PROBABILITY ANALYSIS OF A COMPLEX SYSTEM WORKING IN A SUGAR MILL WITH REPAIR EQUIPMENT FAILURE

Complex systems have widely studied in literature of reliability theory as a large number of researchers are making lot of contribution in the field by incorporating some new concepts. In order to improve the profit of the organization, researchers are always interested in analyzing the real existing industrial system models. Arora et al. [117] analyzed the system and maintenance management for coal handling system in paper plant, Gupta et al.[118] studied the Stochastic Analysis of an Air Condition Cooling System, Mettas and W. Zhao [119], also studied a complex repairable systems with general repair, Gupta and Kishan [120] analyzed the Complex System with Two Physical Conditions of Repairman . Goyal et al. [121] discussed the profit evaluation of a two unit system with operating and rest period. Two or three-unit standby systems with working or failed have been discussed under various assumptions and situations by numerous researchers including, repair upkeep playing an important role as measures for increasing the reliability of the system, they have assumed that repair equipment can never be failed. But in real world situation repair equipment may also fail when repairman is busy in the repair. They have also supposed that the failure and repair of units are independent of each other but in real world situation this is not so. To have a detailed study on such a real world situation having the above-said aspects, the sugar mills situated at Muzaffar Nagar in Uttar Pradesh were visited and the information was collected on the Sulphated Juice Pump(SJP) System Working therein, this system contains one big unit(B) and two small identical units(A),one of them is in standby. At that point, on the premise of the circumstances saw in this plant, we put a step towards this direction by analyzing the reliability and the profit of more than two unit cold standby SJP System, evaluating various measures of system effectiveness Taking these facts into consideration in this chapter we investigated a complex system model assuming the possibility of repair equipment failure and correlated life time.
5.2 **SYSTEM DESCRIPTION**

Complex System consists of two small identical units of A and one big unit B both are connected in series, but A has two Sub unit connected in Parallel one of them is in standby mode, so both units A and B are initially operating but the operation of only one sub unit of A is sufficient for operating the system. There is a single repair facility. At the point when the repairman is approached to do the task, it takes an irrelevant time to reach at the framework. At the point when repair system falls flat amid the repair of any fizzled unit, repairman begins the repair of repair-system first. The joint distribution of failure and repair times for every unit is taken to be bivariate exponential.

![Transition Diagram](image)

*Figure - 5.1*
5.3 NOTATIONS

For describing the various states of the system we describe the following Notations:

- **A₀**: Unit A is in active mode
- **B₀**: Unit B is in active mode
- **Aᵣ**: Unit A is in failure/down mode
- **Bᵣ**: Unit B is in failure/down mode
- **w**: Fixed rate of failure of repair-equipment
- **θ**: Fixed rate of repair-equipment’s repair
- **A₂**: Unit A in inactive mode and is looking for repairman
- **B₂**: Unit B in down mode but is looking for repairman
- **Aᵣ**: Unit A in inactive mode and is looking for repairman
- **Bᵣ**: Unit B in inactive mode and is looking for repairman
- **Xᵢ(i=1,2)**: R.V. describing the failure times of A and B unit respectively for i=1,2
- **Yᵢ(i=1,2)**: R.V. describing the repair times of A and B unit respectively for i=1,2
- **fᵢ(x,y)**: joint pdf of (xᵢ,yᵢ);i=1,2
  \[ fᵢ(x,y) = βᵢ(1−rᵢ) e^{−αᵢx−βᵢy} Iᵢ(2\sqrt{αᵢβᵢxy}); Xᵢ, Yᵢ, αᵢ, βᵢ > 0; \]
  \[ 0 ≤ rᵢ < 1, \]
  \[ \text{where} \ Iᵢ(2\sqrt{αᵢβᵢxy}) = \sum_{j=0}^{∞} \frac{(αᵢβᵢxy)^j}{(j!)^2} \]
- **kᵢ(Y/X)**: conditional pdf of Yᵢ given Xᵢ=x is given by
  \[ kᵢ(Y/X) = βᵢ e^{−αᵢx−βᵢy} Iᵢ(2\sqrt{αᵢβᵢxy}) \]
- **gᵢ()**: marginal pdf of Xᵢ=αᵢ(1−rᵢ)e^{−αᵢ(1−rᵢ)x}
- **hᵢ()**: marginal pdf of Yᵢ=βᵢ(1−rᵢ)e^{−βᵢ(1−rᵢ)y}
- **qᵢ( ), Qᵢ( )**: p.d.f. & c.d.f. of transition time from regenerative states Sᵢ to Sⱼ.
- **μᵢ**: M.S.T. “Mean sojourn time” in state Sᵢ.
- **Θ**: Notation of Laplace convolution.
\[ A(t) \otimes B(t) = \int_0^t A(t-u)B(u)\,du \]

