CHAPTER - 1

INTRODUCTION

Since time immemorial humanity treasured reliability as a very important human attribute. A person is said to be reliable if he/she is consistent and trustworthy in all of his/her actions. It is the need of the hour today that most of the organization looking for reliable persons. Humanity craves for things which are consistent and predictable. Much as human beings treasure reliability in human behavior, it is not easy to define its characteristics or to be able to measure it with precision. In practice no clear line can be drawn between a person who is reliable and one who happens not to be. On the basis of some defined human functionality we can make a judgment on the reliability of an individual. So a person who is working with an organization described as a reliable individual if he or she is punctual at work or in attending meetings.

Generally, Reliability in a wider sense may be considered as a measure of performance. A reliable person is one who has a good track record of doing what he/she promised to do. If a person continually completes his/her tasks, whatever he/she promised to do, he/she is then described as a reliable individual. If a person says he will show up at a particular time, and he is known to be reliable, you can count on him to be on time. So Reliability of every individual depends on the time at which or in which they perform any particular job which may be taken as a measure.

Concept of Reliability does not apply to the actions of persons only, but also to the equipments they utilize in their day to day life. The reliability has been applied to human’s actions but as far as the equipments are concerned which they have made or invented, his/her expectations regarding the reliability are even higher. That’s why it does not only frustrate his/her feelings but also wastes time, costs money and endangers life. According to Green and Bourne [1] the consequences of unreliability has led to individual’s greater interest in reliability and to acquire more reliable equipments. Industrial Revolution has led to an increase in the number of complex products in addition to the complexity of the products themselves. To fulfill the expectations of consumers the complexity of
the products has been increased significantly. These complications in the products and machinery have attracted a number of researchers and scientists from various disciplines especially the Mathematicians, Engineers (Software/Hardware) and Applied Probabilists. These developments have seen the emergence of reliability theory as an another scientific discipline which deals with various issues such as the understanding of the degradation mechanism, the design of reliable equipments and the Maintenance of unreliable equipments. Gnedenko et al.[2] indicated that reliability theory assigns quantitative indices to qualities of production which are computed from the design stage through manufacturing process to use and storage of manufactured goods and operating systems. Increased reliability of manufactured goods and operating systems is a challenge to Governments, engineers and scientists. According to Lloyd and Lopow [3] unreliability costs money, time wasted and inconveniences of the users. The journal on reliability known as IEEE-Transactions on Reliability was published in the year 1963.

Mathematical modeling is very helpful tool to the system designer who is facing the problems of evaluation of several measures of system performance and methods of improving them. These models explain the theoretical and practical features of the system under consideration taking into account its essential features.

Since frequent failures and less availability of equipment are becoming more and more unacceptable so the demand for equipments that perform without failure in a specified time interval is increasing day by day. Repairing of failed units and redundancy (by using some standby units) are two very important methods of improving the availability of a system.

Reliability Theory is multidisciplinary in nature because for solving the different problem we required no. of probability theory methods, Theory of statistics, Optimization techniques, Stochastic process, LPP, NLPP, logical reasoning, the methods of statistical simulation on electronic computers, demography etc. Reliability Theory has been applied in software Reliability, Space word and Global terrestrial system, Spare stock system, phase dependence of population growth, Warranty claim rate projection, fluctuations in business investments etc.
In addition, Mathematical models based on Probability Theory and stochastic processes. The topic reliability has been adopted as the subject in various universities all over the world in their curricula. Prof. F. Paroschan did a remarkable pioneering work in the field of Reliability Theory and thus he is rightly called **Father of Reliability.**

During the last three decades, the common people have become aware of the uncooperative situations arising due to complexity in system and reliability and have started attracting more and more experts/scholars from all over the world. They contribute in this pioneer field. The researchers not only determined the reliability of the system but also evaluated other measures in their studies.

### 1.2 A BRIEF HISTORY AND REVIEW OF LITERATURE

The importance of reliability and quality control was born out of the demands of modern technology used in the World War II. During the war the Army and the Navy in the USA set up a joint committee known as the Vacuum Tube Development Committee in 1943 to study the failures of vacuum tubes which was considered to be one of the focal points of trouble. Bell Laboratories and Aeronautical Radio Inc. (ARINC) were the two leading organizations among those who contributed heavily in this area. The first major committee on reliability was set up by the US Department of Defense in 1950. This was later called the Advisory Group on Reliability of Electronic Equipment (AGREE). The AGREE published its first report in 1957 which included some reliability specifications such as minimum acceptability limits, reliability test requirements etc. Around 1960 a number of reliability and logistic problems were solved by Black and Proschan[4].

Problems have emerged in the design of highly reliable technical systems which include: the creation classes of probability-statistical models which may be used in description of the reliability behavior of the system, and the development of mathematical methods for the assessment of the reliability characteristics of systems.

These problems encouraged studies into the development of high-accuracy methods of reliability analysis. Gnedenko et al.[2], Barlow et al. [5,6,7], Gaver [7], Gertsbakh [8] and Kovalenko et al. [10] considered redundant systems and

Reliability is applicable in many areas of research, a suitable form of reliability form may be introduced. Stochastic analysis is based on good probability models with the ultimate aim of giving numerical estimates of reliability characteristics. Reliability offers by itself solutions to a number of problems not handled by the usual standard probability theoretical approach. According to Gertsbakh [8] reliability of a system depends on the reliability of its components, provides a mathematical expression of aging process, offers well-developed methods of renewal theory, introduces redundant systems to optimize the performance of standby components Gnedenko et al.[2], provides the theory of optimal preventive maintenance and is also a study of inferential statistics often of censored data. Reliability theory of technical objects and survival analysis of biological entities are similar with the exception of notation. Therefore the term “lifetime” is applicable to engineering systems, components, units etc. and to the disciplines like biological, financial etc. with minor modifications.

Reliability is both desirable as well as a necessary and essential component in the present day innovation for accomplishing sound financial advancement of a country. Presently a days, Reliability is not just a subject of study for researchers and academicians additionally a genuine concern to the practising designers, economists and government pioneers also. Unwavering quality contemplations make more powerful utilization of assets and it brings about an increment in gainfulness and diminishing in wastage of cash, material and labor. In the advancement of better than ever innovative frameworks reliability is obtaining uncommon imperativeness as one among the numerous critical framework measures, for example, execution, cost, and so on. Inconsistency generally results
in high cost of repair, upkeep as well as down time and as such reliability is a financial need, much more required for developing nations.

Reliability has now become a settled and a decently formed exploration of foreseeing, evaluating and upgrading the likelihood of survivability of an equipment, framework or mission. Not much importance was given to reliability engineering in the past years and it was perceived just in subjective sense. It was when certain studies, conveyed out amid the post Second World War period, uncovered numerous astonishing results and the consideration of researchers and specialists was drawn for further genuine examination towards it with innovative headway and increment in multifaceted nature and complexity of framework, reliability has procured prime importance in the present era.

Today is an era of industrial growth. Reliability thought has possessed an undeniably essential place in all engineering branches. As the interest for the framework that performs better and expenses less, has expanded so there is an financial requirement to minimize the probability of failures whether the failures essentially build cost and inconvenience or threaten the public security. Notwithstanding, to think about a system without failure is truly unimaginable. The system may come up short in its operation because of different reasons common, human and mechanical. But even after these lapses, a system can be made solid and reliable to use by giving fitting upkeep and repair facilities at certain point of time of damages. In this manner, true endeavors and precautionary measures can uplift the level of reliability of the system with least costs.

