Chapter 1

Introduction and Literature Review

Mathematics is the study of quantity, structure, space, and change. Mathematicians seek out patterns, formulate new conjectures, and establish truth by rigorous deduction from appropriately chosen axioms and definitions.

Mathematics is used throughout the world as an essential tool in many fields, including natural science, engineering, medicine, and the social sciences. Applied mathematics, the branch of mathematics concerned with application of mathematical knowledge to other fields, inspires and makes use of new mathematical discoveries and sometimes leads to the development of entirely new mathematical disciplines, such as statistics and game theory. Mathematicians also engage in pure mathematics, or mathematics for its own sake, without having any application in mind, although practical applications for what began as pure mathematics are often discovered.
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1.1 Historical Review

1.1.1 Elasticity

One of the most important branches of continuum mechanics is the classical theory of elasticity, which is concerned with the systematic study of the response of elastic bodies to the action of forces which deform it. This response is characterized by the stress and strain distributions inside a body that are developed because of the applied traction or change in temperature. The classical theory of elasticity serves as an excellent model for studying the mechanical behavior of a wide variety of solid material and is used extensively in civil, mechanical and aeronautical engineering design.

The history of the theory of elasticity, which is treated to Robert Hooke and Edmá Mariotte in the 17’th century or, even earlier, to Galileo Galilei in the 16’th century. In the theory of linear elasticity, we are concerned with an ideal material governed by Hooke’s law 1678, which represents a linear relationship between the stresses and strains. Hooke’s law has influenced the scientific thoughts for a considerably long period for the classical linear infinitesimal theory of elasticity and its results agreed with experiments quite well.

The first attempt to deduce general equations of equilibrium and vibration of elastic solids was made by Navier on May 14, 1821. This date marks the birth of the mathematical theory of elasticity.
Navier deduced a set of three macroscopic differential equations for the components of displacement in the interior of an isotropic elastic solid. Navier also obtained the equilibrium equations on the surface of the solid (the boundary conditions) with the aid of Lagranges principle of virtual work. Naviers work attracted the attention of Cauchy 1789-1857, who, proceeding from different assumptions, gave a formulation of the linear theory of elasticity that remains virtually unchanged to the present day.

1.1.2 Thermoelasticity

The term thermoelasticity deals with the effects of the thermal state of an elastic body on the stress distribution, and also with the effects of the stress on the temperature.

Thermoelasticity is the study of the relationship between the elastic properties of a material and its temperature, or between its thermal conductivity and its stresses.

Thermoelasticity is based on temperature changes induced by expansion and compression of the test part. Although this coupling between mechanical deformation and thermal energy has been known for over a century, it has only been recently that this phenomenon has been exploited as a means of experimental stress analysis.
The theory of elasticity was extended to include thermal effects. The
time of thermoelectricity is concerned with the influence of the ther-
mal state of an elastic solid upon the distribution of strain and with
the inverse effect, that of deformation upon the thermal state of an
elastic medium.

Thermoelectricity was stimulated by the various engineering sci-
ences. A remarkable progress in the field of aircraft and machine
structure has given rise to numerous problems in which thermal
stresses play a role of primary importance. Thus, the numerous
problems when the thermal stresses play an important, sometimes
the decisive role, occur in the machine building, particularly in steam
and gas turbines, in aviation structures, chemical engineering and es-
pecially in nuclear engineering. It comprises the heat conduction and
stress and strain that arise due to the flow of heat. Thermoelectricity
makes it possible to determine the stresses produced by the temper-
ature field and to calculate the temperature distribution due to an
action of time dependent forces and heat sources.

The theory of thermoelectricity, was found in 1837 by Duhamel
J.M.C. [20] who derived the formulation of boundary value problem
and also the derivation of equation for the coupling of the tempera-
ture field and the body’s deformation. The formulation of thermoele-
asticity equations is due to Neumann F. [72] in 1985, to Almansi
in 1910. **Signorini A.** [100] of 1930 on finite difference deformations should also be mentioned. In 1935, **Biot M. A.** [8] analyzed the properties of two-dimensional distributions of thermal stresses and **Goodier J. N.** [30] in 1937, introduced the notation of the thermoelastic potential and considered the effect of non-continues temperature fields.

