Chapter Seven

Mathematical Modeling of Hollow Charge Jet Penetration into different Target Materials
CHAPTER SEVEN
Mathematical Modeling of Hollow Charge Jet Penetration into different Target Materials

7.1 Abstract
Mathematical modeling of penetration of hollow charge jet into target of different materials has been undertaken in the present studies. Penetration velocity of the hollow charge has been estimated and computed for its progression into the target at various distances (normalized with standoff) within it. Momentum and energy per unit depth which are of important parameters in the design concept of either the target or the projectile (hollow charge) have also been computed. One of the important parameters in terminal ballistics, “penetration” has been computed for different case studies and for different density combinations of target and liner material. The present model has been compared with other research findings. It is observed that the present model yield lower values in comparison to earlier work.

7.2 Analysis
Work done on the system by impacting moving jet on the target can be represented by following equation:

\[ m_j \frac{d(v-u)}{dt} \cdot x_0 = -A(t) \rho_t x u^2 \]  \hspace{1cm} (7-1)

Where \( A(t) \) is the projectile area, given by following relation [286]

\[ A(t) = \frac{m_j}{\rho_j (V_{tip} - V_{tail}) t} \]  \hspace{1cm} (7-2)

\( v \) is the velocity of jet, \( u \) is the penetration velocity, \( x \) is the distance of
penetration, $\rho_j$ is the density of jet, $\rho_t$ is the density of target, $m_j$ mass of jet $V_{tip}$ and $V_{tail}$ are the velocity of jet at tip and tail respectively.

Assuming $v = u(1 + \gamma)$ from Bernoulli’s Equation [301] where $\gamma = \left(\frac{\rho_t}{\rho_j}\right)^{\frac{1}{2}}$ equation (7-2) can be further simplified to

$$\frac{d(v-u)}{dx} \frac{dx}{dt} = -\frac{\rho_t}{\rho_j(v_{tip} - v_{tail})} x_0 \left(\frac{x}{t}\right)^2 u^2$$  \hspace{1cm} (7-3)

Computational aspect of penetration velocity has been studied for the possible three cases in the proposed model.

**Case (I):** $\frac{dx}{dt} \approx \frac{x}{t} = u$

Using initial condition $u = u_0$ and the relation $v = u(1 + \gamma)$, Equation (7-3) can be simplified to

$$u = \frac{u_0(v_{tip} - v_{tail})}{\gamma u_0 \left(\frac{x}{x_0} - 1\right) + (v_{tip} - v_{tail})}$$  \hspace{1cm} (7-4)

The computed results of this is velocity ($u$) has been compared to that of Lambert [286] model,

$$u = u_0 \left(\frac{x}{x_0}\right)^{-\gamma}$$  \hspace{1cm} (7-5)

**Case (II):** $\frac{dx}{dt} \approx u$ and $\frac{x}{t} = v$

Using the same initial condition as that of case (I), penetration velocity ($u$) for the present case takes the following form;

$$u = \frac{u_0(v_{tip} - v_{tail})}{\gamma(1 + \gamma) u_0 \left(\frac{x}{x_0} - 1\right) + (v_{tip} - v_{tail})}$$  \hspace{1cm} (7-6)

**Case (III):** $\frac{dx}{dt} \approx v - u$ and $\frac{x}{t} = u$
Using \( v = u(1 + \gamma) \), and the same initial condition as of case I and II, penetration velocity simplifies to:

\[
 u = \frac{u_0(V_{\text{tip}} - V_{\text{tail}})}{u_0\left(\frac{x}{x_0} - 1\right) + (V_{\text{tip}} - V_{\text{tail}})}
 \tag{7-7}
\]

### 7.3 Depth of Penetration

Abrahamson and Goodier [303] proposed analytical relation for penetration for non-uniform jet velocity over its length

\[
P = L_j \left[\left(\frac{\rho_j}{\rho_t} + 1\right) \int_0^1 v(\rho_j/\rho_t) \left(x\right)dx - 1\right]
+ S\left[\left(\frac{V_{\text{tip}}}{V_{\text{tail}}}\right)^{\rho_j/\rho_t} - 1\right]
\tag{7-8}
\]

Here velocity of jet \( v \) is taken in two different forms as assumed in case-II and in case-III.