\( \otimes \): Notation of Laplace Stieltjes convolution

\[ A(t) \otimes B(t) = \int_0^t A(t-u)dB(u) \]

5.4 TRANSITION PROBABILITY AND SOJOURN TIMES

The steady state transition likelihood “probability” can be expressed as follows

\[
\begin{align*}
P_{01} &= \frac{\alpha_1(1-r_1)}{\phi} & P_{32} &= 1 \\
P_{02} &= \frac{\alpha_2(1-r_2)}{\phi} & P_{41} &= \frac{\theta}{\phi} \\
P_{10} &= \frac{\beta_1(1-r_1)}{\beta_1(1-r_1) + \omega + \phi} & P_{46} &= P_{45.6} = \frac{\alpha_1(1-r_1)}{\theta + \phi} \\
P_{14} &= \frac{\omega}{\beta_1(1-r_1) + \omega + \phi} & P_{48} &= P_{47.8} = \frac{\alpha_2(1-r_2)}{\theta + \phi} \\
P_{15} &= \frac{\alpha_2(1-r_2)}{\beta_1(1-r_1) + \omega + \phi} & P_{51} &= \frac{\beta_1(1-r_1)}{\omega + \beta_1(1-r_1)} \\
P_{17} &= \frac{\alpha_1(1-r_1)}{\beta_1(1-r_1) + \omega + \phi} & P_{56} &= \frac{\omega}{\omega + \beta_1(1-r_1)} \\
P_{20} &= \frac{\beta_1(1-r_1)}{\beta_2(1-r_1) + \omega} & P_{71} &= \frac{\beta_1(1-r_1)}{\omega + \beta_2(1-r_1)} \\
P_{23} &= \frac{\omega}{\beta_2(1-r_1) + \omega} & P_{78} &= \frac{\omega}{\omega + \beta_2(1-r_1)} \\

P_{01} + P_{02} &= 1 & P_{41} + P_{47.8} + P_{45.6} &= 1 \\
P_{10} + P_{14} + P_{15} + P_{17} &= 1 & P_{20} + P_{23} &= 1 \\
P_{41} + P_{46} + P_{48} &= 1 & P_{71} + P_{78} &= 1 \\
P_{32} = 1, P_{87} = 1, P_{65} = 1 & P_{56} + P_{51} &= 1
\end{align*}
\]

(5.1-5.26)

Mean sojourn times:

\[
\begin{align*}
\mu_0 &= \frac{1}{\phi} & \mu_1 &= \frac{1}{\beta(1-r_1) + \omega + \phi} \\
\mu_4 &= \frac{1}{\theta + \phi}
\end{align*}
\]

(5.27-5.29)
5.5 MTSF (MEAN TIME TO SYSTEM FAILURE)

To determine the MTSF “Mean time to system failure” of the system, we regarded the down state of the system as a retaining state and after implementing the concept of regenerating processes, by probabilistic arguments, we obtained following recursive expressions

\[ \varphi_0(t) = Q_{01}(t) \otimes \varphi_1(t) + Q_{02}(t) \]

\[ \varphi_1(t) = Q_{10}(t) \otimes \varphi_0(t) + Q_{14}(t) \otimes \varphi_4(t) + Q_{15}(t) + Q_{17}(t) \]

\[ \varphi_4(t) = Q_{41}(t) \otimes \varphi_3(t) + Q_{48}(t) + Q_{46}(t) \]

(5.30-5.32)