Reliability turned into a subject of extraordinary engineering interest in the 1950’s because of the failure of American rockets as well as the failure of the first commercial air ship; the British de Havilland comet. Life testing (MTSF) was important part of this engineering interest. Epstein and Sobel’s [17] paper studying the exponential distribution was a landmark contribution. On the other hand it was not until 1961 with the publication of Birnbaum, Esary and Saunders [19, 20] paper on coherent structures that reliability theory started to be dealt with as a different subject. The main attention in this paper was on theory.
The Boeing 707 Plane project was under progress at the time the de Havilland comets were crashing. It was mostly hence that the “Boeing Scientific Research Laboratories” (BSRL) in Seattle started to accentuate “Reliability Theory” in their mathematics division. Z. W. Birnbaum from the University of Washington was an advisor to this gathering. Z. W. had a solid mathematical and scientific background. He studied under Steinhaus and Banach among others. He had an unique ability for getting rapidly to the nub of an issue, particularly in his counseling. Initially life time distribution and repair time distribution were taken as exponential distribution. Gaver [8] was the first to generalize the repair time distributions and used supplementary variable technique to analyze the model. Branson and Shah [21] applied semi Markov Process method. Srinivasan and Gopalan [23] applied regenerative point technique. Nakagawa and Osaki [24,25] analyzed the system taking both the distribution in general. During the last few decades several authors have concentrated on the analysis of repairable systems. These system models are analyzed in respect of their various reliability characteristics viz. mean time to system failure, availability, mean time to repair, maintainability, busy period etc. Generally these operating characteristics are obtained in terms of Laplace Transforms. Earlier contribution in this direction includes BarLow and Proschan [5] Nakagawa and Osaki [25] considered a two unit standby redundant system with repair and preventive maintenance. Nakagawa[26] assumed the substitution of the unit at a certain point of time of damage. Goyal and Murari [27] studied a two unit standby framework with two sort of repair facility. Goel, Sharma and Gupta [28] dealt with a standby system under diverse climate conditions. Goel, Sharma and Gupta [29] discussed the reliability analysis of a model with preventive maintenance and various types of repair. AL-Ali and Murari [30] investigated the reliability of a system subject to irregular shocks and preventive support. Mokadis et al. [31, 32] delivered the profit analysis of two priority system with delay in repair and degradation concept. Tuteja and Taneja [33,34] delivered a paper on profit analysis of standby unit system presenting the concept of two identical repairmen, minor repairs, partial failure and random inspection. Rander, Kumar and Tuteja [35] presented a paper
on a system with major and minor failures and preparation time. Tuteja and Taneja [36] discussed analysis of reliability models with distinct sorts of failure and repair. Siwach et al. [37] studied two-unit cold standby framework with guideline and mishap. Taneja and Naveen [38] discussed reliability models under the assumption of patience time and non-availability of expert repairmen. Taneja and Nanda [39] discussed the assumption of acquiring one of the two repair policies-repeat repair policy or resume repair policy by the specialist repairman after the last job done by the normal repairman.

Generally researchers and engineers have assumed that the underlying failure and repair processes in the system are un-correlated. The assumption of un-correlated failure and repair time’s results in a simpler analysis of the system commonly studied. In practice, it is less justifiable from the view points of modeling realistic systems. Some kind of relationship often exists between failure and repair mechanism. The concept of correlation between failure and repair times has been studied by Goel et al. [40] for this new concept, “multivariate exponential distributions” (M.E.D.) are required. A number of multivariate exponential distributions are known, yet they have not been obtained by methods that shed light on their relevance however Marshall and Olkin [41] introduced some significant inductions and derivations of M.E.D. that serves to indicate constraints under which the multivariate exponential distribution is appropriate.

1.3 RELIABILITY ENGINEERING

Reliability Engineering is the branch of engineering which ensuring that a system (or a gadget/unit) will perform its proposed function(s) in a specified manner for a specified interval of time duration. Reliability engineering is performed all through the whole life cycle of a framework, including designing, advancement, test, generation and operation.

Reliability may be characterized in few ways:

- The thought that something is suitable for some reason w.r.t. time;
- The limit of a gadget or framework to execute as outlined;
- The imperviousness to disappointment of a gadget or framework;
- The capacity of a gadget or framework to perform an obliged capacity under
expressed conditions for a determined period to time;

- The probability that a functional unit will its obliged capacity for a tagged interim under expressed conditions

Reliability Engineers depend vigorously on statistics, likelihood hypothesis, and Reliability Theory. Numerous engineering strategies and techniques are applicable in reliability engineering, such as reliability forecast, Weibull analysis, availability testing and quickened life testing. In view of the substantial number of reliability procedures, their cost and the changing degrees of dependability needed for diverse circumstances, generally ventures create an unwavering quality project plan to determine the dependability assignments that will be performed for that particular framework. The function of reliability engineering is to create the reliability prerequisites for the item, establish an effective reliability program, and accordingly perform analyses and tasks to ensure the item would meet its requirements. These tasks are observed and managed by an engineer, who is specialized in reliability education and training. Reliability engineering is nearly connected with maintainability engineering and logistics designing. Numerous issues from different fields, such as security engineering, can likewise be approached utilizing reliability engineering techniques.

![Fig. 1.1: A Reliability Block Diagram](image-url)
1.4 SCOPE OF RELIABILITY

The scope of reliability can be visualized by the following facts in respect of any equipment or system:

- The work space of the framework/system.
- The need of security view-points for men and material.
- Level of vulnerability about the accomplishment of operation and its enhancements in Framework/system execution.
- Requirement for proficient, monetary and constant running of framework without aggravations.
- A failure of a supplies/framework brings up the issue in the minds of the individuals in regards to its dependability and its further utilization.
- Improvement and change in the certainty of the working personnel especially in the unsafe zone because of safety reasons.

Any gadget/framework made is composed with specific goals to reach its objectives in terms of production/service. In view of distinctive natural conditions the system may not give the sought results over period of time. This sort of system may be dealt with as unreliable. In this way, a reliable product is the particular case that works for a given stipulated time period under given natural conditions without interferences. In general terms, if the failures are unpredicted and frequency is high, the system is said to be problematic and its estimated worth is diminished. But if the frequency of failure is low, the system/framework is reliable and its reasonable worth is high.

To understand the genuine scope of reliability it is required to focus on some technical framework, which required high dependability or reliability. The application of reliability gets to be more predominant where danger element is high and capital inclusion is in question with human life. The primary in this class falls the aviation frameworks chiefly flying machines. It is realized that capital interest in the event of flying machines is very high and furthermore human lives are at hazard, the equipments utilized must be reliable. The other zone where high unwavering quality is needed is the Nuclear Power Plant. In the event where such plants are not accessible for power generation, heavy loss per
day will occur there in the plants and the goodwill of the concerning individuals will also affected. The radiations from such plants can bring about devastation, due consideration must be taken amid the release of their by-products. The other exceptionally reliable systems incorporate substance plants, process commercial ventures and electric supply frameworks where the failures can result in high income misfortunes (high revenue losses).

1.5 OBJECTIVES OF RELIABILITY

During the initial phase of manufacturing of any product/system, it is required that a particular system should meet every execution standards as expected not surprisingly inside the detailed constraints. These constraints incorporate expenses of gear/item, natural conditions and accessibility of material/parts and so on. A framework /equipment basically contains numerous units/parts and their interconnection and combination ought to result in attractive execution and satisfactory performance. The numerous varieties of parts and units make a system complex and in this manner, system is subject to multifaceted nature of the working of these units. It is further more hard to attain to acceptable execution from such system/equipment. Consequently, the goals of reliability are many fold and some of them are mentioned below:

- Disappointment free operations/functioning of system/equipment
- The effective performance for a specified interval of time
- The item should work with pre-defined ecological constraints
- Maximization of availability of unit/system and
- Maintainability/sustainability of framework or equipment.

1.6 MEANING OF RELIABILITY

Any equipment/system after its production is tried under genuine working conditions and where real conditions are impractical the idea of simulation is applied. It is much required from any product that they should perform well under the specified interval of time and environment. On the off chance that the item survives the tests, the reliability “R” of the item can be characterized as the likelihood that the item keeps on meeting the given determinations over a given
interval of time w.r.t. to different environmental conditions. However, if the item does not meet the determinations as sought over a period of time it is considered as fizzled in its central goal or mission. The unreliability ‘F’ of the item can be characterized as the likelihood that the item neglects to meet the specifications over a given time period under the given environmental constraints. The failures can happen because of numerous reasons known or obscure, for example, wear and tear, mechanical breakdown, synthetic consumption and chemical corrosion so on.

The reliability and unreliability are time dependent. At time zero when the product/equipment is put into service its reliability is unity, after certain time interim it might be 0.7 and zero (when the item/product fails). But unreliability does not decrease with time i.e., at t=0 unreliability of item/system is zero.

After some time it may move to 0.7 and at the end to 1 when the item/system fails.

Since, at any time ‘t’ the item has either survived or fizzled, consequently the sum of reliability and unreliability should be ‘1’ i.e., both of the events are mutually exclusive and can be described by the following mathematical expression

\[ R(t) + F(t) = 1 \]

Here ‘t’ is the time period of item being used under the specified environmental conditions.

1.7 IMPORTANCE OF RELIABILITY

There are various reasons why reliability is a crucial attribute for a unit product, including:

- **Reputation:** An organization's reputation is nearly identified with the dependability of its items. The more reliable an item is, the more probable the organization is to have a positive reputation.