During and after the Second World War, the requirements associated with new technologies contributed to a wave of research on thermal stresses. Gaining knowledge of the distribution of temperature in specific situations, finding the thermal stresses in parts of complex mechanical systems, assessment of allowed stresses in various materials and in various loading conditions, matters of stability, problems of viscoelasticity, of fatigue, and thermal shock, became topics of active research both theoretical and experimental.

In 1949, **Lighthill J. and Bradshaw J.** [59] discussed on thermal stresses in turbine blades, by **Manson S. S.** [62] of 1947 on gas turbine disks, and by **Aleck J.** [2] of 1949 on thermal stresses in rectangular plate. In 1950, **Mindlin D.** and **Cheng D. H.** [66] discussed the steady-state problems of thermal stresses in a half-space and **Sen B.** [95] in 1951, introduced the notations of nucleus of thermoelastic displacements, a fundamental development in the theory of steady-state problems. In 1957, **Nowacki W.** [78] considered a problem with discontinuous boundary conditions for temperature on
the surface of the half space. Three years latter, in 1960, Sneddon I. N. and Lockett [101] in which steady state problems for a half space and layer were considered. A dynamic counterpart to the static nucleus was proposed by Ignaczak J. [40].

A pioneering work on dynamic thermoelasticity was proposed by Danilovskaya V. I. [16] of 1950, where she solved a one dimensional problem of stresses in a half-space due to a thermal shock applied to the bounding plane. Similar problems subsequently considered by Mura T. [70] and later by Sternberg and Chakravorty [106] in 1959, analyzed the behavior of a stress wave as a result of a change of boundary conditions for temperature. In the same category work by Ignaczak J. [39] of 1957 and by Boley B. A. and Barber A. D. [11] of 1957.

In 1960, Boley B. A. and Weiner J. H. [12] have studied thermally induced beam and plate vibrations. They solved the problem of normal deflection of an axisymmetric heated circular plate in the case of fixed and simply supported edge.

1.1.3 Direct Thermoelastic Problem

The direct thermoelastic problems are consist of temperature, heat transfer conditions prescribed on the surfaces of the body and the
conditions at interior points are to be determined. The direct thermoelastic problems are more useful in engineering problem particularly, in the determination of the state of strain in a rectangular plate constituting foundations of containers for hot gases or liquids, in the foundations for furnaces etc.

In 1956, Biot M. A. [9, 10] developed the coupled theory of thermoelasticity. In this theory, the equations of elasticity and of heat conduction are coupled which deals with the first defect of the uncoupled theory.

In 1957, Nowacki N. W. [79–81] developed the theory of thermoelastic diffusion. In this theory, the coupled thermoelastic model is used. This implies infinite speeds of propagation of thermoelastic waves.

In 1959, Florence A. and Goodier J. [22] discussed a uniform flow of heat is disturbed by an insulated cavity or an insulated hole in an elastic plate, thermal stresses are produced in the plate.

In 1967, Lord and Shulman [60] derived the theory of generalized thermoelasticity, which is also referred as extended thermoelasticity theory, by modifying the Fouriers law of heat conduction with the introduction of a thermal relaxation time parameter. This theory was extended by Dhaliwal and Sherief [19] to include the anisotropic case.
In 1973, Roy Choudhuri [92] discussed the quasi-static thermal deflection of a thin clamped circular plate due to ramp type heating of a concentric circular region of the upper face, with the lower surface is kept at zero temperature and circular edge thermally insulated.

In 1982, Wankhede P. C. [116] determined the arbitrary temperature on the upper face with the lower face at zero temperature and the fixed circular edge thermally insulated.

In 1982, Hata T. [36] derived the thermal stresses distribution in a nonhomogeneous thick elastic plate under steady distribution of the surface temperature.

In 1990, Furukawa A.T., Noda N. and Ashida F. [23] studied the generalized thermoelasticity for an infinitely body with a circular cylindrical hole.


In 1995, Ootao Y., Akai T. and Tanigawa Y. [82] studied the theoretical analysis of a three dimensional transient thermal stress problem is developed for a nonhomogeneous hollow circular cylinder due to a moving heat source in the axial direction from the inner and outer surfaces.

problems making use of the strain increment theorem. The methods of generalized finite Fourier transform and finite Hankel transform has been used for finding the temperature field.