After taking “Laplace Stieltjes Transforms” (L.S.T.) on both sides of these recursive relations and solving for \( \Phi_0^*(s) \),

\[ \Phi_0^*(s) = \frac{N(s)}{D(s)} \]

(5.33)

Where

\[ N = \mu_0(1 - P_{14}P_{41}) + \mu_1(P_{01} + P_{02}P_{41}) + \mu_2(P_{01}P_{14} + P_{02}) \]

\[ D = (1 - P_{14}P_{41}) - P_{01}P_{10} + P_{02}P_{41}P_{10} \]

(5.34-5.35)

5.6 AVAILABILITY ANALYSIS

Suppose \( A_i(t) \) be the probability/likelihood that the system is in active mode or in up-state at a specific moment/time “t”, provided that the system entered into regenerative methodology the point wise availability \( A_i(t) \) is seen to hold the under mentioned recursive expressions

\[ A_0(t) = M_0(t) + q_{00} \oplus A_0(t) + q_{02} \oplus A_2(t) \]

\[ A_1(t) = M_1 + q_{10} \oplus A_0(t) + q_{14} \oplus A_4(t) + q_{15} \oplus A_5(t) + q_{17} \oplus A_7(t) \]

\[ A_2(t) = q_{20} \oplus A_0(t) + q_{23} \oplus A_3(t) \]

\[ A_3(t) = q_{32} \oplus A_2(t) \]
\[ A_i(t) = M_i + q_{i1} \otimes A_i(t) + q_{i5.6} \otimes A_i(t) + q_{i7.8} \otimes A_i(t) \]
\[ A_b(t) = q_{31} \otimes A_i(t) + q_{56} \otimes A_b(t) \]
\[ A_c(t) = q_{65} \otimes A_c(t) \]
\[ A_d(t) = q_{a1} \otimes A_d(t) + q_{a3} \otimes A_d(t) \]
\[ A_e(t) = q_{a3} \otimes A_e(t) \]

(5.36-5.44)

After taking Laplace transformation “L.T.” on both sides of the above expressions for availability analysis and solving them for \( A_0^*(s) \), where \( A_0^*(s) \) is the L.T. of \( A_0(s) \), we obtained

\[ A_0^*(s) = \frac{N_1(s)}{D_1(s)} \]

(5.45)

In the steady-state, availability is given by

\[ A_0 = \lim_{s \to 0} (sA_0^*(s)) = \frac{N_1}{D_1} \]

(5.46)

Where

\[ N_1 = \mu_0 ([1 - P_{23}][P_{71}P_{18.7}(1 - P_{56}P_{65}) + P_{78}P_{51}(P_{45.6}P_{14} + P_{15}) - P_{78}(1 - P_{11.7} - P_{14}P_{41}) + P_{78}P_{65}P_{56}(1 - P_{11.7} - P_{14}P_{41})] + (1 - P_{23})P_{78}P_{01}[(\mu_1 + \mu_4P_{14}) + P_{56}P_{65}(\mu_1 + \mu_4P_{14})]) \]
\[ D_1 = \mu_0P_{78}P_{10}P_{20}P_{51} - \mu_0P_{78}P_{01}P_{20}P_{51} + \mu_s\mu_4(\mu_7P_{51}(1 - P_{14}P_{48} - P_{02}P_{10}) + P_{17}P_{51} - P_{78}P_{10}) + (\mu_1 + \mu_4P_{41})P_{56} \]

(5.47-5.48)
5.7 BUSY PERIOD ANALYSIS OF THE REPAIRMAN

We are assuming that at moment “t”, $B_i(t)$ be the probability that the repairman is busy for repairing the failed unit, provided that at $t=0$, the system entered in a particular regenerative state $i$. We obtained the under mentioned recursive expressions for $B_i(t)$.

