters for the design of weapon and targets have also been estimated and found their variations with flow parameters. The results indicate that, the present model yield lower values in comparison to the one published in the literature [Lambert].

Kinetic Energy projectile impacting on hard targets has been modelled in **Chapter eight**. One of the objectives of the proposed model is to account terms such as dynamical force, yield stress in the momentum force equation. Detailed analysis has been carried out for accounting these two terms [dynamical & yield stress] independently, and also accounting both the terms together in the model. Analytical relation of penetration with time, and distance of penetration, velocity with time and depth, momentum and energy per unit distance have been obtained. The results indicate that, the present model yields higher values for the case of dynamical terms and lower for terms accounting both of them (dynamical & yield stress). The present model has also been compared to that of simulation results obtained by Rosenberg. It is observed that, the present model yields lower values in comparison to Rosenberg’s simulation results. A graphical method has been suggested for the computation of quasi static term $Q$ in yield stress. This $Q$ is required for the computation of all the flow parameters (penetration, velocity, and so forth). The results obtained by the graphical method observed to yield lower values. Energy and momentum per unit distance for dynamical terms has also been computed in the present model. Computational aspect of the model has been
extended for various nose shapes of projectile (ogive, spherical, flat) and interaction with other target materials (steel and aluminum).

Estimation of striking, residual and limit velocity for a projectile impacting onto various target has been undertaken in Chapter nine. One of the main themes behind undertaking this study is to estimate residual energy or energy absorption in the target. Such studies help in estimating the strength of the target (resistive power to the weapon) or effectiveness of the weapon. Residual velocity and energy has been computed for various combinations of target and projectile densities. Results indicate that, targets with lower density have higher energy absorption. This observation is quite contrary to the one observed with respect to different densities of penetrators [higher absorption of energy for higher density projectiles]. Results on residual velocity indicate that, residual velocity with striking velocity increases exponentially initially and founds to settle (asymptotically) at higher velocities.

The last chapter (Chapter ten) deals with mathematical modeling of a bullet impacting onto human body and its applications in bio-engineering and in medical sciences. Flow parameters such as striking velocity, trajectory of bullet, and volume of the temporary cavity has been obtained analytically and computed it for various combinations of bullet and human body (impacting on soft tissue, bones, and so forth). The results have been compared with the other models available in the literature. The results indicate that, the present computed model yield lower values in comparison to the one observed in the literature.

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1 “Mathematical Modeling of Hollow Charge penetration in to the Target and Estimation of Flow Parameters”, 2\textsuperscript{nd} National conference on Engineering Applications of Mathematics MAE, Alandi, 22-23 may 2008,


4 “Analysis of temperature distribution and other flow parameters during the formation of jet in hollow charge (HEAT) projectile”, 35 \textsuperscript{th} FMFP-2008, P.E.S Institute of technology Bangalore, 11-13 Dec 2008.

5 “Estimation of penetration velocity and the distance of penetration of a hollow charge impacting on different targets”, International Conference on Arts, Science, Management and Engineering- Goa, April 23-25, 2009 [Accepted for the Presentation].

7 “Mathematical modeling of Hollow charge penetration into different target materials”, 40th Annual Iranian mathematics conference, Sharif university of technology, Tehran, Iran, 17-20 august 2009


11 “Mathematical Modeling of Kinetic Energy Projectiles impacting on hard Targets”, National conference on computational engineering ,Modelling ,Simulation and optimization (CEMSO-2010), Defence Institute of Advance Technology, Girinagar, 8-10 Dec 2010

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