ANALYSIS OF TWO DISSIMILAR PARALLEL UNIT SYSTEM WITH SERVER VACATION AND CORRELATED LIFE TIME

In the literature of reliability, most of the authors have examined two unit standby redundant systems. Gupta and Kumar [71] provided a study on profit estimation of two unit maintenance redundant system. Gupta et al.[72-74] discussed the profit of two unit standby system. Gupta and Varshney [75-76] also analyzed the reliability of two non identical unit parallel system with repair time distributions, Guo Tong De (1989) studied the behaviour of two unit cold standby system with preparation time for repair, Gupta and Sharma [77], discussed two non identical unit system for optimizing the reliability. Taneja and Nanda [39] studied two unit cold stand by system with resume and repair policies. Jacob et al.[78] analyzed the two unit deteriorating standby system with repair.

Mine and Kaiwal[79] discussed the repair priority effect on availability of two unit system. Sehgal [80] studied some reliability models with various type of repair. Singh et al.[81,82,83] discussed the two unit standby system with different operating mode. Kumar et al.[84,85] gave the probabilistic analysis of two unit system with instructions at need. Rizwan and Taneja [86,87] gave the reliability analysis of two unit with two repairmen. Two unit stand by systems have been widely studied in literature of reliability theory Gopalan et al.[88,89], Gupta and Taneja [90-91] and Gupta et al.[92], all analyzed a two unit repairable system under the assumption that the operator of the system does not need rest but in real life it is not so, A person cannot work continuously on a particular machine and he/she needs rest after working a random amount of time.

The purpose of the this chapter is to investigate a real life model, running in a Sugar Mill situated at Yamuna Nagar, where we observed that only one repairman was working there so he had to do all the repair work if needed because of only one repairman’s availability. The system’s repairman needed some rest. In this chapter we will study the effect of the appearance and disappearance of repairman on the reliability of the system, so rest is required by repairman instead of the system. There is a system which consists of two dissimilar parallel units. Only one
of them is sufficient for operating the system. System fails completely if both the units fail simultaneously.

Here in this section we have extended the idea of irregular appearance and disappearance of repairperson in a two dissimilar parallel unit framework with failure and repair times of every unit as correlated arbitrary variables having their joint distribution as bivariate exponential. Taking into account the above considerations, the present chapter is devoted to analyze two reliability models i.e. Model-I and Model-II. In both models same system of above said Sugar Mill had been analyzed in different modes.

Here in this chapter we are giving two models, in the first model preference of repair has been given to unit A but in the second model there is no preference of repair, so repairman can choose any unit (A or B) randomly for repair, if both units failed.

Here we are using the joint distribution of failure and repair times is taken to be bivariate exponential of the form suggested by Paulson [45] valid for \((0 \leq r < 1)\). The p.d.f. of the B.V.E. distribution is

\[
f(x, y) = \alpha \beta (1-r) e^{-\alpha x - \beta y} \ I_0(2 \sqrt{\alpha \beta xy})
\]

\(x, y, \alpha, \beta > 0; 0 \leq r < 1\)

where

\[
I_0(z) = \sum_{k=0}^{\infty} \frac{\left( \frac{z}{\alpha} \right)^k}{(k!)^2}
\]

is the modified Bessel function of type I and order zero.

### 2.2 SYSTEM DESCRIPTION AND ASSUMPTION FOR MODEL-I

- Both Models comprises of two dissimilar units A and B.
- Both units are initially operating.
- The operation of only one unit is also sufficient for operating the
system.

• There is single repairman facility which goes on vacation (appears in and disappears) from the system randomly.

• Once the repairman begins the repair of a fizzled unit he doesn't leave the system till all the units are repaired that fizzled amid his stay in the system.

• Service pattern of repair person in the system is of First Come First Serve”FCFS” bases.

• The joint distribution of failure and repair times for each unit is taken to be Bivariate exponential having density function.

• If both units are in failure mode then preference has been given to unit A for repairing first and each repaired unit works as good as new.

2.3 NOTATIONS AND STATES OF THE SYSTEM

For describing the various states of the model we are defining following symbols:

A₀: Unit A is in active mode
B₀: Unit B is in active mode
Afr: Unit A is in failure/down mode
Bfr: Unit B is in down mode
ω: Constant rate of repairman’s availability
θ: Constant rate of repairman’s unavailability
AF: Unit A in failure mode but waiting for repairman
a: P (picking unit A for repair, if A and B failed).
b: P (picking unit B for repair, if A and B failed).
Yᵢ(i=1,2): “Random variables” (R.V) depicting the repair times of both the units A and B respectively for i =1,2
Xᵢ(i=1,2): “Random variables” (R.V) depicting the failure/down times of both the units A and B respectively for i =1,2
\( f_i(x,y)\): Joint pdf of \((x_i, y_i)\); \(i=1,2\)
\[
= \alpha_i \beta_i (1 - r_i) e^{-\alpha_i x_i - \beta_i y_i} I_0(2 \sqrt{(\alpha_i \beta_i r_i)}); X, Y, \alpha_i, \beta_i > 0; 0 \leq r_i < 1
\]
where
\[
I_0(2 \sqrt{(\alpha_i \beta_i r_i)} xy) = \sum_{j=0}^{\infty} \frac{(\alpha_i \beta_i r_i xy)^j}{(j!)^2}
\]