- **Customer Satisfaction:** While a good reliable item may not significantly influence consumer loyalty in a positive way, an untrustworthy item will adversely influence consumer loyalty extremely. Hence high reliability is an obligatory necessity for consumer loyalty.
- **Warranty Costs:** If an item fails to perform its function inside the guarantee period, the substitution and repair expenses will adversely influence benefits, and in addition increase undesirable negative consideration. Presenting reliability investigation is a critical venture in making restorative move, at last prompting an item that is more reliable.

- **Repeat Business:** A concentrated exertion towards enhanced reliability shows existing clients that a maker is serious about its item, and focused on consumer loyalty. This sort of mentality has a positive effect on future business.

- **Cost Analysis:** Manufacturers may take reliability information and consolidate it with other expense data to outline the expense viability of their items. This life cycle cost investigation can demonstrate that in spite of the fact that the introductory expense of an item may be higher, the general lifetime expense is lower than that of a competitor's on account of their item obliges less repairs or less support.

- **Clients Requirements:** Many clients in today's business sector demands that their suppliers have a powerful reliability program. These clients have learned the profits of reliability analysis from experience.

- **Competitive Advantage:** Many Manufacturers published their predicted reliability analysis to obtain a Competitive advantage over their rivals.

### 1.8 FACTORS INFLUENCING SYSTEM EFFECTIVENESS

The product/system adequacy is by implication related the reliability of that product on the grounds that reliable system will help the working individual more adequately. In the event that the system is effective/viable its performance can be enhanced for satisfaction of the clients. Product in working conditions may be showing sub-standard execution won’t be acknowledged only due to its poor equipment adequacy/effectiveness. During these days, the clients wish to improve on their speculations with all conceivable viability or effectiveness of the system. The system/product will be affected by the accompanying components.

- Availability of the system/equipment
- Complexity of devices/components
- Maintainability
- Maintenance system practiced
- Maintenance support system
- Failure history of the equipment/system
- Supporting staff and working personnel should have good Coordination and cooperation between them.

1.8.1 Evaluation and Assessment of System Effectiveness
In case of complex systems, redundant units are inside inherent to help the whole system to execute their basic task. Few of them can execute the similar job by utilizing different sources. For this situation failure of a unit can prompt diminished execution yet whole complex system does not fall flat.

Effectiveness can be utilized to gauge the execution of the framework in quantitative way. For instance, nature of fuel utilized as a part of IC motors can lower its execution, or low quality administration in cafeteria can bring down the number of clients. If effectiveness index number is high, the system can be known as more effective/adequate, which may be different for different sort of framework/systems.

1.9 COST AND RELIABILITY
It is a fact that the cost of item enhanced relying upon its reliability requirements. Accordingly, a cost-benefit analysis is key to plan and assembling item at an ideal expense or at an optimal cost. It is a well known truth that dependability expense increments with higher prerequisites of reliability. Every failure has a reason and it is vital to know, what is the expense of anticipating or correcting the error, compared with the expense of doing nothing. Starting high cost of attaining reliability can be repaid by minimizing the failures.

Achieving reliable designs and products requires a coordinated approach, including configuration, training, testing, preparing and in addition the reliability program exercises.

1.10 FAILURE
Gertsbakh (1989) defined failure as a result of a joint action of many unpredictable, random processes going on inside the operating system as well as
in the environment in which the system is operating. Failure is stochastic in nature and its operation gets seriously impeded or completely stopped at a certain point in time.

Determination of failure may be easily detected in some cases just through observation but in others it’s very difficult since these units deteriorate continuously and the actual moment of failure is not as easy to determine. We assume that failure is exactly observable in this study and failure is known as a disappointment or a death. When a system fails it enters a down state which may also be called a system breakdown (Finkelstein[16]). According to Zacks[42] data is of two types: from continuously monitored units for failure and from observations of failure made at discrete points in time. Villemeur [43] cited a number of possible failures and their causes, they fall in two categories: random individual independent failures and interdependent failures. Failures are either catastrophic or drift depending on whether their parameters fall shapely or gradually as a result of wear and fatigue.

1.10.1 Failure Rate

Failure rate is the number of failure occurred in the system per unit time. It is often denoted by the Greek letter ,Lambda and it plays a very important role in reliability engineering. Generally, the inverse rate Mean Time between Failures is used for systems. Failure rate is typically time subordinate, and a natural culmination is that both rates change over the long haul versus the normal life cycle of a framework. For instance, as an auto becomes more older, the reliability in its sixth year may be many times lower than its reliability during its first year – one essentially does not hope to supplant a fumes channel, upgrade the brakes, or have significant force plant-transmission issues in another vehicle. So in the uncommon situation when the probability of disappointment stays steady as for time (for instance, in some item like a brick or ensured steel pillar), failure rate is just the opposite of the Mean Time between Failures (MTBF), expressed in failure/hours. Mean Time between Failures (MTBF) is an important specification parameter in all
aspects of high importance engineering design – such as Navy, Aerospace Engineering, and Automobile engineering etc.

The failure rate is characterized for non repairable populations as the (prompt) rate of failure for the survivors to time $t$ during the next moment of time. It is a rate per unit of time e.g. reading a car speedometer at a particular instant and seeing 37 mph. The next instant the failure rate may change and the units that have officially fizzled assume no further part since just the survivor's count.

The failure rate (or hazard rate) is denoted by $h(t)$ and calculated from

$$h(t) = \frac{f(t)}{1 - F(t)} = \frac{f(t)}{R(t)}$$

the instantaneous (conditional) failure rate.

In some special cases the failure rate is called a "conditional/restrictive failure rate" since the expression $1 - F(t)$ i.e. “the population survivors” converts the whole ratio into a conditional rate, given survival past time $t$.

Since $h(t)$ is also equal to the negative of the derivative of $\ln{R(t)}$, we have the useful identity:

$$F(t) = 1 - \exp\left\{-\int_{0}^{t} h(t) \, dt \right\}$$

If we let

$$H(t) = \int_{0}^{t} h(t) \, dt$$

be the Cumulative Hazard Function, we then have $F(t) = 1 - \exp(-H(t))$.

Two other useful identities that follow from these formulas are:

$$h(t) = -\frac{d\ln{R(t)}}{dt}$$

$$H(t) = -\ln{R(t)}.$$

1.10.2 Failure Rate in the Discrete Sense

Failure rate can also be defined as “The total number of failures within an item population, divided by the total time expended by that population, during a particular measurement interval under stated conditions”.
Failure rate $F(t)$ can be thought of as the likelihood that a failure occurs in a specified time interval, given no failure occurs before specific moment “$t$”. Failure rate can be expressed with the help of the reliability / survival function $R(t)$, the probability of number failure before specific moment “$t$” as:

$$F(t) = \frac{R(t) - R(t + \Delta t)}{\Delta t R(t)}$$

Where $\Delta t = (t_1 - t_2)$, $t_1$ (or $t$) and $t_2$ are respectively the starting and last point of a specified interval of time denoted by $\Delta t$. So it is a conditional probability, hence the $R(t)$ is in the denominator.

1.10.3 Failure Rate in the Continuous Sense (Instantaneous Hazard Rate)

By evaluating the failure rate for littler interims of time $\Delta t$, these intervals may becomes very small. This gives the hazard function, which is the “instantaneous failure rate” at any point in time:

$$r(t) = \lim_{\Delta t \to 0} \frac{R(t) - R(t + \Delta t)}{\Delta t R(t)}$$

Continuous failure rate related with the failure distribution, $F(t)$, which is a “c.d.f” that gives the probability of failure before time $t$,

$$P(T \leq t) = F(t) = 1 - R(t), \ t > 0$$

where $T$ is the failure time. The failure distribution function can be expressed as the integral of the failure density function “p.d.f”, $f(x)$,

$$F(t) = \int_0^t f(x) \, dx \quad r(t)=f(t)/R(t)$$

There are number of failure distributions. An important distribution is the exponential failure distribution.

$$F(t) = \int_0^t \lambda e^{-\lambda x} \, dx = 1 - e^{-\lambda t}$$

This is based on the exponential density function and leads to a constant hazard rate. For other distributions, such as The Weibull distribution, Rayleigh distribution, the hazard function is variable, which means that the failure rate varies with time.
1.10.4 Bathtub Curve

A plot of the failure rate over the long haul for most items yields a bend that resembles a drawing of a bathtub.