In 1999, Adams, R. and Bert, C. [1] discussed the response of a rectangular, simply supported, symmetrically laminated, cross-ply composite plate subjected to a thermal shock is developed.

In 2000, Ng, T., Lam, K. and Liew, K. [73] discussed a semi analytical method and Galerkin technique were employed to predict the nonlinear vibration behavior of FGM-laminated plates.

In 2001, Woo, J. and Meguid, S. [118] studied the nonlinear deflection of FGM plates and shells under transverse mechanical loads and a temperature field.


In 2002, Kim, K. and Noda, N. [51] studied two-dimensional unsteady thermoelastic problems of the FGM infinite hollow cylinder and the deflection of a functionally graded plate under transient thermal loading.

In 2004, Qian, L. and Batra, R. [89] solved the transient thermoelastic deformations of a thick functionally graded plate with edges held at a uniform temperature and either simply supported or clamped.

In 2004, Hany H. Sherief, Elmisierya A. E. M., Elhagary M. A. [34] have studied one-dimensional problem for an infinitely long hollow cylinder in the context of the theory of generalized thermoelasticity with one relaxation time.

In 2006, Ootao Y. and Tanigawa Y. [83] analyzed the theoretical treatment of transient thermoelastic problem involving a functionally graded hollow cylinder due to uniform heat supply.

In 2007, Imrak C. E. and Gerdemeli I. [41] analysis the deflections of a rectangular fixed thin plates under uniformly distributed loads.


In 2010, Dange, W., Khobragade, N. W., Durge, M. [15] determined the deflection, thermoelastic displacement, and stresses of
a thin equilateral triangular plate with the help of trilinear coordinates.

In 2010, **Yepeng, X., Ding, Z.** and **Kefu L.** [119] determined the three-dimensional thermoelastic analysis of rectangular plates with variable thickness subjected to thermo-mechanical loads.

In 2010, **Parihar K.** and **Patil S.** [87, 88] determined the thermoelastic problems of thin circular and rectangular plates.

In 2011, **Zhengzhu, D., Jun, L.** and **Fashan, L.** [121] discussed the thermal bending of circular plates for non-axisymmetrical problems.

### 1.1.4 Inverse Thermoelastic Problem

The thermoelasticity is an approach to derived the temperature, displacement and thermal stresses in different types of solid due to heating under various initial and boundary conditions.

The inverse thermoelastic problems consist of the determination of the heating medium, the heat flux at the boundary surfaces of the solids when the conditions of the displacement and stresses were known at the same points of the solid under consideration. The inverse problem was very important in view of its relevance to various industrial machines subjected to heating such as the main shaft of lathe and turbine and the role of rolling mill, turbines subjected to
heating and cooling mediums. Also it is useful in measurement of aerodynamic heating.

The investigation of one-dimensional transient thermoelastic problem was done by Cialkowski and Grysa [14], Grysa, Cialkowski and Kaminski [32], Grysa and Kozlowski [33], and Kozlowski and Grysa [53]. The heating temperature and the heat flux on the surface of an isotropic infinite slab was derived by them.

In 1989, Noda N., Ashida F. and Tsuji T. [75] discussed an analytical method for an inverse problem of three-dimensional transient thermoelasticity in a transversely isotropic solid by applying the Laplace and Fourier transforms with newly designed potential function and the practically applicability of the methods in engineering problems of great importance was illustrated.


In 2005, Gaikwad M. N. [24] solved the inverse problem of thermoelasticity in a thin isotropic circular plate by determining the unknown temperature gradient, temperature distribution and the thermal deflection on the edge of the circular plate.


In 2009, Kozlov V. A., Mazya V. G. and Fomin A. V. [52] solved the inverse problem of coupled thermoelasticity is considered in the static, quasi-static, and dynamic cases.

In 2010, Meshram, S. A. and Salve, P. M. [64] solved a two-dimensional inverse transient thermoelastic problem relating to the
axisymmetric dynamic temperature in a short length circular cylinder, the surface of which is traction free.