For case-II: \( v \) is taken as [using \( v = u(1 + \gamma) \) and \( u \) from Eqn. (7-6)]

\[
v = \frac{u_0(V_{\text{tip}} - V_{\text{tail}})(1 + \gamma)}{\gamma(1 + \gamma)u_0\left(\frac{x}{x_0} - 1\right) + (V_{\text{tip}} - V_{\text{tail}})}
\tag{7-9}
\]

For case-III: \( v \) is taken as [using \( u \) from Eqn. (7-7)]

\[
v = \frac{u_0(V_{\text{tip}} - V_{\text{tail}})(1 + \gamma)}{u_0\left(\frac{x}{x_0} - 1\right) + (V_{\text{tip}} - V_{\text{tail}})}
\tag{7-10}
\]

Penetration derived for case-II and case-III respectively are given below:

\[
P_{II} = L_j \left[\left(\frac{(\gamma^{-2} + 1)}{v_{\text{tip}}^{\gamma^{-2}}}\right)\left(A\right)^{\gamma^{-2}}\left(\frac{\gamma^2}{\gamma^2 - 1}\left[\left(1 - x_0 + \frac{c}{B}\right)^{-\gamma^{-2} + 1} - \left(-x_0 + \frac{c}{B}\right)^{-\gamma^{-2} + 1}\right]\right) - 1\right]
+ S\left[\left(\frac{V_{\text{tail}}}{V_{\text{tip}}}\right)^{\gamma^{-2}} - 1\right]
\tag{7-11}
\]
\[ P_{III} = L_j \{ \frac{(y^{-2} + 1)}{v_{tip}^{y^{-2}}} \} \left( \frac{A y (1+y)}{B} \right)^{y^{-2}} \frac{y^2}{y^2 - 1} \left[ \left( 1 - x_0 + \frac{y(1+y)C}{B} \right)^{-y^{-2} + 1} - \left( -x_0 + \frac{y(1+y)C}{B} \right)^{-y^{-2} + 1} \right] \]

\[ -1 + S \left[ \frac{v_{tail}}{v_{tip}} \right]^{y^{-2}} - 1 \] (7-12)

Where \( A = u_0 C (1 + \gamma) \), \( B = \frac{y(1+y)u_0}{x_0} \), and \( C = (V_{tip} - V_{tail}) \).

Here, \( S \) is the standoff distance. Marvin et al [304] proposed relation for penetration for uniform jet velocity in terms of yield strength of the material.

\[ P = L_j y^{-1} \left( \frac{1}{\sqrt{y^{-2} - 1} - \frac{2\sigma Yt}{\rho_j v_j^2 (1-y^{-2})}} \right) \] (7-13)

Here "\( v_j \)" is replaced by "\( v_{tip} \)" in the computational aspect.

### 7.4-Momentum and Energy per Unit Depth

Momentum and energy per unit depth\([dM/dp, dE/dp]\), are important parameters from diagnostic and design aspect point of view. Energy can be linked to understand how much energy absorbed or dissipated in the material and momentum to tolerance which a target can possess after the impact of the projectile. Hence these parameters\([dM/dp, dE/dp]\) are derived by using following relations;

\[ M = \rho_j A l V, \quad E = \frac{1}{2} \rho_j A l V^2 \] (7-14)

Hence

\[ \frac{dM}{dp} = \frac{m_j u_0^2}{x_0 [y (1+y)u_0 (\frac{x}{x_0} - 1) + (V_{tip} - V_{tail})]} \right(1 + y)\gamma \left( \frac{x}{x_0} \right)^{(1+y)} \] (7-15)
\[
\frac{\mathrm{d}E}{\mathrm{d}p} = \frac{m_j u_0^3 (V_{\text{tip}} - V_{\text{tail}})(1+\gamma)^2 \gamma}{2x_0 \left[ \gamma(1+\gamma)u_0 \left( \frac{x}{x_0} - 1 \right) + (V_{\text{tip}} - V_{\text{tail}}) \right]^2 \left( \frac{x}{x_0} \right)^{-(1+\gamma)}}
\]  
(7-16)

The data used for the computation propose have been shown in Table 1.

