Chapter nine

Estimation of Striking, Residual and Limit Velocity for a Projectile Penetration into Different Targets
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9.1 Abstract

Terminal ballistic is a study of dynamical interaction of interacting materials. One of the main objectives of the studies is to consider damage caused to the target as a result of interaction of impacting material (projectile) on the target. It is equally important to establish the counter attack (defense), a criteria for protecting against the projectile. The studies in the present model can be classified into the first category where damages to the target requires assessment [282&278]. In a classical report on towards standardization in terminal ballistic testing, Lambert and Jonas [335] propounds a theory for the estimation of residual velocity ($v_r$) by knowing striking velocity and limit velocity ($v_L$). Method of regression analysis has been adapted for the parameter estimation purpose. The approach adopted in the report is of purely analytical in nature. This approach has been re-visited and studied from the concept of momentum equation in the present model and also accounts recently propounded another theory by Lambert [286] for the projectile penetration into the target by using force balance equation and Bernoulli’s principle.

Estimation of striking, residual and limit velocity for a projectile penetrating different targets has been carried out in the present investigations. Basic concepts propounded by Lambert and Jonas [335] for the ballistic testing measurements has been extended to the present model by combining with momentum equation. The studies has been extended for incorporating
into estimation of residual energies which is a vital parameter for the studies pertaining to fracture mechanics.

9.2 Analysis

The basic damage of projectile impacting on the target is shown in Fig.1.

Assumptions on which the model is developed are

\[ v_L = \max \{ v_s : v_r = 0 \} = \inf \{ v_s : v_r > 0 \} \quad (9-1) \]

Equivalently

\[ \begin{cases} v_r = 0 & 0 < v_s < v_L \\ v_r \geq 0 & v_s \geq v_L \end{cases} \quad (9-2) \]

Where, \( v_s \) is striking velocity, \( v_L \) is limit velocity and \( v_r \) is the residual velocity. Here \( v_s \geq v_L \). If \( v_s < v_L \) no penetration, \( v_r \) is also estimated by

\[ v_r = \begin{cases} 0 & 0 < v_s < v_L \\ a(v_s^2 - v_L^2)^{1/2} & v_s \geq v_L \end{cases} \quad (9-3) \]

\[ v_r = \begin{cases} a v_s & \text{for} \quad v_s \to \infty \\ 0 & \text{for} \quad v_s \to 0 \end{cases} \quad (9-4) \]

\[ \frac{dv_r}{dv_s} = \begin{cases} a & \text{for} \quad v_s \to \infty \\ \infty & \text{for} \quad v_s = v_L \end{cases} \quad (9-5) \]

\( v_r = av_s \quad (v_s \to \infty) \) can be viewed as an asymptotic line to a hyperbola in the first quadrant with \( a \) as its slope. Here \( a \) is the constant associated with the
material property. Recht and Ipson [336] propounded empirical relation for a relating to material property:

\[ a = (1 + r)^{-1} \]  
\[ r = \frac{\text{ejected plug mass}}{\text{mass of projectile}} \]  

9.3 Residual Energy

In terminal ballistics residual energy \( e_r \) play an important role in knowing how much energy is being absorbed by the material as a result of an impact. This residual energy can be estimated by using following relation

\[ e_r = a^2 (e_s - e_L) \]  
Where \( e_s = \frac{1}{2} m v_s^2 \), \( e_L = \frac{1}{2} m v_L^2 \)

9.4 Estimation of Limit Velocity

Applying Bernoulli’s equation at the interface, we can write equation

\[ \frac{1}{2} \rho_p (v_s - v_r)^2 = \frac{1}{2} \rho_t v_r^2 \]  
Where \( \rho_p \) and \( \rho_t \) are the density of projectile and target respectively. Equation (9-10) can be simplified to yield

\[ v_s = v_r (1 + \gamma) \]  
Where \( \gamma = \left( \frac{\rho_r}{\rho_t} \right)^{\frac{1}{2}} \)

Using relation (9-3), and (9-11), we can obtain

\[ v_L = v_s \left(1 - \frac{1}{a^2(1+\gamma)^2}\right)^{\frac{1}{2}} \]  

9.5 Estimation of Striking Velocity

We can write Force balance equation at the interface
\[ m \frac{d(v_s - v_r)}{dt} = -\rho_t A v_r^2 \]  
(9-14)