Over numerous years, and over a wide mixture of mechanical and electronic parts and frameworks, individuals have figured/calculated empirical population failure rates as units age over time and obtained a graph similar to the one given below. Because of the shape of this failure rate curve, it has become widely known as the "Bathtub" curve.

The beginning area that starts at time zero when a client first starts to utilize the item is portrayed by a high yet quickly diminishing failure rate. This area is known as the Early Failure Period (also known as Infant Mortality Period, from the actuarial origins of the first bathtub curve plots). This diminishing disappointment rate commonly endures a few weeks to a couple of months.

Next, the failure rate level off and remains generally consistent for (assuredly) most of the helpful life of the item. This long stretch of a level failure rate is known as the Intrinsic Failure Period (also called the Stable Failure Period) and the constant failure rate level is called the Intrinsic Failure Rate. Note that most frameworks spend the majority of their lifetimes working in this portion of the bathtub curve.

At last, if the units from the populace stay being used long enough, the failure rate begins to increase as materials destroy and degradation failures occur at an ever increasing rate. This is the Wear out Failure Period.

![The Bathtub Curve](image-url)
1.11 QUALITY VERSUS RELIABILITY
The regular use term "Quality of an Item" is approximately taken to mean its inherent level of brilliance. However in industry, this is made more exact by characterizing quality to be "conformance to necessities toward the begin of utilization ".

Letting that item specifications effectively catch client necessities, the quality level can now be exactly measured by the part of units dispatched that meet specifications.

Despite the fact that an item has a reliable design, when it is produced and utilized, its reliability may be unacceptable. The reason for this low reliability may be that the item was ineffectively fabricated. Along these lines, despite the fact that the item has a reliable design it is problematic when fielded which is really the consequence of a substandard assembling procedure.

As an example, cold solder joints could pass testing at the producer's end, however come up short in the field as the after effect of warm cycling or vibration. This kind of failure does not happen on the grounds that of an improper design, yet rather it is the consequence of a mediocre assembling or manufacturing process. So while this item may have a reliable design, its quality is inadmissible in light of the assembling methodology.

Reliability is the science of predicting, estimating, or optimizing the life distribution of components of systems. So we can say that Reliability is closely associated with the quality of a product.

Quality can be defined qualitatively as the level by which the product satisfies the users (customers) requirements. It depends on the production system and adherence to manufacturing procedures and tolerances.

On the other hand reliability is concerned with how long the product continues to function once it becomes operational. A poor-quality product will likely have poor reliability, and a high-quality product will have a high reliability.
1.12 REPAIRABLE SYSTEMS, NON-REPAIRABLE POPULATIONS AND LIFETIME DISTRIBUTION MODELS

Although failed units of a system may be replaced with new ones, repair is always more feasible because of the costs involved in buying new ones. Some systems are repairable while others are not. Repairable (or renewable) systems are those systems (or units) which may be made repairable by a repair facility once it is in a down state as a result of a failure. A renewed system has its service time increased as a result of increased reliability. In case the repair facility is not free then the down units lined up for repair. In this study the lifetime of a unit while on line, standby or repair are considered as independent variables. We assume that the distributions of these random functions are known with probability density functions. Investigations of repairable systems have been in existence for ages.

A repairable framework is one which can be restored to satisfactory operation by any activity, including parts substitutions or changes to movable settings. At the point when talking about the rate at which failures occur amid system operation time (then go for repair) we will define a “Rate of occurrence of failure” (ROCOF). It would be inaccurate to discuss about failure rate or hazard rate for repairable systems, as these terms apply just to the first failure occasions for a population of non repairable segments.

A non-repairable population is one for which single unit that come up short are expelled forever from the whole system. While the system may be repaired by supplanting fizzled units from either a comparative or an alternate population.

The Theoretical population models used to depict unit lifetimes are known as Lifetime Distribution Models. A Lifetime Distribution model can be any probability density function (or PDF),f(t) defined over the range of time from t = 0 to t = infinity. The corresponding cumulative distribution function (or CDF), F(t) is a very useful function, as it gives the probability that a randomly selected unit will fail by time t. The figure below shows the relationship between f(t) and F(t) and gives three descriptions of F(t).
i) $F(t) =$ The area under the PDF $f(t)$ to the left of $t$.

ii) $F(t) =$ The probability that a single randomly chosen new unit will fail by time $t$.

iii) $F(t) =$ The proportion of the entire population that fails by time $t$.

The figure above also shows a shaded area under $f(t)$ between the two times $t_1$ and $t_2$.

This area is $[F(t_2) - F(t_1)]$ and it represents the proportion of the population that fails between times $t_1$ and $t_2$ (or the probability that a brand new randomly chosen unit will survive till time $t_1$ but fail before time $t_2$).

Note that the PDF $f(t)$ has only non-negative values and eventually either becomes 0 as $t$ increases, or decreases towards 0. The CDF $F(t)$ is monotonically increasing and goes from 0 to 1 as $t$ approaches infinity. In other words, the total area under the curve is always 1.

1.12.1 Survival Function

*Survival is the complementary event to failure.*

The Reliability Function $R(t)$, is also known as the Survival Function $S(t)$ and is defined by:

$$R(t) = S(t) = \text{the probability a unit survives beyond time } t.$$  

Since a unit either fails, or survives, and one of these two mutually exclusive alternatives must occur, we have

$$R(t) = 1 - F(t), \quad F(t) = 1 - R(t)$$
Calculations using $R(t)$ often occur when building up from single components to subsystems with many components. For example, if one microprocessor comes from a population with reliability function $R_m(t)$ and two of them are used for the CPU in a system, then the system CPU has a reliability function given by

$$R_{cpu}(t) = R_m(t)$$

Very limited lifetime distribution models have enjoyed great practical success.

1.13 BASIC LIFETIME DISTRIBUTION MODELS

In the theory of reliability, the failure times, the repair times etc are random variables. A random variable is completely characterized by the distribution function which is defined in terms of probability parameters.

The distribution function $F(x)$ is defined as

$$F(x) = \Pr\{X \leq x\} = \int_0^x f(t)dt.$$ 

Where $f(t)$ is known as probability density function. Some important distributions which are commonly used in reliability analysis are:

1.13.1 Exponential Distribution

A continuous random variable $t$ assuming non-negative values is said to have an exponential distribution with parameter $\lambda > 0$ if it has the probability density function of the form

$$f(t) = \begin{cases} \lambda e^{-\lambda t} & 0 < t < \infty \\ 0 & t < 0 \end{cases}$$

The corresponding distribution function is

$$F(t) = \begin{cases} 1 - e^{-\lambda t} & 0 \leq t < \infty \\ 0 & t < 0 \end{cases}$$

The reliability function of this distribution are given as

$$R(T) = e^{-\lambda T}$$

$r(t) = \lambda$(constant)
\[ E(t) = \int_{0}^{\infty} t\lambda e^{-\lambda t} \, dt = \int_{0}^{\infty} R(t) \, dt = \frac{1}{\lambda} \]

Exponential distribution has a very important role in reliability studies. Other than various numerical properties, it has a very important memory less property i.e. if a structure has not failed up to time t, the probability distribution of its future life span will be the same as if the structure were quite new and has just been placed in use at time t. For example, an electric fuse (assuming it cannot melt partially) whose future life distribution is practically unchanged as long as it has not yet failed.

Note that the failure rate reduces to the constant \( \lambda \) for any time. The exponential distribution is the only distribution to have a constant failure rate. Also, another name for the exponential mean is the Mean Time To Fail or MTTF and it is represented as \( 1/\lambda \).

The cumulative hazard function for the exponential is just the integral of the failure rate or \( H(t) = \lambda t \).

*The Exponential models, the flat part of the well known “bathtub curve” - where many systems spend most of their ‘lives’.*

Uses of the Exponential Distribution Model

- Because to its constant failure rate property, it is an excellent model for the long flat portion of the Bathtub Curve. Since no. of equipments and systems spend most of their lifetimes in this portion of the Bathtub Curve, this justifies frequent use of the exponential distribution (when early failures or wear out is not a considered).

- This Exponential distribution is a two-parameter distribution since the mean and standard deviations are equal. We can approximate any model by piecewise exponential distribution because it is very useful to approximate any curve by piecewise straight line.
1.13.2  The Weibull Distribution

The Weibull distribution can be shaped to represent many distributions i.e. it has no specific characteristic shape. Its probability density function is defined by

\[ f(t) = \alpha t^{\alpha-1} e^{-\alpha t^\alpha}; \quad \text{for } \alpha > 0, t > 0 \]
\[ = 0, \quad \text{elsewhere} \]
\[ r(t) = \alpha t^{\alpha-1}, \quad R(t) = \exp(-\alpha t^\alpha): \alpha > 0 \]

Where \( R(t) \) is a Reliability function, and \( \alpha \) is a shape parameter. The Weibull distribution is very flexible and also has theoretical justification in many applications.