This thesis “A STUDY OF SOME ASPECTS OF THERMOELASTIC PROBLEM” is an attempt made for solution of various thermoelastic problems by mathematical methods. We investigate the some direct and inverse thermoelastic problem in a thick circular plate and thick annular disc under steady state and unsteady state field and discussed their thermoelasticity. We discuss the thermoelastic problem of an infinitely long circular cylinder and discussed their thermoelasticity. We investigate an inverse thermoelastic problem in a thin clamped circular plate and discussed their thermoelasticity. We also solved some nonhomogeneous thermoelastic problem in a cartesian and cylindrical geometry and discussed their thermoelasticity. To improve the accuracy of our results by any other finite difference method or numerical method, the temperature distribution function is found by solving the governing heat conduction by using integral transform technique. The results have been computed numerically and illustrated graphically.
1.2 Useful Definitions and Standard Results

1.2.1 Laplace Transform

Let $f(x)$ be a function of $x$ defined for $x > 0$, then the Laplace transform of $f(x)$ is defined by

$$\mathcal{L}(f(x)) = \int_0^{\infty} e^{-px} f(x) dx$$

(1.2.1)

Thus $\mathcal{L}(f(p)) = L[f(x)]$, then

$$f(x) = L^{-1}[\mathcal{L}(f(p))]$$

$f(x)$ is called the inverse Laplace transform of $\mathcal{L}(f(p))$, where $p$ is the Laplace transform parameter.

**Theorem 1.1.** If $\mathcal{L}(f(p))$ is an analytic function of the complex variable $p$ and is of order $0(p^{-k})$ in some half plane $R(p) \geq c$, where $c, k$ are real and $k > 1$, then the integral

$$\lim_{\beta \to \infty} \frac{1}{2\pi i} \int_{\gamma-i\beta}^{\gamma+i\beta} e^{px} \mathcal{L}(f(p)) dp$$

(1.2.2)

along any line $R(p) = \gamma \geq c$ converges to a function $f(x)$ which is independent of $\gamma$ and whole Laplace transform is $\mathcal{L}(f(p))$. Furthermore, the function $f(x)$ is continuous for each $x \geq 0$, $f(0) = 0$ and $f(x)$ is of order $0(e^{-\gamma x})$ for all $x \geq 0$. 
Theorem 1.2. Convolution Theorem for Laplace Transform

If \( \mathcal{F}(p) \) and \( \mathcal{G}(p) \) are the Laplace transform of \( f(x) \) and \( g(x) \) then

\[
\frac{1}{2\pi i} \int_{\gamma-i\beta}^{\gamma+i\beta} \mathcal{F}(p) \mathcal{G}(p) e^{px} dp = \frac{1}{2\pi i} \int_{\gamma-i\beta}^{\gamma+i\beta} e^{px} \mathcal{F}(p) dp \int_0^\infty g(y) e^{-py} dy = \int_0^\infty g(y) dy \times \frac{1}{2\pi i} \int_{\gamma-i\beta}^{\gamma+i\beta} e^{p(x-y)} \mathcal{F}(p) dp
\]

(1.2.3)

But

\[
\frac{1}{2\pi i} \int_{\gamma-i\beta}^{\gamma+i\beta} e^{p(x-y)} \mathcal{F}(p) dp = f(x-y)
\]

so that

\[
\frac{1}{2\pi i} \int_{\gamma-i\beta}^{\gamma+i\beta} \mathcal{F}(p) \mathcal{G}(p) e^{px} dp = \int_0^\infty g(y) f(x-y) dy
\]

Now \( f(x-y) = 0 \) if \( x-y < 0 \), that is, if \( y > x \) and hence the integrand vanishes if \( y > x \) and one obtains finally

\[
\frac{1}{2\pi i} \int_{\gamma-i\beta}^{\gamma+i\beta} \mathcal{F}(p) \mathcal{G}(p) e^{px} dp = \int_0^\infty g(y) f(x-y) dy \quad (1.2.4)
\]

Changing the variable in the integon the right hand side of equation (1.5.4) from \( y \) to \( \eta = x-y \), one obtains

\[
\frac{1}{2\pi i} \int_{\gamma-i\beta}^{\gamma+i\beta} \mathcal{F}(p) \mathcal{G}(p) e^{px} dp = \int_0^\infty f(\eta) g(x-\eta) d\eta \quad (1.2.5)
\]

1.2.2 Finite Fourier Transorm

(A) Finite Sine Fourier Transorm:

