Chapter 3

An Inverse Quasi-Static Thermoelastic Problem in a Thick Circular Plate

3.1 Introduction

clamped circular plate. Also, Kulkarni et al. [6] studied quasi-static thermal stresses in a thick circular plate. Recently, Dange et al. [7] studied two dimensional transient problems for a thick annular disc in thermoelasticity.

In this chapter, an attempt is made to solve the inverse quasi-static thermoelastic problem in a thick circular plate and determine the unknown temperature, displacement and thermal stresses on the upper surface of a thick circular plate subjected to an arbitrary known interior temperature under a steady state field. The fixed circular edge thermally insulated and the temperature of the lower half of the plate is kept at zero. The governing heat conduction equation has been solved by using the Hankel transform technique. The results are obtained in series form in terms of Bessels functions and these have been computed numerically and illustrated graphically.

3.2 Formulation of the problem

Consider a thick circular plate of radius $a$ and thickness $2h$ occupying space $D: 0 \leq r \leq a, -h \leq z \leq h$. Initially the plate is at zero temperature. Let the plate be subjected to an arbitrary known temperature $f(r)$ with in region $-h \leq \xi \leq h$. The lower surface ($z = -h$) is kept at zero temperature and the fixed circular edge
(r = a) thermally insulated. Assume that the boundary of the circular plate is free from traction.

Under these more realistic prescribed conditions, the unknown temperature \( g(r) \) which is at the upper surface of the plate and the quasi-static thermal stresses due to unknown temperature \( g(r) \) need to be determined.

The differential equation governing the displacement potential function \( \phi(r, z) \) is given in [8] as,

\[
\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} = K \tau \tag{3.2.1}
\]

where \( K \) is the restraint coefficient and the temperature change is given by \( \tau = T - T_i \), where \( T_i \) is the initial temperature. The displacement function \( \phi \) is known as Goodier’s thermoelastic potential.

The steady-state temperature of the plate satisfies the heat condition equation,

\[
\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = 0 \tag{3.2.2}
\]

subject to the boundary conditions,

\[
\frac{\partial T}{\partial r} = 0 \quad \text{at } r = a, -h \leq z \leq h \tag{3.2.3}
\]

\[
\frac{\partial T}{\partial z} + h s_1 T = f(r) \quad \text{(known)} \quad \text{at } z = \xi, 0 \leq r \leq a \tag{3.2.4}
\]
\[
\frac{\partial T}{\partial z} - h_s^2 T = 0 \quad \text{at } z = -h, \ 0 \leq r \leq a \quad (3.2.5)
\]

and

\[
T = g(r) \ (\text{unknown}) \quad \text{at } z = h, \ 0 \leq r \leq a \quad (3.2.6)
\]

where \(h_{s1}\) and \(h_{s2}\) are relative heat transfer coefficients on the upper and the lower surface of the thick circular plate.

The displacement function in the cylindrical coordinate system are represented by the Michell’s function defined in [8] as,

\[
u_r = \frac{\partial \phi}{\partial r} - \frac{\partial^2 M}{\partial r \partial z} \quad (3.2.7)
\]

\[
u_z = \frac{\partial \phi}{\partial z} + 2 (1 - \nu) \nabla^2 M - \frac{\partial^2 M}{\partial z^2}. \quad (3.2.8)
\]

The Michell’s function \(M\) must satisfy

\[
\nabla^2 \nabla^2 M = 0, \quad (3.2.9)
\]

where

\[
\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}. \quad (3.2.10)
\]

The components of the stresses are represented by the thermoelastic displacements potential \(\phi\) and Michell’s function \(M\) as

\[
\sigma_{rr} = 2G \left[ \frac{\partial^2 \phi}{\partial r^2} - K \tau + \frac{\partial}{\partial z} \left( \nu \nabla^2 M - \frac{\partial^2 M}{\partial r^2} \right) \right] \quad (3.2.11)
\]
\[ \sigma_{\theta \theta} = 2G \left[ \frac{1}{r} \frac{\partial \phi}{\partial r} - K \tau + \frac{\partial}{\partial z} \left( \nu \nabla^2 M - \frac{1}{r} \frac{\partial M}{\partial r} \right) \right] \]  
\hfill (3.2.12)

\[ \sigma_{zz} = 2G \left[ \frac{\partial^2 \phi}{\partial z^2} - K \tau + \frac{\partial}{\partial z} \left( (2 - \nu) \nabla^2 M - \frac{\partial^2 M}{\partial z^2} \right) \right] \]  
\hfill (3.2.13)

and

\[ \sigma_{rz} = 2G \left[ \frac{\partial^2 \phi}{\partial r \partial z} + \frac{\partial}{\partial z} \left( (1 - \nu) M - \frac{\partial^2 M}{\partial z^2} \right) \right] \]  
\hfill (3.2.14)

where \( G \) and \( \nu \) are the Shear modulus and Poisson’s ratio respectively.