<table>
<thead>
<tr>
<th>Materials</th>
<th>Density g/cm(^3)</th>
<th>Dynamic Yield Strength Mbar</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>2.7</td>
<td>0.0042</td>
</tr>
<tr>
<td>Steel</td>
<td>7.85</td>
<td>0.012</td>
</tr>
<tr>
<td>Tantalum</td>
<td>16.66</td>
<td>0.009</td>
</tr>
<tr>
<td>Uranium</td>
<td>18.9</td>
<td>0.011</td>
</tr>
</tbody>
</table>

Table 7.1: Values of density and dynamical yield strength for different Materials [Ref [305]]

7.5 Results and Discussions

7.5.1 Penetration Velocity

Penetration velocity "\(u\)" has been computed for all the three cases referred above. Also for different combinations of jet and target densities, and for different initial velocities which have been chosen from Refs. [281,386]. The computed results have been shown in Figs 1-7. The results indicate that, for case (I) and case (III), computed results are closer to each other whereas for the case II, it is observed that, case II values are significantly lower in comparison to case I and case III [Fig 6]. Penetration velocity for different target density found to decrease with increase in target density [Fig.4] whereas for different jet density it is found that, penetration velocity decreases with decrease in jet density [Fig.5]. These results on penetration velocity are found to agree with the general physical observations since for higher target density; higher penetration velocity (with higher density of the jet) is required for
penetrating the target. The case for different initial velocity chosen from Refs [281 & 386] has been shown in Fig. 7. The results indicate that, higher is the initial velocity, higher will be the penetration velocity [Fig.7]. In all the cases discussed above, the present model has been compared with Lambert [286] model and found that, the present model yields lower values when compared to Lambert [286] model.

7.5.2 Momentum and Energy Per Unit Depth

Momentum and Energy per unit depth has been computed for its variation with different standoff distance and for different jet density [Fig.8a& 9a]. It is observed that, the momentum and Energy per unit depth decrease with increase in jet density. The comparative study with Lambert [286] model [Fig.8b & 9b] indicates that, the present model yield higher values when compared to Lambert [286] model. Also it indicates that, the results of the present model decay faster with standoff in comparison to Lambert [286] model [Fig 286 & 303]. As stated earlier, energy and momentum are linked to design aspect and tolerance limits of the target respectively, it could be viewed here that, higher values of momentum and energy when compared to Lambert [286] model and higher decay (per unit depth) with standoff distance implies that the present model exhibits more capability with either jet or target performance [jet or target can be specified only after the failure analysis]. Nevertheless the model exhibits better performance when compared with earlier works [286].

7.5.3 Penetration

Penetration has been computed for its variation with length of jet for
three different cases; (i) for case II, (ii) for case III and (iii) for case of uniform jet velocity obtained by Marvin et al [304]. The results obtained for case II and case III has been named as $P_{II}$ and $P_{III}$ in our studies; and for the case of uniform jet velocity obtained by Marvin et al [304], it has been named as ‘$P$’.The computed results have been shown in Figs.[10-15]. The results indicate that $P_{III}$ are higher than $P_{II}$ for ‘$V_{tip}$ ’velocity which are less than 10 km/sec [Fig.10]. For ‘$V_{tip}$ ’grater than 10 km/sec, it is observed that [Fig.11], $P_{II}$ overlaps on $P_{III}$. It is also observed that, $P_{III}$ values are higher than $P$ [Figs. 12-14] for the three different samples [Aluminum, Steel, and Tantalum]. Initially it appears that $P$ values are higher but it is only in the length of jet 0.1 to 0.15 however, $P_{III}$ overshoots $P$ afterwards. It may be said that, this much distance (0.1 to 0.15) metal may requires to overcome its yield value. In case of Uranium [Fig15], it is found that, the initial range of $P$ are higher (0.3 to 0.4) beyond which $P_{III}$ overshoots $P$. The reasons for this delay in overshooting could be due to higher yield strength coupled with higher density for Uranium in comparison to all the three cases mentioned earlier. In all the cases it has been observed that, the present model ($P_{III}$) yields higher values of penetration in comparison to the one proposed in earlier studies (Results obtained for $P$ by using Ref. [304]). It is observed that, the results of the present model appears to be closer (analytical) to the one found in the literature [308].