If \( f(x) \) satisfies Dirichelet’s conditions in the interval \((0, a)\) and if for
that range its finite Fourier Sine transform is defined to be

\[ \bar{f}_s(m) = \int_{0}^{a} f(x) \sin \left( \frac{m\pi x}{a} \right) dx \quad (1.2.6) \]

then at each point of \((0, a)\) at which \(f(x)\) is continuous,

\[ f(x) = \frac{2}{a} \sum_{n=1}^{\infty} \bar{f}_s(m) \sin \left( \frac{m\pi x}{a} \right) \quad (1.2.7) \]

(B) Finite Cosine Fourier Transorm:

The finite Fourier cosine transform of \(f(x)\) in the interval \((0, a)\) is defined to be

\[ \bar{f}_c(m) = \int_{0}^{a} f(x) \cos \left( \frac{m\pi x}{a} \right) dx \quad (1.2.8) \]

then at each point of \((0, a)\) at which \(f(x)\) is continuous,

\[ f(x) = \frac{2}{a} \sum_{n=1}^{\infty} \bar{f}_c(m) \cos \left( \frac{m\pi x}{a} \right) + \frac{\bar{f}_c(0)}{a} \quad (1.2.9) \]

Properties of Sine and Cosine Transform:

(1) \[ \int_{0}^{a} \frac{\partial f}{\partial x} \sin \left( \frac{m\pi x}{a} \right) dx = \frac{-m\pi}{a} \bar{f}_c(m) \]

(2) \[ \int_{0}^{a} \frac{\partial f}{\partial x} \cos \left( \frac{m\pi x}{a} \right) dx = (-1)^m f(a) - f(0) + \frac{m\pi}{a} \bar{f}_s(m) \]

(3) \[ \int_{0}^{a} \frac{\partial^2 f}{\partial x^2} \sin \left( \frac{m\pi x}{a} \right) dx = \frac{m\pi}{a} \left[ (-1)^{m+1} f(a) - f(0) \right] - \frac{m^2\pi^2}{a^2} \bar{f}_s(m) \]

(4) \[ \int_{0}^{a} \frac{\partial^2 f}{\partial x^2} \cos \left( \frac{m\pi x}{a} \right) dx = (-1)^m f'(a) - f'(0) - \frac{m^2\pi^2}{a^2} \bar{f}_c(m) \]
where \( \mathcal{F}_s(m) \) and \( \mathcal{F}_c(m) \) are the finite Fourier Sine and Cosine transform.

### 1.2.3 Finite Hankel Transform

1. If \( f(x) \) satisfies Dirichlet’s conditions in the interval \((0, a)\) and if for that range its finite Hankel transform in that range is defined to be

\[
\mathcal{H}_\mu(\xi_i) = \int_0^a x f(x) J_\mu(x \xi_i) dx \tag{1.2.10}
\]

where \( \xi_i \) is the root of the transcendental equation

\[
J_\mu(a \xi_i) = 0 \tag{1.2.11}
\]

then at any point of \((0, a)\) at which the function \( f(x) \) is continuous,

\[
f(x) = \frac{2}{a^2} \sum_i \mathcal{H}_\mu(\xi_i) \frac{J_\mu(x \xi_i)}{[J_\mu(a \xi_i)]^2} \tag{1.2.12}
\]

where the sum is taken over all the positive roots of the equation \((1.5.11)\).

2. **Property of the Hankel transform:**

If \( f(x) \) satisfies Dirichlet’s conditions in the interval \([0, a]\) then

(a) Finite Hankel transform of \( \frac{\partial f}{\partial x} \), i.e.

\[
H_\mu \left[ \frac{\partial f}{\partial x} \right] = \int_0^a \frac{\partial f}{\partial x} x J_\mu(x \xi_i) dx
\]
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\[= \frac{\xi_i}{2\mu}[(\mu - 1)H_{\mu+1}f(x) - (\mu + 1)H_{\mu-1}f(x)]\]

\[(b) \quad H_{\mu}\left[\frac{\partial^2 f}{\partial x^2} + \frac{1}{x} \frac{\partial f}{\partial x}\right] = \frac{\xi_i}{2}\left[-H_{\mu-1}\frac{\partial f}{\partial x} + H_{\mu+1}\frac{\partial f}{\partial x}\right]\]