\( k_i(Y/X)\): Conditional pdf of \(Y_i\) given \(X_i=x\) is given by
\[
= \beta_i e^{-\alpha_i x_i - \beta_i y_i} I_0(2 \sqrt{(\alpha_i \beta_i r_i)} xy)
\]

\( g_i(.)\): Marginal pdf of \(X_i=\alpha_i (1 - r_i) e^{-\alpha_i x_i} x_i\)

\( h_i(.)\): Marginal pdf of \(Y_i=\beta_i (1 - r_i) e^{-\beta_i y_i} y_i\)

\( q_{ij}(.), Q_{ij}(.)\): P.D.F. & C.D.F. of transition time from \(S_i\) to \(S_j\).

\( \mu_i\): M.S.T. “Mean sojourn time” in state \(S_i\).

\( \bigoplus\): Symbol/Notations of Laplace convolution
\[
A(t) \bigoplus B(t) = \int_0^t A(t-u)B(u)du
\]

\( \otimes\): Symbol/Notations of Laplace stieltjes convolution
\[
A(t) \otimes B(t) = \int_0^t A(t-u)dB(u)
\]

**Up states**:
- \(S_0\): \((A_0, B_0)\)
- \(S_1\): \((A_f, B_0)\)
- \(S_2\): \((A_0, B_f)\)
- \(S_5\): \((A_0, B_0)\)
- \(S_6\): \((A_0, B_F)\)
- \(S_7\): \((A_F, B_0)\)

**Down states**:
- \(S_3\): \((A_{fR}, B_F)\)
- \(S_4\): \((A_F, B_{fR})\)
- \(S_8\): \((A_F, B_F)\)
- \(S_9\): \((A_{fR}, B_F)\)
It is clear from the model that six states $S_0$, $S_1$, $S_2$, $S_5$, $S_6$, and $S_7$ are regenerative states on the other hand four states $S_3$, $S_4$, $S_8$, $S_9$ are non regenerative.

**TRANSITION DIAGRAM MODEL-I**

![Transition Diagram Model-I](image)

Fig-2.1 : (Model-I)
2.4 TRANSITION PROBABILITY AND SOJOURN TIMES

The steady state transition likelihood “probability” can be expressed as follows

\[ P_{ij} = \lim_{t \to \infty} Q_{ij}(t) \]

\[ p_{01} = \frac{\alpha_1 (1-r_1)}{\phi + \theta} \]

\[ p_{05} = \frac{\theta}{\phi + \theta} \]

\[ p_{12.3} = \frac{\alpha_2 (1-r_2)}{\alpha_2 (1-r_2) + \beta_i (1-r_i)} \]

\[ p_{21.4} = \frac{\alpha_i (1-r_i)}{\beta_i (1-r_2) + \alpha_i (1-r_i)} \]

\[ p_{56} = \frac{\alpha_z (1-r_z)}{\phi + \omega} \]

\[ p_{62.89} = \frac{\alpha_i (1-r_i)}{\alpha_i (1-r_i) + \omega} \]

\[ p_{71} = \frac{\omega}{\alpha_z (1-r_z) + \omega} \]

\[ p_{21.4} + p_{20} = 1 \]

\[ p_{10} + p_{12.3} = 1 \]

\[ p_{50} + p_{56} + p_{57} = 1 \]

\[ p_{62} + p_{62.89} = 1 \]

\[ p_{71} + p_{72.89} = 1 \]

It can be verified that

\[ (2.1-2.14) \]

The mean sojourn time \((\mu_i)\) in the regenerative state ‘i’ are as follows
\[ \mu_0 = \frac{1}{\phi + \theta}, \]
\[ \mu_1 = \frac{1}{\beta_1(1 - r_1) + \alpha_2(1 - r_2)}, \]
\[ \mu_2 = \frac{1}{\beta_2(1 - r_2) + \alpha_1(1 - r_1)}, \]
\[ \mu_5 = \frac{1}{\omega + \phi}, \]
\[ \mu_6 = \frac{1}{\omega + \alpha_1(1 - r_1)}, \]
\[ \mu_7 = \frac{1}{\omega + \alpha_2(1 - r_2)}, \]

(2.15-2.26)