\[
\begin{align*}
B_i(t) &= W_i + q_{i0} \otimes B_0(t) + q_{i4} \otimes B_4(t) + q_{i5} \otimes B_5(t) + q_{i7} \otimes B_7(t) + q_{i17} \otimes B_{17}(t) \\
B_2(t) &= W_2 + q_{20} \otimes B_0(t) + q_{25} \otimes B_5(t) \\
B_3(t) &= W_3 + q_{32} \otimes B_2(t) \\
B_4(t) &= W_4 + q_{43} \otimes B_3(t) \\
B_5(t) &= W_5 + q_{54} \otimes B_4(t) \\
B_6(t) &= q_{61} \otimes B_1(t) + q_{65} \otimes B_5(t) \\
B_7(t) &= W_7 + q_{77} \otimes B_7(t)
\end{align*}
\]

for $B^*_0(s)$, we get

\[
B^*_0(s) = \frac{N_2(s)}{D_1(s)}
\]

Taking “Laplace Transformation” (L.T) of the above expressions (on both sides) of busy period of repairman of failed units and solving them for $B^*_0(s)$, where $B^*_0(s)$ is the L.T. of $B_0(s)$, we obtained

\[
B_0 = \lim_{s \to 0} (sB^*_0(s)) = \frac{N_2}{D_1}
\]

Where

\[
\begin{align*}
N_2 &= \mu_0 (1 - P_{14}P_{41} - P_{20}P_{23}) + \mu_5 (P_{50}P_{56} + P_{52}P_{41}) + \mu_4 P_{17} (P_{01}P_{14} + P_{02}P_{51}) + (1 - P_{30}P_{31}) + P_{41}P_{40}) \\
D_1 &= \text{has already been evaluated.}
\end{align*}
\]
5.8 EXPECTED NUMBER OF VISITS BY THE REPAIRMAN

For repairing and replacement we assumed $V_i(t)$ as the expected number of visits by the repairman in the interval $(0,t]$, provided the system initially begins from $S_i$ ("a regenerative state"). By utilizing the concept of regenerative point, we obtained the below mentioned relations for $V_i(t)$ which are recursive in nature:

$$
V_0(t) = Q_{00}(t) \otimes (1+V_1(t)) + Q_{01}(t) \otimes (1+V_2(t))
$$

$$
V_i(t) = Q_{ik}(t) \otimes V_k(t) + Q_{i+1k}(t) \otimes V_{i+1}(t) + Q_{i+2k}(t) \otimes V_{i+2}(t)
$$

$$
V_3(t) = Q_{20}(t) \otimes V_0(t) + Q_{21}(t) \otimes V_1(t)
$$

$$
V_4(t) = Q_{30}(t) \otimes V_0(t) + Q_{31}(t) \otimes V_1(t) + Q_{32}(t) \otimes V_2(t)
$$

$$
V_5(t) = Q_{40}(t) \otimes V_0(t) + Q_{41}(t) \otimes V_1(t) + Q_{42}(t) \otimes V_2(t) + Q_{43}(t) \otimes V_3(t)
$$

$$
V_6(t) = Q_{50}(t) \otimes V_0(t) + Q_{51}(t) \otimes V_1(t) + Q_{52}(t) \otimes V_2(t) + Q_{53}(t) \otimes V_3(t) + Q_{54}(t) \otimes V_4(t)
$$

$$
V_7(t) = Q_{60}(t) \otimes V_0(t) + Q_{61}(t) \otimes V_1(t) + Q_{62}(t) \otimes V_2(t) + Q_{63}(t) \otimes V_3(t) + Q_{64}(t) \otimes V_4(t) + Q_{65}(t) \otimes V_5(t)
$$

$$
V_8(t) = Q_{70}(t) \otimes V_0(t) + Q_{71}(t) \otimes V_1(t) + Q_{72}(t) \otimes V_2(t) + Q_{73}(t) \otimes V_3(t) + Q_{74}(t) \otimes V_4(t) + Q_{75}(t) \otimes V_5(t) + Q_{76}(t) \otimes V_6(t)
$$

(5.61-5.69)

After having "Laplace Stieltjes Transform" L.S.T. on both sides of the above written expressions of expected number of visits and evaluating them for $V_0'''(s)$, we are having:

$$
V_0'''(s) = \frac{N_3(s)}{D_1(s)}
$$

(5.70)

In steady state

$$
V_0 = \lim_{s \to 0} (sV_0'''(s)) = \frac{N_3}{D_1}
$$

(5.71)