Uses of the Weibull Distribution Model

- Because of its flexible shape and ability to model a wide range of failure rates, the Weibull Distribution has been used successfully in many applications as a purely empirical model.
- Understanding the shape parameter of this distribution provides the maintenance Engineer a tool for predicting the nature of equipments and it also provide the help for finalizing the strategy of maintenance and repair of systems.
- Special case arises when we put shape parameter is equal to 2. Then The distribution is called the Rayleigh Distribution (RD),here we can see

\[ f(t) = 2t e^{-t^2} \quad \text{Where shape parameter is 2.} \]

1.13.3  Erlang Distribution

Let \( t_i \) (where \( i = 1, 2, 3, 4, \ldots, k \)) be a sequence of independent and identically distributed random variables with exponential distribution, with mean \( \frac{1}{k\lambda} \) consider \( t = \sum_{i=1}^{k} t_i \), then \( t \) is known as Erlang variable and its probability density function is given by

\[
 f(t) = \begin{cases} 
 \frac{(\lambda k(\lambda k t)^{k-1} e^{-\lambda k t}}{(k-1)!} & ; \quad t \geq 0, \lambda > 0, k \geq 1 \\
 0 & ; \quad t < 0 
\end{cases}
\]
\[ r(t) = \frac{\lambda k e^{-\lambda k t} t^{k-1}}{(k-1)!}; \quad k \geq 0 \]

and

\[ MTSF = \frac{1}{\lambda^k} \]

If \( k = 1 \) it gives

\[ f(t) = \begin{cases} \lambda e^{-\lambda t} & ; \quad t > 0, \; \lambda > 0 \\ 0 & ; \quad t < 0 \end{cases} \]

This is the p.d.f. of the exponential distribution. i.e. Erlang distribution become exponential distribution.

### 1.13.4 Rayleigh Distribution

This is a special case of Weibull distribution and similar to exponential distribution. This distribution is defined as

\[ f(t) = k t e^{-\frac{kt^2}{2}} \]

The reliability function of this distribution are given as

\[ R(t) = e^{-\frac{kt^2}{2}} \]

and

\[ r(t) = kt \]

### 1.13.5 Bivariate Exponential Distribution

Bivariate exponential distributions have been extended to multivariate exponential distributions. Together with other types of multivariate distributions (Weibull, Gamma, phase-type, etc.), multivariate exponential distributions have been widely used in statistics, reliability, and risk analysis.

A number of methods have been introduced for the construction of bivariate exponential distributions. Most of them impose some restrictions on the correlation coefficient. For example, Kibble [44], Marshall and Olkin [41] restrict the correlation to be non-negative. Such a restriction limits the applications of bivariate exponential distributions. In addition, some of the constructions are too complex for applications.
The joint distribution of failure and repair times is taken to be bivariate exponential of the form suggested by Paulson, A.S. [45] valid for \( 0 \leq r < 1 \). The p.d.f. of the B.V.E. distribution is

\[
f(x, y) = \alpha \beta (1-r) e^{-(\alpha x + \beta y)} I_0 (2 \sqrt{\alpha \beta r xy})
\]

\( x, y, \alpha, \beta > 0; 0 \leq r < 1 \)

Where

\[
I_0(z) = \sum_{k=0}^{\infty} \left( \frac{z}{\alpha} \right)^k \frac{(k!)^2}{(k!)^2}
\]

is the modified Bessel function of type I and order zero.

If \( g(x) \) and \( h(x) \) is the marginal p.d.f. of \( x \) and \( y \) respectively then

\[
g(x) = \alpha (1-r) e^{-\alpha (1-r)x}
\]

\[
h(x) = \beta (1-r) e^{-\beta (1-r)y}
\]

And conditional p.d.f. of \( y \) given \( x \) is

\[
h(y/x) = \beta e^{-(\beta y + \alpha rx)} I_0 (2 \sqrt{\alpha \beta r xy}).
\]

### 1.14 PERFORMANCE MEASURES

In this section, we concentrate and explain some of the performance measures of a system which are of interest to the system’s designing and analysis.

(a) **Reliability**

Reliability is the likelihood of an unit functioning for a given period of time interval without failure. All the more for the most part, reliability is the functioning of units, components, gadgets, products and systems to perform their prescribed functions for specified periods of time with full functioning, in specified environments and with desired confidence.

Mathematically, if a random variable \( T \) representing the life time of the system, then the reliability function of the framework/system at instant “\( t \)” is given by

\[
R(t) = P[T > t] = \int_{t}^{\infty} f(u) \, du
\]
\[ A(t) = \begin{cases} 1 - F(t) & \text{if } t > 0 \\ 1 & \text{if } t = 0 \end{cases} \]

Where, \( f(\cdot) \) and \( F(\cdot) \) are the p.d.f. and c.d.f. of the life time of the system ‘T’ respectively.

The reliability is always a function of time. It also depends on environmental conditions, which may or may not vary with time. The following assumptions are made,

- \( R(0) = 1 \), since the device is assumed to be perfect at time \( t = 0 \).
- \( R(\infty) = 0 \), since no device can work forever without failure.
- \( R(t) \) is a non-increasing function between time limits 0 to 1.

(b) Availability

Availability (Accessibility) is settled in the writing/literature of stochastic demonstrating and ideal upkeep/maintenance. Barlow and Proschan [46] characterize Availability of repairable framework as “Probability or likelihood of the framework is functioning properly at a specified time \( t \)” and in reliability theory, the term availability has the following meanings:

The extent to which a framework, subsystem or equipment is operable and in a committable state at the begin of a mission, when the mission is called for at an obscure, i.e. an arbitrary time. Basically, availability is the interval of time a system is in a operating condition. Availability is the likelihood that an item will be able to functioning (i.e. not failed or undergoing repair) when called upon to do so. This measure considers an item’s unwavering quality (how rapidly it fizzes) and its viability (how rapidly it can be repaired). It is defined as the probability that the system will be able to operate within tolerance limits at a given instant \( t \) and is also called operational readiness.

Symbolically, if \( X(t) \) is a binary variate which takes values 1 and 0 respectively when the system operates and does not operate at epoch ‘t’ then point wise availability is defined as,

\[ A(t) = P[X(t) = 1] \]

We also note that reliability is a uniformly non-increasing function defined in an interval while availability is a function defined at a time epoch.
(c) **Interval Availability**
The expected fraction of a given interval of time for which the system is able to operate within tolerances is known as the interval availability. For the interval \((0, t]\), the interval availability of the system can be obtained by using its point wise availability as under

\[
\bar{A}(t) = \frac{1}{t} \int_{0}^{t} A(u) du = \frac{\mu_{up}(t)}{t}
\]

Where \(A(u)\) represents point wise availability of the system at epoch \(u\) and \(\mu_{up}(t)\) is the expected up time of the system during \((0, t]\).

(d) **Asymptotic or Steady-State Availability**
Steady-state availability is the probability that in the long-run, the system operates satisfactorily. Symbolically, the steady-state availability is

\[
A(\infty) = \lim_{t \to \infty} A(t) = \lim_{t \to \infty} \mu_{up}(t)
\]

This measure is suitable for those systems which are operating continuously e.g. radar system.

The difference between the measures of reliability and availability are as follows:

- The reliability is an interval function while availability is a point function describing the behavior of the system at a specified epoch.
- The reliability function precludes the failure of the system during the interval under consideration, while availability functions does not impose any such restriction on the behavior of the system.

(e) **Mean Sojourn Time in a State**
The expected time taken by the system in a particular state before transiting to any other state is known as Mean Sojourn Time or Mean Survival Time in that state.

Let \(T_i\) be the Sojourn Time in any state \(S_i\)(say), then Mean Sojourn Time \(\mu_i\) in state \(S_i\) is given by,

\[
\mu_i = \int P[T_i > t] \, dt
\]
(f) **Mean Time to System Failure (MTSF)**

The time taken by a system to reach into the failed state first time is known as Time to System Failure (TSF) and its expected value is termed as mean time to system failure. Sometimes it is also known as mean time to first failure of the system.

To obtain it we regard the failed state as absorbing state i.e. once the system enters into the failed state(s) it remains there forever.