2.5 MEAN TIME TO SYSTEM FAILURE

To estimate the MTSF “Mean time to system failure” of the system, we regarded the down state of the system as a retaining state and after implementing the concept of regenerating processes, by probabilistic arguments, we obtained following recursive expressions

\[ \phi_0(t) = Q_{01}(t) \otimes \phi_1(t) + Q_{02}(t) \otimes \phi_2(t) + Q_{05}(t) \otimes \phi_5(t) \]
\[ \phi_1(t) = Q_{10}(t) \otimes \phi_0(t) + Q_{13}(t) \]
\[ \phi_2(t) = Q_{20}(t) \otimes \phi_0(t) + Q_{24}(t) \]
\[ \phi_3(t) = Q_{30}(t) \otimes \phi_0(t) + Q_{57}(t) \otimes \phi_7(t) + Q_{36}(t) \otimes \phi_6(t) \]
\[ \phi_6(t) = Q_{68}(t) \otimes \phi_2(t) + Q_{68}(t) \]
\[ \phi_7(t) = Q_{71}(t) \otimes \phi_6(t) + Q_{78}(t) \]

(2.27-2.32)

After taking “Laplace Stieltjes Transforms” (L.S.T.) on both sides of these recursive relations and solving for \( \phi_0^*(s) \),

\[ \phi_0^*(s) = \frac{N(s)}{D(s)} \]
Where

\[ N = \mu_0 + \mu_1 (p_{01} + p_{05} P_{71} P_{57}) + \mu_2 (p_{02} + p_{05} P_{56} P_{62}) + \mu_5 P_{05} + \mu_6 p_{05} P_{57} + \mu_8 P_{05} P_{56}. \]

\[ D = 1 - p_{01} P_{10} - p_{02} P_{20} - p_{05} + p_{05} P_{57} (1 - P_{71} P_{10}) + p_{05} P_{56} (1 - p_{62} P_{20}) \]

(2.33-2.34)

### 2.6 AVAILABILITY ANALYSIS

Suppose \( A_i(t) \) be the probability/likelihood that the system is in active mode or in up-state at a specific moment/time “t”, provided that the system entered into regenerative state i at \( t=0 \). Using the concept of the theory of a regenerative methodology the point wise availability \( A_i(t) \) is seen to hold the under mentioned recursive expressions

\[
A_0(t) = M_0(t) + q_{01}(t) \oplus A_1(t) + q_{02}(t) \oplus A_2(t) + q_{05}(t) \oplus A_5(t)
\]

\[
A_1(t) = M_1(t) + q_{10}(t) \oplus A_0(t) + q_{1214}(t) \oplus A_1(t)
\]

\[
A_2(t) = M_2(t) + q_{20}(t) \oplus A_0(t) + q_{214}(t) \oplus A_1(t)
\]

\[
A_3(t) = M_3(t) + q_{30}(t) \oplus A_0(t) + q_{36}(t) \oplus A_6(t) + q_{57}(t) \oplus A_7(t)
\]

\[
A_6(t) = M_6(t) + q_{6289}(t) \oplus A_2(t) + q_{62}(t) \oplus A_4(t)
\]

\[
A_7(t) = M_7(t) + q_{7289}(t) \oplus A_2(t) + q_{71}(t) \oplus A_4(t)
\]

(2.35-2.40)

After Taking Laplace transformation “L.T.” on both sides of the above expressions for availability analysis and solving them for \( A_0^*(s) \), where \( A_0^*(s) \) is the L.T. of \( A_0(s) \), we obtained

We are having

\[
A_0^*(s) = \frac{N_1(s)}{D_i(s)}
\]

(2.41)

The steady state availability is

\[
A_b = \lim_{t \to \infty} (s A_0^*(s)) = \frac{N_1}{D_i}
\]

(2.42)
Where

\[ N_1 = (1 - p_{12.3} p_{21.4})[\mu_0 + \mu_1(p_{01} + p_{05} p_{57} p_{71}) + p_{05}(\mu_5 + \mu_6 p_{56} - \mu_7 p_{57})] + (\mu_2 + \mu_4 p_{21.4})[p_{12.3} p_{01} + p_{02} + p_{05}(p_{56} + p_{57} + p_{57} p_{72.89} + p_{57} p_{71} p_{12.3})] \]

\[ D_i = \mu_0(p_{10} + p_{20} - p_{10} p_{20}) + \mu_i(1 - p_{20} + p_{01} p_{20}(1 - p_{20})p_{05} p_{50} + p_{05} p_{57} p_{71} p_{20}) + \mu_2(1 - p_{01} p_{10} - p_{05} p_{50} + p_{05} p_{57} p_{71} p_{10}) + \mu_5(p_{05} p_{20} + (1 - p_{20}) p_{05} p_{10}) + \mu_6(p_{05} p_{56} + (p_{20} + p_{21.4} p_{10}) + \mu_7(p_{05} p_{57} (p_{20} + p_{21.4} p_{10})) \]

(2.43-2.44)

2.7 BUSY PERIOD ANALYSIS OF THE REPAIRMAN

We are assuming that at moment “t”, \( B_i(t) \) be the probability that the repairman is busy for repairing the failed unit, provided that at \( t=0 \), the system entered in a particular regenerative state \( i \). We obtained the under mentioned recursive expressions for \( B_i(t) \).