**7.5.4 Comparative Studies**

In order to get deeper insight of the present works, the results of the present model had been compared with other experimental and theoretical
(numerical simulation) models available in the literature. The comparative statements had been made vis-à-vis with the present model. The results have been shown in Table.7.2. Though it was difficult to get exact matching with the present works however results on penetration (range from 0.05 – 0.92m) with present model found to be in agreement [for jet velocity less than 5.6 Km/sec] with works in Ref. [308 - 310].

7.6 Conclusions

Mathematical modeling of hollow charge jet into different target materials has been undertaken in the present studies. The model has been developed with an aim to compute parameters such as penetration, penetration velocity, energy and momentum per unit depth which are of importance in the design of either target material or projectile development. In view of its importance, the model has been analyzed for various combinations of target and jet densities interactions. The results indicate that, penetration velocity computed from the present model are lower in comparison to the one published in the literature [286]. The results computed for momentum and energy per unit depth are quite useful from the diagnostic point of view since; higher energy absorption indicates positive side of projectile (hollow charge) performance and the converse from target point of view. It is found in our model that momentum and energy per unit depth yield higher values in comparison to Lambert [286] model. The results on penetration computed from the present model indicate that, the results are in agreement with the one published in the literature [308 - 310].
Table 2: Comparison of the present model with other models

<table>
<thead>
<tr>
<th>Density of Jet (kg/m³)</th>
<th>Initial velocity (km/sec)</th>
<th>Penetration (m)</th>
<th>Diameter (m)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>8940(copper) [target steel having density 7850]</td>
<td>10</td>
<td>1.80 - 2.50</td>
<td>--</td>
<td>Present model</td>
</tr>
<tr>
<td>8940(copper) [target Aluminum having density 2700]</td>
<td>5.525</td>
<td>0.30 – 2.60 (P_{III} relation) 0.60 – 0.90 (P relation)</td>
<td>--</td>
<td>Present model</td>
</tr>
<tr>
<td>8940(copper) [target steel having density 7850]</td>
<td>5.525</td>
<td>0.11 – 1.10 (P_{III} relation) 0.20 – 0.62 (P relation)</td>
<td>--</td>
<td>Present model</td>
</tr>
<tr>
<td>8940(copper) [target tantalum having density 1660]</td>
<td>5.525</td>
<td>0.08 – 0.62 (P_{III} relation) 0.09 – 0.51 (P relation)</td>
<td>--</td>
<td>Present model</td>
</tr>
<tr>
<td>8940(copper) [target Uranium having density 1890]</td>
<td>5.525</td>
<td>0.05 – 0.58 (P_{III} relation) 0.10 – 0.56 (P relation)</td>
<td>--</td>
<td>Present model</td>
</tr>
<tr>
<td>--</td>
<td>--</td>
<td>0.0775 – 0.2325</td>
<td>0.155</td>
<td>Walters Williams [308]</td>
</tr>
<tr>
<td>--</td>
<td>--</td>
<td>0.356</td>
<td>RPG-7 Grenade (0.440 Kg shape charge explosive)</td>
<td>Shape Charge [309]</td>
</tr>
<tr>
<td>--</td>
<td>--</td>
<td>0.68 – 0.92</td>
<td>0.20 – 1.0</td>
<td>Walters et al [310]</td>
</tr>
</tbody>
</table>
Chapter 7

Fig 10 variation of penetration with Length of jet(in).

Fig 12 variation of penetration with Length of jet(in).

Fig 14 variation of penetration with Length of jet(in).

Fig 15 variation of penetration with Length of jet(in).