Using equation (9-11), equation (9-14) can be solved for striking velocity

\[ v_s = v_0 e^{-k(x-x_0)} \]  
(9-15)

Where

\[ k = A \left( \frac{\rho_t \rho_p}{m} \right)^{\frac{1}{2}} \]  
(9-16)

\(x_0\) is the standard stand-off distance for striking an object (target), and \(v_0\) is the initial velocity of the projectile (can be treated as muzzle velocity at the time of launch). Having known striking velocity, residual velocity \((v_r)\) can be estimated by knowing material parameter "a" and using relation (9-4)

\[ v_r = a v_s \]

'\(a\)' is generally observed to vary within the limits of 0 and 1 \((0 < a < 1)\) and its value is observed to decay with increase in thickness of the target. Relation for limit velocity \((v_L)\) and residual velocity \((v_r)\) are obtained from the above analysis,

\[ v_L = v_0 \left( 1 - \frac{1}{a^2(1+y)^2} \right)^{\frac{1}{2}} e^{-k(x-x_0)} \]  
(9-17)

\[ v_r = \frac{v_0}{(1+y)} e^{-k(x-x_0)} \]  
(9-18)

Residual energy is obtained by using relation [Eqn. (9-9)]

\[ e_r = \frac{1}{2} ma^2 v_0 e^{-k(x-x_0)} \left( \frac{1}{a^2(1+y)^2} \right) \]  
(9-19)

### 9.6 Result and Discussions

Variation of striking velocity \((v_s)\), limit velocity \((v_L)\) and residual velocity \((v_r)\) have been computed for different projectile densities \((\rho_p)\) and for different target densities \((\rho_t)\). The computed results have been shown in Figs 1-3. The experimental data required for the computational purpose has been taken from Lambert [286] and from Ref [337]. The results indicate that,
all the three velocities decay faster for the higher densities in comparison to the lower densities. The same trend has been observed in case of residual energy also [Fig.4]. The results appears to be in qualitative agreement with the physical observation since higher density projectiles impart higher amount energy in the projectile and hence will leave with less residual energy. It is interest is note that for higher striking velocities of projectiles, residual velocity is observed to follow a linear function of striking velocity \(v_r = av_s\) (an asymptote curve to hyperbola in the first quadrant), here “\(a\)" can be visualized as a slope [Fig.5].

9.7 Conclusions

Mathematical model for the estimation of residual \((v_r)\), striking \((v_s)\) and limit velocity \((v_L)\) has been proposed in the present studies. The flow parameters \((v_r, v_s and v_L)\) have been computed for different projectile and target densities. Residual energy has also been computed in the present model. These flow parameters have vital applications in the terminal ballistics as the results of velocity and residual energy can be used for designing a better armor for the protection against a target also as an offensive measure, the information can be used for estimating whether a particular projectile can damage the intended target under the investigations.
Fig 1. Variation of striking velocity $v_s$ with inside target distance ($x$)

(a): For different penetrator

$$v_s = v_0 \exp(-k(x-x_0))$$

$v_0 = 800$ $x_0 = 1$ $m = 0.1$ $\rho_p = 7850$

$$k = A \left(\frac{\rho_t \rho_p}{m}\right)^\frac{1}{2}$$

$$A = 54.082 \times 10^{-6}$$

(b): For different target

$$v_s = v_0 \exp(-k(x-x_0))$$

$v_0 = 800$ $x_0 = 1$ $m = 0.1$ $\rho_t = 7850$

$$k = A \left(\frac{\rho_t \rho_p}{m}\right)^\frac{1}{2}$$

$$A = 54.082 \times 10^{-6}$$

$p_p$ - Density of penetrator

- $19300$ kg/m$^3$ (W)
- $11340$ kg/m$^3$ (Lead)
- $10188$ kg/m$^3$ (Mo)
- $8940$ kg/m$^3$ (Cu)
- $7850$ kg/m$^3$ (Fe)
- $2700$ kg/m$^3$ (Al)
- $1840$ kg/m$^3$ (Beryllium)
- $1738$ kg/m$^3$ (Mg)