\[ \begin{align*}
B_0(t) &= q_{00}(t) \oplus B_1(t) + q_{02}(t) \oplus B_2(t) + q_{05}(t) \oplus B_5(t) \\
B_1(t) &= W_1(t) + q_{10}(t) \oplus B_0(t) + q_{12.3}(t) \oplus B_2(t) \\
B_2(t) &= W_2(t) + q_{20}(t) \oplus B_0(t) + q_{21.4}(t) \oplus B_1(t) \\
B_5(t) &= q_{50}(t) \oplus B_0(t) + q_{57}(t) \oplus B_1(t) + q_{56}(t) \oplus B_6(t) \\
B_6(t) &= q_{62}(t) \oplus B_2(t) + q_{62.89}(t) \oplus B_3(t) \\
B_7(t) &= q_{71}(t) \oplus B_1(t) + q_{72.89}(t) \oplus B_2(t)
\end{align*} \]

(2.45-2.50)

Taking “Laplace Transformation” (L.T) of the above expressions (on both sides) of busy period of repairman of failed units and solving them for \( B_0^*(s) \), where \( B_0^*(s) \) is the L.T. of \( B_0(s) \), we obtained

\[ B_0^*(s) = \frac{N_2(s)}{D_1(s)} \]

(2.51)

In the steady state

\[ B_0 = \lim_{s \to 0} (sB_0^*(s)) = \frac{N_2}{D_1} \]

(2.52)
Where

\[ N_2 = \mu_1 (p_{01} + p_{21.4} p_{02} + p_{05} p_{21.4} p_{56} - p_{05} p_{21.4} p_{57} - p_{05} p_{71} p_{57}) \]
\[ \mu_2 (p_{12.3} p_{01} + p_{02} + p_{05} p_{56} + p_{05} p_{72.89} + p_{05} p_{12.3} p_{57} p_{71}) \]

(2.53)

\[ D_1 \] is has already been evaluated.

### 2.8 EXPECTED NUMBER OF VISITS BY THE REPAIRMAN

For repairing and replacement we assumed \( V_i(t) \) as the expected number of visits by the repairman in the interval \((0,t]\), provided the system initially begins from \( S_i \) ("a regenerative state"). By utilizing the concept of regenerative point, we obtained the below mentioned relations for \( V_i(t) \) which are recursive in nature

\[
V_0(t) = Q_{01}(t) \otimes (1 + V_1(t)) + Q_{02}(t) \otimes (1 + V_2(t)) + Q_{05}(t) \otimes V_5(t)
\]
\[
V_1(t) = Q_{10}(t) \otimes V_0(t) + Q_{12.3}(t) \otimes V_2(t)
\]
\[
V_2(t) = Q_{20}(t) \otimes V_0(t) + Q_{21.4}(t) \otimes V_1(t)
\]
\[
V_5(t) = Q_{50}(t) \otimes V_0(t) + Q_{57}(t) \otimes V_5(t) + Q_{56}(t) \otimes V_6(t)
\]
\[
V_6(t) = Q_{62}(t) \otimes V_2(t) + Q_{62.89}(t) \otimes V_2(t)
\]
\[
V_7(t) = Q_{71}(t) \otimes V_1(t) + Q_{72.89}(t) \otimes V_2(t)
\]

(2.54-2.59)

After having “Laplace Stieltjes Transform” L.S.T. on both sides of the above written expressions of expected number of visits and evaluating them for \( V_0^{**}(s) \), we are having

\[
V_0^{**}(s) = \frac{N_3(s)}{D_1(s)}
\]

(2.60)

In steady state

\[
V_0 = \lim_{s \to 0} (sV_0^*(s)) = \frac{N_3}{D_1}
\]

(2.61)
Where

\[ N_3 = (1 - p_{05})(1 - p_{12.3}p_{21.4}) \]  

(2.62)

D_1 is has already been evaluated.

2.9 PROFIT ANALYSIS

The expected benefit (financially) incurred to the system in steady state is given by

\[ P = C_0A_0 - C_1B_0 - C_2V_0 \]  

(2.63)

Where

- \( C_0 = \) Revenue per unit uptime of the system
- \( C_1 = \) Cost (expenses) per unit time under which repairman is busy
- \( C_2 = \) Cost (expenses) per visit for the repairman

Particular case

According to the numerical values, taken as

\[ \alpha_2 = .005, \beta_1 = .03, \beta_2 = .02, r_2 = 0.6, r_{11} = 0.25 \]

Mean Time to System Failure (MTSF) = 693.20

Availability (A_0) = 0.3346

Busy Period of Repairman (B_0) = 0.1403

Expected no. of visits by the repairman (V_0) = 0.002246

Profit = 278.51

2.10 GRAPHICAL STUDY OF SYSTEM BEHAVIOR

For finding out the more clear picture of the system’s characteristics with respect to the different parameters. We plot graphs for Mean Time to System Failure “MTSF” and profit function in figure-2.2 and figure-2.5 with respect to the failure parameter (\( \alpha_i \)) of sub-unit A for three distinct values of correlation coefficient (\( r_{11} = 0.25, r_{12} = 0.50, r_{13} = 0.75 \)), for distinct values of correlation
coefficient ‘r’, between above defined random variable X and Y, however the other parameters are assumed constant as

\[ \alpha_2 = .005, \beta_1 = .03, \beta_2 = .02 \]

\[ C_0 = 1000, C_1 = 400, C_2 = 80 \]

From the Figure - 2.2 it is cleared that Mean Time to System Failure diminishing as failure rate moving right side of the graph regardless of other parameters. The graph also indicates that for the fixed failure rate, Mean Time to System Failure is higher for higher values of correlation coefficient (r). So we are in the position to conclude that the high estimation of r between failure and repair has a tendency to expand the normal life time of the system.