Let ‘T’ be the survival or life time of the system and \( f(\cdot), F(\cdot) \) be the p.d.f. and c.d.f. respectively of life time \( T \), then

\[
\text{MTSF} = E[T] = \int_0^\infty t \, f(t) \, dt = \int_0^\infty dF(t) = \int_0^\infty t \, d[1 - R(t)] = -\int_0^\infty t \, dR(t)
\]

\[
= -t \, R(t) \bigg|_0^\infty + \int_0^\infty R(t) \, dt
\]

\[
= \int_0^\infty R(t) \, dt
\]

Where, \( R(t) = 1 - F(t) \) is the reliability function of the system at time \( t \).

If \( R^*(s) \) is the Laplace transform of the reliability function \( R(t) \), then MTSF is given by,

\[
E(t) = \lim_{s \to 0} \int_0^\infty e^{-st} R(t) \, dt
\]

\[
= \lim_{s \to 0} R^*(s) = \lim_{s \to 0} \left[ \frac{1}{s} - F^*(s) \right]
\]

\[
= \lim_{s \to 0} \left[ \frac{1 - F^{**}(s)}{s} \right]
\]

\[
= -\frac{d}{ds} f^{**}(s) \bigg|_{s=0} \quad \{ \therefore F^{**}(s)|_{s=0} = 1 \}
\]

Where, \( F^*(s) \) and \( F^{**}(s) \) denote respectively the Laplace and Laplace Stieltjes transforms of c.d.f. of time to system failure \( T \).
1.15 SOME SYSTEM CONFIGURATIONS

A system is an arbitrary device consisting of parts, components or units having known reliabilities. It is assumed that components fail independently of each other. The configuration and nature of the system must be known so that the effect of failure of each component or unit on the system can be determined.

Suppose that a system with life time $T$ consists of $n$-different components $C_1, C_2, \ldots, C_n$ with life time $T_i$ (of the $i$th component). At time $t$ the system reliability is,

$$R(t) = P[T > t]$$

and reliability of $i$th component is,

$$R_i(t) = P[T_i > t]$$

The system structures generally considered are given below:

1.15.1 Series Model

The Series Model is used to develop from segments to sub-assemblies and systems. It is just applicable to non replaceable populations (or to first failures of framework). The presumptions and methods for the Series Models are indistinguishable to those for the Competing Risk Model, with the $k$ failure modes inside a segment supplanted by the $n$ components within a framework/system.

The accompanying two presumptions are required:

i) Every unit works or fails individually/autonomously; at least until the first unit failure happens.

ii) The whole system is said to be in failure mode when the first unit failure happens.

Each and every segment of $n$ (possibly distinctive) components in the framework has a life time distribution model $F_i(t)$, e.g. Exponential, The Weibull distribution etc.

In a Series Model we have following relations between failure and reliability:
\[ R_S(t) = \prod_{i=1}^{n} R_i(t) \]
\[ h_S(t) = \sum_{i=1}^{n} h_i(t) \]
\[ F_S(t) = 1 - \prod_{i=1}^{n} (1 - F_i(t)) \]

In above expressions subscript S is referring to the whole system and the “i” referring to the \( i^{th} \) unit.

Note that these expressions hold for any unit life distribution models, as long as "independence" and "very first segment failure causes the system to fail" hold. The analogy/similarity to a series arrangement is very helpful. The whole system has “n” units in this arrangement. The system fails independently of all the other units. The schematic diagram shown below demonstrates a framework with five units in arrangement "supplanted" by one (for reliability) framework with one and only one component.

**Series System Reduced to Equivalent One Component System**

\[ R_1(t) \quad R_2(t) \quad R_3(t) \quad R_4(t) \quad R_5(t) \]

\[ R_5(t) = R_1(t) \times R_2(t) \times R_3(t) \times R_4(t) \times R_5(t) \]

**Fig. 1.4 : Series Combination**
1.15.2 Parallel or Redundant Model

In parallel model all n units operate freely/independently and the system operates properly as long as minimum one unit still works.

The inverse of a series model, for which the first part disappointment causes the whole system to fizzle, is a parallel model for which all the segments need to fizzle before the system falls flat. If there are n units in the system, n-1 of them are called redundant to the left one (regardless of the fact that the units are all distinctive). At the point when the framework is turned on, all the parts work until they fizzle. The framework achieves disappointment at the time of the last segment failure.

The assumptions for a parallel model are:

- Every unit works independently (for reliability point of view).
- The system works as long as minimum one unit is still working. System fails at the time of the last unit failure.

The c.d.f. of each unit is known. For a parallel model, the c.d.f. “F_s(t)” for the system is just the product of the c.d.f.’s F_i(t) of each unit or we can write, Mathematically

\[ F_s(t) = \prod_{i=1}^{n} F_i(t) \]

R_s(t) and h_s(t) can be evaluated using basic definitions, once we have F_s(t).

The diagram shown below describe a parallel model including five units and the (reliability) equivalent one unit system with a c.d.f. “F_s(t)” equal to the product of the five unit c.d.f.’s.
**Parallel System and Equivalent Single Component**

![Diagram of Parallel System](image)

\[ F_d(t) = F_1(t) \times F_2(t) \times F_3(t) \times F_4(t) \times F_5(t) \]

**Fig. 1.5: A Parallel combination**

**Availability analysis in parallel system:** As stated above, the combination of system (consisting parallel units) is considered failed when all units fail. The consolidated framework is operational if either is accessible. From this it follows that for a two unit system the combined availability is \(1 - (\text{both parts are unavailable})\). The consolidated availability is described by the following expression:

\[
A = 1 - (1 - A) \times (1 - A) \]

The conclusion of the above discussion is that the joint availability of n units connected in parallel is greater than the availability of its any single unit.

**1.15.3 Complex Systems**

Numerous complex models can be diagrammed as mix arrangements of Series, Parallel units, R out of N units and redundant units. By utilizing the methods for these systems, subsystems or segments of the main system can be supplanted by an "equivalent" one component with predefined Reliability function. Proceeding like this, it is possible to finally reduce the whole system to one equivalent unit. Following diagram is a model of a
A complex system including of both components in parallel and in series is initially reduced to a series system and after that to a single-unit system.

**Complex System Reduced to Equivalent One Component System**

![Diagram of complex system]

\[ R_s = R_A \times R_B \times R_0 = (1 - F_1 \times F_2 \times F_3)(1 - F_4 \times F_5)(R_0) \]

Fig. 1.6 : A Complex Combination

### 1.16 MORE ON REDUNDANCY AND DIFFERENT TYPES OF REDUNDANT SYSTEMS

Redundancy is introduced in a system by building into it more units than is actually necessary for the system to properly perform. There are two forms of redundancy, namely parallel and standby (sequential) redundancy. Parallel redundancy occurs when the units form part of the system from the beginning but if series redundancy occurs, a standby system does not form part of the system until it is required.

**Redundant Systems**

A parallel redundant system is defined as one with \( n \) units which are all operating simultaneously, despite the fact that system operation needs at least one unit to be
in operation. In this case system failure occurs only when all the components have failed. Let \( k \) be a non-negative integer, such that \( k \leq n \), counting the number of units in an \( n \)-unit system. This system is normally referred to as a \( k \)-out-of-\( n \) system.

**\( k \)-out-of-\( n \) : F-system**

If the system only fails when \( k \) units fail in a \( k \)-out-of-\( n \) system, it is known as an F-system. Papastavridis [47, 48], Malon [49], Shanthikumar [50], Fu J.C. [51], Griffith and Govindarajulu [52], Hwang [53] pointed out that the functioning of a minimum number of units ensures that the system is operating; these systems were surveyed by Chao et al. [54].

**\( k \)-out-of-\( n \) : G-system**

If and only if at least \( k \) units out of the \( n \) units of the system are operational the system is operational, it is known as a G-system. Zhang [55], Zuo, M. [56] and Kuo, W. [57] have studied such systems, for example a radar network has \( n \) radar control stations covering a certain area in which the system can be operable if and only if at least \( k \) of these stations are operable. In this case a minimum number of units, \( k \) is essential for the functioning of the system.

Attention has shifted to load-sharing \( k \)-out-of-\( n \) : G systems of late, where serving units share the load and the failure rate of a component is affected by the magnitude of the load it shares.

**n-out-of-\( n \) : G-system**

An n-out-of-n system is basically a series system that consists of \( n \) units and failure of any one unit causes the system to fail. This type of system is not really redundant since all the units are in series and have to be operational for the unit to operate however; it is still called a special case of a \( k \)-out-of-\( n \) system.