$p_t$ - Density of target

- $19300$ kg/m$^3$ (W)
- $11340$ kg/m$^3$ (Lead)
- $10188$ kg/m$^3$ (Mo)
- $8940$ kg/m$^3$ (Cu)
- $7850$ kg/m$^3$ (Fe)
- $2700$ kg/m$^3$ (Al)
- $1840$ kg/m$^3$ (Beryllium)
- $1738$ kg/m$^3$ (Mg)
\[ V_L = V_0 \left( 1 - \left( \frac{1}{(a^2)(1+\gamma^2)} \right)^{1/2} \right) \exp(-k(x-x_0)) \]

- \( V_0 = 800 \quad x_0 = 1 \quad m = 0.1 \quad \rho_P = 7850 \)
- \( k = A \left( \frac{\rho_L \rho_P}{m} \right)^{1/2} \)
- \( \gamma = \left( \frac{\rho_L}{\rho_P} \right)^{1/2} \quad a = 0.7 \)

- Density of penetrator: 
  - 19300 kg/m³ (W)
  - 11340 kg/m³ (Lead)
  - 10188 kg/m³ (Mo)
  - 8940 kg/m³ (Cu)
  - 7850 kg/m³ (Fe)
  - 2700 kg/m³ (Al)
  - 1840 kg/m³ (Beryllium)
  - 1738 kg/m³ (Mg)

- Density of target: 
  - 19300 kg/m³ (W)
  - 11340 kg/m³ (Lead)
  - 10188 kg/m³ (Mo)
  - 8940 kg/m³ (Cu)
  - 7850 kg/m³ (Fe)
  - 2700 kg/m³ (Al)
  - 1840 kg/m³ (Beryllium)
  - 1738 kg/m³ (Mg)
1. Variation of residual velocity with inside target distance (x)

(a): for different penetrator

\[ \begin{align*}
    v_r &= \frac{v_0}{1 + \gamma} \exp(-k(x - x_0)) \\
    v_0 &= 800 \text{ m/s}, \quad x_0 = 1 \text{ m}, \quad \rho_t = 7850 \text{ kg/m}^3 \\
    k &= A \left( \frac{\rho_t \rho_p}{\mu} \right)^{1/2} \\
    \gamma &= \left( \frac{\rho_t}{\rho_p} \right)^{1/2} \\
    A &= 54.082 \times 10^{-6}
\end{align*} \]

(b): for different target

\[ \begin{align*}
    \rho_t &= \text{density of target} \\
    \gamma &= \left( \frac{\rho_t}{\rho_p} \right)^{1/2}, \quad \alpha = 0.5 \\
    A &= 54.082 \times 10^{-6} \\
    v_0 &= 800 \text{ m/s}, \quad x_0 = 1 \text{ m}, \quad \rho_p = 7850 \text{ kg/m}^3
\end{align*} \]


\[ e_r = \frac{1}{2} \frac{ma^2}{m} e^{k(x-x_0)} \left( \frac{1}{a(1+y)^2} \right) \]

\[ k = A \left( \frac{\rho_t}{\rho_p} \right)^{1/2} \]

\[ \gamma = (\rho_t/\rho_p)^{1/2} \]

\[ A = 54.082 \times 10^{-6} \]

\[ v_0 = 800 \quad x_0 = 1 \quad m = 0.1 \quad \rho_t = 7850 \]

\[ \rho_p \] - density of penetrator

\[ \rho_t \] - density of target

19300 kg/m$^3$ (W)
11340 kg/m$^3$ (Lead)
10188 kg/m$^3$ (Mo)
8940 kg/m$^3$ (Cu)
7850 kg/m$^3$ (Fe)
2700 kg/m$^3$ (Al)
1840 kg/m$^3$ (Beryllium)
1738 kg/m$^3$ (Mg)

Fig. 4 variation of residual energy $e_r$ with inside target distance ($x$)

(a): for different penetrator

(b): for different target
\[ v_r = av_s \]

\[ v_r = a(v_s^{p} - v_L^{p})^{1/p} \]

\[ p = 2.6 \quad a = 0.75 \quad c = 387 \quad 387 \leq x \leq 400 \]

Fig 5. (a) variation of residual velocity with striking velocity

Fig 5. (b) variation residual velocity with striking velocity

\[ v_r = a(v_s^{p} - v_L^{p})^{1/p} \]

\[ p = 2.6 \quad a = 0.25 \quad c = 387 \quad 387 \leq x \leq 400 \]