Figure - 2.3 shows that busy period of the repairman going upward direction as the Failure rate moving right side of X-axis.

Figure - 2.4 shows the pattern of the availability of the system with respect to the failure rate as availability decreases as the failure rate increases.

From the Figure - 2.5 it is clear that profit decreases as failure rate increases. Also for the fixed value of failure rate, the profit is higher for high correlation (r). So finally we conclude that the high correlation between failure and repair of the system yields the better system performance.
Figure - 2.3

Busy period vs Failure Rate

Figure - 2.4

Availability vs Failure Rate
Figure - 2.5

MODEL-II
2.11 SYSTEM DESCRIPTION AND ASSUMPTION FOR MODEL-II

- System consists of two non-identical (dissimilar) units A and B, both units are initially operating but the operation of only one unit is also sufficient for operating the system.

- There is single repair facility which appears in and disappears from the system randomly.

- Once the repairman begins the repair of a fizzled unit he doesn't leave the system till all the units are repaired that fizzled amid his stay in the system.

- Service pattern of repair person in the system is of First Come First Serve”FCFS” bases.

- The joint distribution of failure and repair times for each unit is taken to be bivariate exponential.

- If Both Units are in failure mode then preference has not been given to unit A for repairing first (as in Model-I) but repairman will pick the unit for repair at randomly.
Probability of picking unit A is “a” and probability of picking unit B is “b”, where \( a + b = 1 \). Each repaired unit works as good as new.
2.12 TRANSITION PROBABILITY AND SOJOURN TIMES

The steady state transition probability can be as follows

\[ P_{01} = \frac{\alpha_2(1-r_2)}{\phi + \theta} \]
\[ P_{02} = \frac{\alpha_2(1-r_2)}{\phi + \theta} \]
\[ P_{05} = \frac{\theta}{\phi + \theta} \]
\[ P_{12.3} = \frac{\alpha_2(1-r_2)}{\alpha_2(1-r_2) + \beta_1(1-r_1)} \]
\[ P_{20} = \frac{\beta_2(1-r_2)}{\beta_2(1-r_2) + \alpha_1(1-r_1)} \]
\[ P_{21.4} = \frac{\alpha_1(1-r_1)}{\beta_2(1-r_2) + \alpha_1(1-r_1)} \]
\[ P_{57} = \frac{\alpha_1(1-r_1)}{\phi + \omega} \]
\[ P_{56} = \frac{\alpha_2(1-r_2)}{\phi + \omega} \]
\[ P_{50} = \frac{\omega}{\phi + \omega} \]
\[ P_{62.89} = \frac{a\alpha_1(1-r_1)}{\alpha_1(1-r_1) + \omega} \]
\[ P_{61.89} = \frac{b\alpha_1(1-r_1)}{\alpha_1(1-r_1) + \omega} \]
\[ P_{63} = \frac{\omega}{\alpha_1(1-r_1) + \omega} \]
\[ P_{71} = \frac{\omega}{\alpha_2(1-r_2) + \omega} \]
\[ P_{72.89} = \frac{a\alpha_2(1-r_2)}{\alpha_2(1-r_2) + \omega} \]
\[ P_{71.89} = \frac{b\alpha_2(1-r_2)}{\alpha_2(1-r_2) + \omega} \]

(2.64-2.80)

Here we get,
\[ P_{01} + P_{02} + P_{05} = 1 \]
\[ P_{21.4} + P_{20} = 1 \]
\[ P_{10} + P_{12.3} = 1 \]
\[ P_{50} + P_{56} + P_{57} = 1 \]
\[ P_{62} + P_{62.89} + P_{61.89} = 1 \]
\[ P_{71} + P_{72.89} + P_{71.89} = 1 \]

Mean sojourn time “M.S.T.” are as follows

\[ \mu_0 = \frac{1}{\phi + \theta} \]
\[ \mu_2 = \frac{1}{\beta_2(1-r_2) + \alpha_1(1-r_1)} \]
\[ \mu_6 = \frac{1}{\omega + \alpha_1(1-r_1)} \]
\[ \mu_1 = \frac{1}{\beta_1(1-r_1) + \alpha_2(1-r_2)} \]
\[ \mu_5 = \frac{1}{\omega + \phi} \]
\[ \mu_7 = \frac{1}{\omega + \alpha_2(1-r_2)} \]