A series system is equivalent to a 1-out-of-n: F system and to an n-out-of-n:G system while parallel system is equivalent to an n-out-of-n:F system and to a 1-out-of-n:G system.

The k-out-of-n system structure is a very popular type of redundancy in fault-tolerant systems. It finds wide applications in both industrial and military systems.
1.16.1 Standby Redundancy
Standby redundancy comprises of an attachment to an operating unit one or more redundant (standby) units, which can, on failure of the operating unit, be switched online (if operable). These units may be classified as cold, warm or hot (Gnedenko et al. (1969)).

i) A **cold standby** is a method of redundancy in which standby unit is completely in inactive (non operative) mode, it cannot (in theory) fail until it is put to use by replacing a primary unit.

ii) A **warm standby** is a method of redundancy in which standby unit is partially energized which has diminished load. The operative unit and the standby unit are not subject to the same loading conditions. The probability failure of the warm standby unit is smaller than the probability of failure of the on-line unit.

iii) A **hot standby** is also a method of redundancy in which standby unit is fully energized and in active mode and the possibility of failure of a hot standby is the same as that of an operating unit in the standby state.

1.17 RELIABILITY IMPROVEMENT
The manufacturers as well as the user of a system always desire a high reliability. Reliability of a system can be improved upon in several ways. One way of improving reliability of a unit is either duplicate some of the parts/components of the unit or the unit as a whole. The other way is to provide repair and maintenance to the system at the time of need. Some important methods are given below:

1.17.1 Redundancy
In redundant system some additional parts are created for performing the system function. Although either one of the component/unit is sufficient for the successful operation of the system, we deliberately use some more components/units so as to increase the probability of success, thus causing the system to be redundant. Thus, redundancy is the creation of new
parallel paths in a system structure to improve the system reliability. The redundancy may be classified as:

- **Active Redundancy**
  An active redundant system with \( n \) units/components is the one in which all the units function simultaneously and system operates even when one unit/ component operates. In active redundant system all the units fail independently. For example – the human body is two organ active redundant system in respect of many functions e.g. seeing, hearing, breathing, walking, manual work etc.

- **Passive (Standby) Redundancy**
  A standby redundant system is the one in which one unit/component is operative at a time and others are kept in spare known as standby. An \( n \) independent units/components standby system operates in the following manner: component \( C_1 \) operates until its failure, then component \( C_2 \) is switched on ........... component \( C_i \) operates until its failure... Component \( C_n \) operates until its failure and the system is declared failed. Such a system is sometimes referred to as a sequentially redundant system of order \( n \). This type of system is similar to parallel system of order \( n \) except that each component is used one at a time rather than using all of them simultaneously. Aggrawal [58] studied the Redundancy Optimization in general system and Arora[59] discussed the Reliability of several standby priority redundant systems.

1.17.2 **System Maintenance**
All recoverable systems which are utilized for continuous service for specified interval of time are subject to maintenance. Maintenance or up-keep policies can be classified in the following categories,

- Preventive Maintenance (P.M.)
- Corrective Maintenance (C.M.)
- Repair Maintenance (R.M.)
Preventive Maintenance (P.M.) is a special type of repair policy that is used prior to the real failure of a unit. A periodic policy for “Preventive Maintenance” P.M. is generally adopted. But in real situations, it is not always possible to perform this action exactly at the desired time and hence one can expect the time at which this P.M. occurred to be a R.V. with little dispersion about the time. During this sort of P.M. the system can be either in an active mode or it can be in an inactive mode for some period. It can be supposed that after each P.M. operation, system or unit works as good as new. Corrective Maintenance deals with the system performance when it gives wrong results. Repair Maintenance is concerned with increasing system availability by implementation of major changes in the failed components of a unit. The reliability of the system can also be increased by Repair Maintenance if the system consists of at least two units in parallel configuration. On failure, a unit is sent to a repair facility, if available i.e. free, otherwise it queues up for repair. The time taken for preventive maintenance, corrective maintenance and repair maintenance is random variable and it is assumed that the distribution functions of these variables are known and have certain probability density functions.

There are many systems in which some units are given preference for operation as well as for repair. Such systems are called priority systems and consist of two sub-classes of units. One is the class of priority (p) units and the other is of ordinary (o) units. The p-unit is never kept as a standby. When the p-unit fails, it goes into repair immediately if repair facility is free. However if an o-unit is under repair and the p-unit fails, the following policies can be adopted:

1. **Pre-emptive priority**: The repair of the o-unit is interrupted and its repair is continued as soon as the repair of the p-unit is completed. The resumed repair o-unit can follow any one of the following rules.
   - **Pre-emptive resume**: The repair of the o-unit is continued from where it was left.
- **Pre-emptive repeat:** The repair of the o-unit is started fresh i.e. the time already spent in the repair of o-unit has no-influence on its repair time now.

(2) **Non pre-emptive priority:** The repair of the o-unit is continued and the repair of the p-unit is started only when the repair of the o-unit is completed.

### 1.17.3 Inspection

The inspection of the system at random epochs is useful to trace out the fault in redundant system, particularly in deteriorating stand by systems. When in service, maintenance is impossible or unit failure indication is impractical, units in parallel redundant system must be periodically inspected to assure that none of them have failed and that the system still has its existing reliability. Peng, W. et al.[60] discussed the Reliability Based Optimal Preventive Maintenance Policy of Series-parallel systems.

### 1.18 STOCHASTIC PROCESSES

A Stochastic Process \{X(t), t\in T\} is a collection of random variables i.e. for each \( t \in T \), \( X(t) \) is a random variable. The random variable \( X(t) \) is called the state of the system at time ‘t’. For example, \( X(t) \) might be equal to the total number of incoming calls at a telephone exchange up to time ‘t’, the number of consumers that arrive in a super market up to time ‘t’. If ‘t’ assumes discrete values then the process \{X(t), \( t = 0, 1, 2, \ldots \)\} is called a discrete parametric stochastic process. If ‘t’ assumes continuous values then the process \{X(t), \( t \geq 0 \)\} is called as a continuous parametric stochastic process. The set of all possible values of \( X(t) \) is called the parametric space (S) and the set of all possible values of ‘t’ is called the parametric space or the index set (T). Goel et al [61-70] did lot of work in the field of reliability, based on stochastic process.

Generally, the stochastic processes encountered in the analysis of a specified complex system, can be put into one of the following classes:
(a) **Markov Process**

A stochastic process \( \{X(t), t \in T\} \) is said to be Markov process if for every \( n \) and arbitrary time points \( t_1 < t_2 < \ldots < t_n \), we have

\[
P[X(t_n) \leq x_n | X(t_1) = x_1, \ldots, X(t_{n-1}) = x_{n-1}] = P[X(t_n) \leq x_n | X(t_{n-1}) = x_{n-1}]
\]

It is a random process whose future probabilities are determined by its most recent values.

In other words, a Markov process is a process having the Markovian property that the conditional probability of the future event depends on present and not on past.

A Markov process whose state space is discrete is called a Markov chain.

A Markov chain is called a Discrete Parameter Markov chain or aContinuous Parameter Markov chain as the parametric space is discrete or continuous.

(b) **Poisson Process**

The Poisson process, named after the French mathematician Siemon-Denis Poisson (1781 – 1840), is a stochastic process which is defined in terms of the occurrences of events.

A stochastic process \( \{N(t), t \geq 0\} \) is said to be a counting process if \( N(t) \) represents the number of events that have occurred by the time ‘t’, e.g. number of birth in a locality by the instant ‘t’, number of goals that a given soccer player has scored by the epoch ‘t’ etc. For a counting process, \( N(t) \) must satisfy

(i) \( N(t) \geq 0 \)

(ii) \( N(t) \) is integer valued

(iii) If \( s \leq t \), then \( N(s) \leq N(t) \) and \( N(t) - N(s) \) represents the number of events that have occurred in the interval \( (s, t) \) and has a Poisson distribution.

The stochastic process \( \{N(t), t \geq 0\} \) is said to be a Poisson Process having with intensity rate \( \lambda > 0 \), if the following postulates are satisfied:

(i) \( N(0) = 0 \)
The process has stationary and independent increments.

\[ P[N(\Delta t) = 1] = \lambda \Delta t + O(\Delta t) \]

There are some examples of Poisson process:

- The number of phone calls receiving by a switchboard, or at a programmed telephone exchanging framework, may be portrayed by a Poisson process.