3. If \(f(x)\) satisfies Dirichlet’s conditions in the interval \([0, a]\) and if its finite Hankel transform in that range is defined to be

\[H[f(x)] = \overline{f}_{\mu}(\xi_i) = \int_0^a x f(x)\left[J_\mu(x, \xi_i)G_\mu(a, \xi_i) - J_\mu(a, \xi_i)G_\mu(x, \xi_i)\right]dx\]

(1.2.13)

where \(\xi_i\) is the root of the transcendental equation

\[J_\mu(\xi_i b)G_\mu(\xi_i a) - J_\mu(\xi_i a)G_\mu(\xi_i b) = 0\]

(1.2.14)

then at which the function \(f(x)\) is continuous,

\[f(x) = \sum_i \frac{2\xi_i^2J_\mu^2(\xi_i b)\overline{f}_{\mu}(\xi_i)}{[J_\mu^2(\xi_i a) - J_\mu^2(\xi_i b)]}\times[J_\mu(x, \xi_i)G_\mu(a, \xi_i) - J_\mu(a, \xi_i)G_\mu(x, \xi_i)]\]

(1.2.15)

4. Property of the Hankel transform defined in (1.2.13)

\[\int_a^b \left\{\left[\frac{\partial^2 f}{\partial x^2} + \frac{1}{x} \frac{\partial f}{\partial x}\right] \times [J_\mu(x, \xi_i)G_\mu(a, \xi_i) - J_\mu(a, \xi_i)G_\mu(x, \xi_i)]\right\}dx\]

\[= -\xi_i^2\overline{f}_{\mu}(\xi_i) + a\{J_\mu(x, \xi_i)G_\mu(a, \xi_i) - J_\mu(a, \xi_i)G_\mu(x, \xi_i)\}_{x=a}\]

\[+ b\{J_\mu(x, \xi_i)G_\mu(a, \xi_i) - J_\mu(a, \xi_i)G_\mu(x, \xi_i)\}_{x=b}\]

\[= -\xi_i^2\overline{f}_{\mu}(\xi_i)\]
1.3 Homogeneous and Nonhomogeneous Boundary Value Problems of Heat Conduction

In mathematics, a boundary value problem consists of a differential equation to be satisfied at all points in the interior of an interval or a region and a set of boundary conditions specifying the values of the solution or some of its derivatives everywhere on the boundary of the interval or region. Boundary value problems may be posed for ordinary differential equations as well as partial differential equations.

In this work we have been used following types of boundary value problems of heat conduction.

1.3.1 Homogeneous problem

The time-dependent boundary value problem of heat conduction will be referred to as a homogproblem when both the differential equation and the boundary conditions are homogeneous.

\[ \nabla^2 T = \frac{1}{\alpha} \frac{\partial T}{\partial t}, \quad \text{in region } R, \ t > 0 \quad (1.3.1) \]

\[ k_i \frac{\partial T}{\partial n_i} + h_i T = 0, \quad \text{on the boundary surface } s_i, \ t > 0 \quad (1.3.2) \]

\[ T = f(r), \quad \text{in region } R, \ t = 0. \quad (1.3.3) \]
1.3.2 Nonhomogeneous problem

The time-dependent boundary value problem of heat conduction will be referred to as a homogproblem when both the differential equation, or the boundary conditions or both are nonhomogeneous.

\[
\nabla^2 T + \frac{g(r, t)}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}, \quad \text{in region } R, \ t > 0
\]  

(1.3.4)

\[
k_i \frac{\partial T}{\partial n_i} + h_i T = f_i(r, t), \quad \text{on the boundary surface } s_i, \ t > 0
\]

(1.3.5)

\[
T = f(r), \quad \text{in region } R, \ t = 0.
\]

(1.3.6)

is nonhomogeneous because function \(g(r, t)\) and \(f_i(r, t)\) do not include \(T\) as a product.

The boundary value problem of heat conduction in the form

\[
\nabla^2 T + \frac{g(r, t)}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}, \quad \text{in region } R, \ t > 0
\]  

(1.3.7)

\[
k_i \frac{\partial T}{\partial n_i} + h_i T = 0, \quad \text{on the boundary surface } s_i, \ t > 0
\]

(1.3.8)

\[
T = f(r), \quad \text{in region } R, \ t = 0.
\]

(1.3.9)

is nonhomogeneous because the differential equation is nonhomogeneous.
Bibliography


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