(2.81-2.93)
2.13 MEAN TIME TO SYSTEM FAILURE

To evaluating the Mean time to system failure “MTSF” of the system, we supposed that the failed state of the system is an retaining state, by using the concept of probability arguments, we obtained

\[
\begin{align*}
\phi_0(t) &= Q_{01}(t) \otimes \phi_1(t) + Q_{02}(t) \otimes \phi_2(t) + Q_{05}(t) \otimes \phi_5(t) \\
\phi_1(t) &= Q_{10}(t) \otimes \phi_0(t) + Q_{12}(t) \\
\phi_2(t) &= Q_{20}(t) \otimes \phi_0(t) + Q_{24}(t) \\
\phi_5(t) &= Q_{50}(t) \otimes \phi_0(t) + Q_{57}(t) \otimes \phi_7(t) + Q_{56}(t) \otimes \phi_6(t) \\
\phi_6(t) &= Q_{68}(t) \otimes \phi_2(t) + Q_{68}(t) \\
\phi_7(t) &= Q_{71}(t) \otimes \phi_1(t) + Q_{76}(t)
\end{align*}
\]

(2.94-2.99)

After taking “Laplace Stieltjes Transforms” (L.S.T.) on both sides of these recursive relations and solving for \(\phi_0^*(s)\),

\[
\phi_0^*(s) = \frac{N(s)}{D(s)}
\]

(2.100)

Where

\[
\begin{align*}
N &= \mu_0 + \mu_1(p_{01} + p_{05}p_{71}p_{57}) + \mu_2(p_{02} + p_{05}p_{56}p_{62}) + \mu_5p_{05} + \mu_7p_{05}p_{57} + \mu_6p_{05}p_{56} \\
D &= 1 - p_{01}p_{10} - p_{02}p_{20} - p_{05} - p_{05}p_{57}(1 - p_{71}p_{10}) + p_{05}p_{56}(1 - p_{62}p_{20})
\end{align*}
\]

(2.101-2.102)

2.14 AVAILABILITY ANALYSIS

For a repairable framework, availability is playing a significant role in achieving a high or needed level of reliability of that framework. Availability of a framework is basically measured as an important characteristic of reliability of that framework/system, as reliability is directly proportional to the availability of the product.

Suppose \(A_i(t)\) be the probability/likelihood that the system is in active mode or in up-state at a specific moment/time “t”, provided that the system entered into regenerative state i at t=0. Using the concept of the theory of a regenerative
methodology the point wise availability \( A_i(t) \) is seen to hold the under mentioned recursive expressions

\[
A_0(t) = M_0(t) + q_{01}(t) \oplus A_1(t) + q_{02}(t) \oplus A_2(t) + q_{05}(t) \oplus A_5(t)
\]

\[
A_1(t) = M_1(t) + q_{10}(t) \oplus A_0(t) + q_{12.3}(t) \oplus A_2(t)
\]

\[
A_2(t) = M_2(t) + q_{20}(t) \oplus A_0(t) + q_{21.4}(t) \oplus A_4(t)
\]

\[
A_5(t) = M_5(t) + q_{50}(t) \oplus A_0(t) + q_{56}(t) \oplus A_6(t) + q_{57}(t) \oplus A_7(t)
\]

\[
A_6(t) = M_6(t) + q_{62.89}(t) \oplus A_2(t) + q_{63}(t) \oplus A_3(t) + q_{61.89}(t) \oplus A_1(t)
\]

\[
A_7(t) = M_7(t) + q_{72.89}(t) \oplus A_2(t) + q_{71.72}(t) \oplus A_7(t) + q_{71.89}(t) \oplus A_1(t)
\]

(2.103-2.108)

Taking Laplace transformation “L.T.” on both sides of the above expressions for availability analysis and solving them for \( A_i^*(s) \), where \( A_i^*(s) \) is the L.T. of \( A_0(s) \), we obtained

\[
A_i^*(s) = \frac{N_i(s)}{D_i(s)}
\]

(2.109)

In the steady-state, availability is given by

\[
A_i = \lim_{s \to 0} (sA_i^*(s)) = \frac{N_i}{D_i}
\]

Where

\[
N_i = (1 - p_{12.3})P_{21.4}[\mu_0 + \mu_1(P_{01} + P_{05}P_{57}P_{71}) + P_{05}(\mu_5 + \mu_6P_{56} - \mu_7P_{57})]
+(\mu_2 + \mu_1P_{21.4})[P_{12.3}P_{01} + P_{02} + P_{05}(P_{56} + P_{57} + P_{57}P_{72.89} + P_{57}P_{61.89}P_{12.3})]
\]

\[
D_i = (p_{10} + p_{20} - p_{10}p_{20})\mu_0 + \mu_1(P_{05} + \mu_6P_{56}P_{05} + \mu_7P_{57}P_{05}) + \mu_1[p_{57}P_{05}(1 - p_{20}P_{72.89})
+p_{05}P_{56}(1 - p_{20} + p_{20}P_{61.89}) + (p_{01} + p_{02} - p_{01}p_{20})] + \mu_2[p_{57}P_{05}(1 - p_{10}P_{61.89})
+p_{05}P_{57}(1 - p_{10} + p_{10}P_{72.89}) + (p_{01} + p_{02} - p_{01}P_{10})]
\]