- The quantity of photons hitting a photograph finder, when hit by a laser source, may be described by a non heterogeneous Poisson process. Different sources show either a grouping or a bunching of these photons.

- The quantity of website page solicitations landing at a server may be described by a Poisson procedure aside from surprising circumstances, for example, coordinated denial of service attacks.

- The quantity of raindrops falling over a wide spatial zone may be described by a spatial Poisson process.

- The entry of "clients" is normally displayed as a Poisson transform in the investigation and study of simple queuing systems.

(c) **Renewal Process**

Suppose we have a reparable system which starts operation at \( t = 0 \). Let \( X_i \) denotes the time between \((i-1)\)th renewal to \(i\)th failure and \( Y_i \) denotes the time between \(i\)th failure to \(i\)th renewal, then \( t_i = X_i + Y_i \) is the time between \((i-1)\)th and \(i\)th renewal \((i = 1, 2, 3, \ldots)\).

If we define

\[ S_0 = 0 \]
\[ S_n = t_1 + t_2 + \ldots + t_n = \text{epoch of the nth renewal during (0, t)} \]

and

\[ N(t) = \max \{ n : S_n \leq t \} \]

\[ = \text{maximum number of renewals during (0, t)} \]

Then, the process \([N(t), t \geq 0]\) is called a renewal process.
For example, we have an infinite supply of light bulb whose life time is independent and identically distributed. Further, suppose that a single bulb is used at a time and when it fails, it is immediately replaced with a new one.

If $X_1, X_2, \ldots, X_n$ denote the respective life times of the bulbs and $S_n = 0$, then

$$S_n = \sum_{i=1}^{n} X_i \; ; \; n > 1$$

= epoch of the replacement of $n^{th}$ bulb.

Then $[N(t), t \geq 0]$ is a renewal process where $N(t)$ represents the number of bulbs that have failed by time $t$.

(d) Markov – Renewal Process

Let $E = (0, 1, 2 \ldots)$ denote the state of a Markov chain $\{X_n\}$, where $X_n$ is the state visited at epoch $t_n + \text{i.e. just after the transition at epoch } t_n, \quad (n = 0, 1, 2\ldots)$.

If,

$$P[X_{n+1} = j, \; t_{n+1} - t_n < t \mid X_0, X_1, X_2, \ldots, X_n, t_1, t_2, \ldots, t_n] = P[X_{n+1} = j, \; t_{n+1} - t_n < t \mid X_n, t_n] \; \forall \; n \in \mathbb{N}, j \in E, t > 0$$

then $\{X_n, t_n\}$ is said to constitute a Markov renewal process with state space $E$ and embedded Markov chain $\{X_n\}$. If the probabilities

$$P[X_{n+1} = j, \; t_{n+1} - t_n < t \mid X_n = i] = Q_{ij}(t) \; ; \; i, j \in E, t > 0$$

are independent of $n$, the process is said to be time homogeneous.

The probabilities $Q_{ij}(t)$ are said to constitute a semi-Markov kernel over $E$ and the matrix,

$$P = (p_{ij}) = [Q_{ij}(\infty)]$$

Represents the transition probability matrix (t.p.m.) of the Markov chain generated by $\{X_n\}$. 

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Regenerative Process
Let X(t) be the state of the system at epoch t. If \( t_1, t_2, \ldots, t_n \) are the epochs at which the process probabilistically restarts, then these epochs are called regenerative epochs and the process [X(t), \( t = t_1, t_2, \ldots, t_n \)] is called regenerative process. Here, the process beyond \( t_1 \) is a probabilistic replica of the whole process starting at \( t = 0 \). A regenerative state has the property that as soon as the system enters it, its further development is independent of the past history. This process was introduced by Smith.

1.19 BUSY PERIOD OF THE REPAIRMAN WITH THE SYSTEM
Let B(t) be the probability that a repairman/server is busy with the system in the interval (0, t], then in the long run the total fraction of time for which a repairman is busy is given by:

\[
B = \lim_{t \to \infty} B(t)
\]

1.20 EXPECTED NUMBER OF VISITS BY THE SERVER
Let N(t) be a random variable representing the number of times, the repairman has visited the system in the interval (0, t], then the expected number of visits by the repairman to the system in (0, t], is \( E[N(t)] \) and in the long run this number per unit time is given by

\[
N = \lim_{t \to \infty} \frac{E[N(t)]}{t}
\]

1.21 PROFIT ANALYSIS
Any Manufacturing Industry is basically a profit making organization and no organization can survive for long without minimum financial returns for its investment. There must be an optimal balance between the reliability aspect of a product and its cost. The major items contributing to the total cost are Research and Development, production, spares and maintenance. How the cost of these individual items varies with reliability is shown in fig. 1.7. In order to increase the reliability of the products, we would require a correspondingly high investment in the research and development activities. The production cost also would increase with the requirement of greater reliability.
The revenue and cost function lead to the profit function of a firm, as the profit is excess of revenue over the cost of production. The profit function in time \( t \) is given by:

\[
P(t) = \text{Expected revenue in } (0, t] - \text{Expected total cost in } (0, t]
\]

In general, the optimal policies can more easily be derived for an infinite time span or compared to a finite time span. The profit per unit time, in infinite time span is expressed as

\[
\lim_{t \to \infty} \frac{P(t)}{t}
\]

Fig. 1.7: Reliability vs. Cost
i.e. profit per unit time = total revenue per unit time – total cost per unit time.

Considering the various costs, the profit equation is given as

\[ P = K_1 A_0 - K_2 B_0 - K_3 N_0 \]

Where \( P \) = Profit per unit time incurred to the system

\( K_1 \) = Revenue per unit up time of the system

\( A_0 \) = Total fraction of time for which the system is up

\( K_2 \) = Cost per unit time for which server is busy

\( B_0 \) = Total fraction of time for which the server is busy

\( K_3 \) = Cost per visit by the server

\( N_0 \) = Expected number of visits per unit time for the server

1.22 TRANSFORMS AND CONVOLUTIONS

Laplace Transforms

A transform is merely a mapping or function from one space to another. While it may be very difficult to solve certain equations directly for a particular function of interest, it is often easier to solve a corresponding equation in terms of a transform of the function and then invert the transform to obtain the function. One particular transform, that is, very useful for solving some types of differential equations as well as certain integral equations, is the Laplace transform (L.T.).

Let \( f(t) \) be a function of positive real variable \( t \). Then the Laplace transform of \( f(t) \) is denoted by \( f^*(s) \) and defined as

\[ L[f(t)] = f^*(s) = \int_0^\infty e^{-st} f(t) \, dt \]

For the range of value of \( s \) for which the integral exists. Here \( f(t) \) is called an inverse Laplace transform of \( f^*(s) \) and we write \( f(t) = L^{-1}\{f^*(s)\} \). The following are some important properties of Laplace transform:-

i) \[ L[\sum_{i=1}^n c_i f_i(t)] = \sum_{i=1}^n c_i f_i^*(s) \]

ii) \[ L[t^n f(t)] = \frac{d^nf(s)}{ds^n} \]

iii) \[ L\left[ \int_0^t f(u) \, du \right] = \frac{f^*(s)}{s} \]
Laplace Stieltjes Transforms

Let $X$ be a non-negative random variable with distribution function

$$F(x) = \Pr [X \leq x]$$

then Laplace Stieltjes transform of $F(x)$ is defined for $s > 0$ by

$$F^{**}(s) = \int_0^\infty e^{-sx} dF(X)$$

under certain regular conditions, we have

$$F^{**}(s) = s \int_0^\infty e^{-sx} F(X) dX = sF^*(s) \quad \text{and} \quad F^{**}(s) = \int_0^\infty e^{-sx} f(X) dX = f^*(s) \quad \text{Where } f(X) = \frac{dF(X)}{dx}$$

Convolution

Let $f(t)$ and $g(t)$ be two real valued non-negative continuous functions $t$, then the integral

$$\int_0^t f(t-u)g(u) du = \int_0^t g(t-u)f(u) du$$

$$= f(t) \circledast g(t)$$

$$= L^{-1}[f^*(s)g^*(s)]$$

is called Laplace convolution of the functions $f(t)$ and $g(t)$. If $F(t)$ and $G(t)$ be two real valued distribution functions defined for $t \geq 0$, the resulting convolution is again a distribution and integral

$$\int_0^t F(t-u) dG(u) = \int_0^t G(t-u) dF(u) = F(t) \circledast G(t)$$

is known as Stieltjes convolution of $F(t)$ and $G(t)$. 

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