Where $N_3 = \mu \left(1 - P_{14}P_{41} + P_{01} + P_{02}P_{41}\right) - P_{01}P_{14} + P_{02}$

(5.72)

$D_1$ is has already been evaluated.
5.9 PROFIT ANALYSIS

The expected total profit incurred to the system in steady state is given by

\[ P = C_0 A_0 - C_1 B_0 - C_2 V_0 \]

(5.73)

Where

\[ C_0 = \text{Revenue per unit uptime of the system} \]
\[ C_1 = \text{Cost (expenses) per unit time for which repairman is busy} \]
\[ C_2 = \text{Cost (expenses) per visit for the repairman} \]

5.10 Particular Case

According to the numerical values taken as

\[ \alpha_1 = .01, \alpha_2 = .003, \beta_1 = .0001, \beta_2 = 0.0002, \theta = 0.03, \omega = .02 \]

Mean Time To System Failure (MTSF) = 230

Availability \((A_0) = 0.7304\)

Busy Period of Repairman \((B_0) = 0.711\)

Expected no. of visits by the repairman \((V_0) = 0.8643\)

Profit=2029.541

If \(r_1 = r_2 = 0\)

Mean Time to System Failure (MTSF) = 153.96

Availability \((A_0) = 0.6712\)

Busy Period of Repairman \((B_0) = 0.769\)

Expected no. of visits by the repairman \((V_0) = 0.881\)

Profit=1097.146

5.11 CONCLUSION

For the more clear perspective of the framework with respect to the different parameters included, we plot the graphs for Mean Time to System Failure “MTSF” and Availability function in the Figure - 5.2 and figure-5.3 respectively with respect to the failure rate \(\alpha\) of a sub-unit A for distinct values of correlation coefficient ‘r’, between above defined random variable X and Y, however the other parameters are assumed constant as
From the Figure - 5.2 it is cleared that Mean Time to System Failure “MTSF” going down side as failure rate moving right side of the graph regardless of other parameters. These graphs also pointing out that for the fixed failure rate, MTSF is much more for higher values of correlation coefficient(r), hence, we are in the position to say that the high estimation of r between failure and repair has a tendency to expand the normal life time of the system. From the figure-5.3 it reveals that availability decreases as failure rate going right side of the x-axis. Also concluded that for the fixed failure rate, the financial benefit is much more for high correlation (r).

Figure - 5.5 shows that for r\(_1\) = 0.25 profit is negative according as failure rate is > or = 0.034. Thus the system is profitable only if failure rate is less than or = 0.034.

- for r\(_2\) = 0.50 profit is negative according as failure rate is > or = 0.05426. Thus the system is profitable only if failure rate is less than or =0.05426.
- for r\(_3\) = 0.75 profit is negative according as failure rate is > or = 0.1031. Thus the system is profitable only if failure rate is less than or = 0.10310.

From Figure - 5.6 it is clearly indicated that MTSF increases as the Repair Rate increases.

and values of MTSF are higher for lower values of correlation coefficients.

Figure - 5.7 depicts the Pattern of the profit with respect to the Revenue per Unit time (C\(_0\)) for different values of correlation coefficients (0.25, 0.50, and 0.75). The monetary benefit going upward with increase in the values of Revenue (C\(_0\)) and profit is much small for lower values of correlation coefficients. We can observe that for a particular value of r = 0.25 cut off value of C\(_0\) is equal to 270 (apprx.) because for more value of C\(_0\) profit is always positive.

From the interpretations as made through various graphs, we can say that the cut off points for various rates/expenses can be obtained which help us to decide the upper and lower acceptable values of rates and costs so that the system is profitable, the upper threshold limit of the failure rate can be evaluated, the lower value of the revenue per unit up time can be estimated. on this basis the organization can fix the cost of the item generated/produced by the company so
that the system can give the profit, the upper threshold limit of the cost per unit time paid for indulging the repairman can also be estimated, the upper and lower limits of different other rates and costs can be estimated. After evaluating these sort of values, different valuable suggestions can be made available to the companies which are very useful to the system managers and engineers.

**Figure - 5.2**

**Figure - 5.3**
Figure - 5.4

Figure - 5.5
Figure - 5.6

Figure - 5.7