(2.110-2.111)

2.15 BUSY PERIOD ANALYSIS OF THE REPAIRMAN

We are assuming that at moment “t”, \( B_i(t) \) be the probability that the repairman is busy for repairing the failed unit, provided that at \( t=0 \), the system entered in a particular regenerative state \( i \). We obtained the under mentioned recursive expressions for \( B_i(t) \).
\[ B_0(t) = q_{01}(t) \oplus B_1(t) + q_{02}(t) \oplus B_2(t) + q_{03}(t) \oplus B_3(t) \]
\[ B_1(t) = W_1(t) + q_{10}(t) \oplus B_0(t) + q_{12.3}(t) \oplus B_2(t) \]
\[ B_2(t) = W_2(t) + q_{20}(t) \oplus B_1(t) + q_{21.4}(t) \oplus B_3(t) \]
\[ B_3(t) = q_{30}(t) \oplus B_0(t) + q_{37}(t) \oplus B_1(t) + q_{56}(t) \oplus B_2(t) \]
\[ B_4(t) = q_{42}(t) \oplus B_2(t) + q_{42.89}(t) \oplus B_2(t) + q_{61.89}(t) \oplus B_1(t) \]
\[ B_5(t) = q_{51}(t) \oplus B_2(t) + q_{57.89}(t) \oplus B_2(t) + q_{71.89}(t) \oplus B_1(t) \]

Taking “Laplace Transformation” (L.T) of the above expressions (on both sides) of busy period of repairman of failed units and solving them for \( B'_0(s) \), where \( B'_0(s) \) is the L.T. of \( B_0(s) \), we obtained

\[ B'_0(s) = \frac{N_2(s)}{D_1(s)} \]

In the steady-state, busy period is

\[ b_0 = \lim_{s \to \infty} \frac{N_2(s)}{D_1} \]

Where

\[ N_2 = \mu_1(P_{01} + P_{02}P_{21.4} + (1 - P_{61.89}) + P_{05}P_{56}P_{21.4} + P_{05}P_{61.89}P_{56} - P_{05}P_{57}P_{21.4} - P_{05}P_{57}P_{71}) \]
\[ + \mu_2[P_{12.3}P_{01} + P_{02} + P_{05}P_{56} + P_{05}P_{57}P_{72.89} + P_{05}P_{57}P_{71}P_{12.3}] \]

(2.115)

\( D_1 \) has already been specified (equation-2.111).

### 2.16 EXPECTED NUMBER OF VISITS BY THE REPAIRMAN

For repairing the failed unit, we assumed \( V_i(t) \) as the expected number of visits by the repairperson in the interval \((0,t]\), provided the system initially begins from \( S_i \) (“a regenerative state”), By utilizing the concept of regenerative point, we obtained the below mentioned relations for \( V_i(t) \) which are recursive in nature

\[ V_0(t) = Q_{01}(t) \otimes (1 + V_i(t)) + Q_{02}(t) \otimes (1 + V_2(t)) + Q_{03}(t) \otimes V_3(t) \]
\[ V_1(t) = Q_{10}(t) \otimes V_0(t) + Q_{12.3}(t) \otimes V_2(t) \]
\[ V_2(t) = Q_{20}(t) \otimes V_0(t) + Q_{21.4}(t) \otimes V_1(t) \]
\[ V_3(t) = Q_{30}(t) \otimes V_0(t) + Q_{37}(t) \otimes V_1(t) + Q_{56}(t) \otimes V_6(t) \]
\[ V_4(t) = Q_{42}(t) \otimes V_2(t) + Q_{42.89}(t) \otimes V_2(t) + Q_{61.89}(t) \otimes V_4(t) \]
\[ V_5(t) = Q_{51}(t) \otimes V_2(t) + Q_{57.89}(t) \otimes V_3(t) + Q_{71.89}(t) \otimes V_5(t) \]
\[ V_6(t) = Q_{62}(t) \otimes V_2(t) + Q_{62.89}(t) \otimes V_2(t) + Q_{61.89}(t) \otimes V_6(t) \]
\[ V_7(t) = Q_{71}(t) \otimes V_2(t) + Q_{72.89}(t) \otimes V_3(t) + Q_{72.89}(t) \otimes V_5(t) \]

(2.116-2.121)
After having “Laplace Stieltjes Transform” L.S.T. on both sides of the above written expressions of expected number of visits and evaluating them for $V_0''(s)$, we are having

$$V_0''(s) = \frac{N_3(s)}{D_1(s)}$$

In steady state, Expected Number of visit by the repairman is,

$$V_0 = \lim_{s\to 0}(sV_0'(s)) = \frac{N_3}{D_1}$$

(2.122)

Where

$$N_3 = (1 - p_{06})(1 - p_{12.3}p_{21.4})$$

(2.123)

$D_1$ has already been estimated (in equation-2.111).

### 2.17 PROFIT ANALYSIS

The expected total profit incurred to the system in steady state is given by

$$P = C_0 A_0 - C_1 B_0 - C_2 V_0$$

(2.124)

Where

- $C_0$ = Revenue per unit uptime/active time of the system
- $C_1$ = Cost/unit time for which repairperson is busy in repair work
- $C_2$ = Cost/visit for the repairperson for repair and replacement of a unit

### 2.18 PARTICULAR CASE

On the basis of the numerical values taken as

$\alpha_1 = .01, \alpha_2 = .005, \beta_1 = .03, \beta_2 = .02, \omega = .002, p(a) = 0.4, p(b) = 0.6, r_2 = 0.6, r_{11} = 0.25$

Mean Time to System Failure (MTSF) = 693.20

Availability ($A_0$) = 0.26

Busy Period of Repairman ($B_0$) = 0.2283

Expected no. of visits by the repairman ($V_0$) = 0.00347
Profit=165.75
If \( r_1 = r_2 = 0 \)
Mean Time to System Failure (MTSF) = 350.48
Availability \((A_0) = 0.2599\)
Busy Period of Repairman \((B_0) = 0.277\)
Expected no. of visits by the repairman \((V_0) = 0.0060\)
Profit=155.75

2.19 GRAPHICAL STUDY OF SYSTEM BEHAVIOR

Behaviour of Mean Time to System Failure and the profit have been observed with respect to various failure rates, probabilities and different costs also plotting graphs. It has been noticed that the MTSF and the profit get diminished with increment in the failure rate.

For an all the more clear perspective of the framework attributes w.r.t. the different parameters included. We plot different bends for Mean Time to System Failure and profit function in Figure-2.7 and Figure-2.10 we plot the curve with different values of failure rate “\( \alpha \)” of sub-unit A and for various values of “\( r \)” \((r_{11} = 0.25, r_{11} = 0.50, r_{11} = 0.75 \) ) while the other parameters are kept constant as \( \alpha_2 = .005, \beta_1 = .03, \beta_2 = .02, \omega = .002, \theta = .005 \)

\( p(a) = 0.4, p(b) = 0.6C_0 = 1000, C_1 = 400, C_2 = 80 \)

From the Figure-2.7 it is depicted that values of MTSF going downward as the failure rate increases regardless of other parameters. This trend also depicted that for any fixed value of failure rate of sub-unit A, mean time to system failure is high for higher values of \( r \),so here we can concluded that the high value of \( r \) tends to increase the reliability of the system. Availability of any framework is basically measured as an important component Figure-2.8 shows the pattern of availability with respect to the failure rate for different values of the correlation coefficients(r). Availability of the system decreases as the failure rate of the unit increases.

Figure-2.9 depicts that busy period of the repairman increases as the Failure rate increases.
From the Figure-2.10 it is clear that profit curve getting downward (linearly) as failure rate moving right hand side of the axis. It can also be seen that for a particular value of failure rate, the profit is higher for higher value of coefficient (r).

Following can also be interpreted from Figure-2.10

- For \( r_{11} = 0.25 \) the profit is depending upon whether the failure rate is \(<\) or \(\geq\) 0.03685, so in this case the system is profitable only if the failure rate is less than 0.03685
- For \( r_{12} = 0.50 \) the profit is depending upon whether the failure rate is \(<\) or \(\geq\) 0.04793, so in this case the system is profitable only if the failure rate is less than 0.04793
- For \( r_{13} = 0.75 \) the profit is depending upon whether the failure rate is \(<\) or \(\geq\) 0.06884, so in this case the system is profitable only if the failure rate is less than 0.06884

So at last we come to the point that the high correlation between failure and repair of the system yields the better system performance.

Figure-2.11 shows that values of MTSF are increases with increase in the values of the repair rate and values of MTSF are higher for the higher values of Correlation coefficients.

Figure-2.12 depicts that the MTSF is decreases with increase in the values of the failure rate for different values of correlation coefficients and it can also be analyzed that for

\[ r_1 = r_2 = 0 \]

values of MTSF are quite low as compared to the high values of correlation coefficients.

Similar trend of the graph is depicted. From Figure-2.13 and Figure-2.14 we can see that Busy period and profit of the system is also very small as compared to the high values of correlation coefficients.

From the interpretations as made through different graphs, we can conclude that the cut off points for various failure rates can be obtained which help us to decide the upper and lower acceptable values of failure rates. so that the system is profitable. the upper limit of the failure rate can be obtained, the lower value of the revenue per unit up time can be obtained on the basis of which the company
can fix the price of the product manufactured by the company so that the system gives the positive profit.

![MTSF vs Failure Rate](image1)

**Figure - 2.7**

![Availability vs Failure Rate](image2)

**Figure - 2.8**
Figure - 2.9

Figure - 2.10
MTSF vs Repair Rate

Figure - 2.11

MTSF vs Failure Rate

Figure - 2.12