The boundary conditions on the traction free surfaces of the circular plate are

\[ \sigma_{rr} = \sigma_{rz} = 0 \quad \text{at} \quad r = a \]  
\hfill (3.2.15)

\[ \sigma_{zz} = \sigma_{rz} = 0 \quad \text{at} \quad z = \pm h. \]

Equations (3.2.1) to (3.2.15) constitute the mathematical formulation of the problem under consideration.
3.3 Solution of the problem

TEMPERATURE

To obtain the expressions for temperature $T(r, z)$ we introduce the finite Hankel transform over the variable $r$ and its inverse transform defined in [9] as

$$
\mathcal{T}(\lambda_n, z) = \int_0^a r J_\nu(\lambda_n r) T(r, z) dr \quad (3.3.1)
$$

$$
T(r, z) = \sum_{n=1}^{\infty} \left( \frac{2 J_\nu(\lambda_n r)}{a^2 J_\nu^2(\lambda_n a)} \right) \mathcal{T}(\lambda_n, z) \quad (3.3.2)
$$

where $\lambda_1, \lambda_2, \ldots$ are the roots of the transcendental equation

$$
J_1(\lambda a) = 0 \quad (3.3.3)
$$

with $J_n(x)$ is Bessel function of the first kind of order $n$.

This transform satisfies the relations

$$
H \left[ \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right] = -\lambda_n^2 \mathcal{T}(\lambda_n, z) \quad (3.3.4)
$$

and

$$
H \left[ \frac{\partial^2 T}{\partial z^2} \right] = \frac{d^2 T}{d z^2}. \quad (3.3.5)
$$

On applying the finite Hankel transform defined in equation (3.3.1) to equation (3.2.2), one obtains
\[ \frac{d^2\bar{T}}{dz^2} - \lambda_n^2 \bar{T} = 0 \quad (3.3.6) \]

where \( \bar{T} \) is the Hankel transform of \( T \).

On solving equation (3.3.6) under the conditions given in equation (3.2.4) and (3.2.5), one obtains

\[
\bar{T} = \sum_{n=1}^{\infty} \bar{f}(\lambda_n) \times \left[ \frac{\lambda_n \cosh [\lambda_n(z + h)] + h_{s_2} \sinh [\lambda_n(z + h)]}{(\lambda_n^2 + h_{s_1}h_{s_2}) \sinh [\lambda_n(\xi + h)] + \lambda_n(h_{s_1} + h_{s_2}) \cosh [\lambda_n(\xi + h)]} \right] 
\quad (3.3.7)
\]

where

\[
\bar{f}(\lambda_n) = \int_0^a r J_0(\lambda_n r) f(r) dr \quad (3.3.8)
\]

Applying the inverse Hankel transform defined in equation (3.3.2) to equation (3.3.7), one obtains the expression for the temperature as

\[
T = \left( \frac{2}{a^2} \right) \sum_{n=1}^{\infty} \bar{T} \times \left[ \frac{\bar{f}(\lambda_n) J_0(\lambda_n r)}{J_0^2(\lambda_n a)} \right] \times \left[ \frac{\lambda_n \cosh [\lambda_n(z + h)] + h_{s_2} \sinh [\lambda_n(z + h)]}{(\lambda_n^2 + h_{s_1}h_{s_2}) \sinh [\lambda_n(\xi + h)] + \lambda_n(h_{s_1} + h_{s_2}) \cosh [\lambda_n(\xi + h)]} \right] 
\quad (3.3.9)
\]

Since \( T_i = 0 \), the temperature change is \( \tau = T - T_i = T \).

\[
(3.3.10)
\]

**UNKNOWN TEMPERATURE** \( g(r) \)

The unknown temperature can be obtained by substituting \( z = h \)
into equation (3.3.9), one obtains

\[
g(r) = \left( \frac{2}{a^2} \right) \sum_{n=1}^{\infty} \left[ \frac{\tilde{f}(\lambda_n) J_0(\lambda_n r)}{J_0^2(\lambda_n a)} \right] \\
\times \frac{\lambda_n \cosh[2\lambda_n h] + h_{s2} \sinh[2\lambda_n h]}{(\lambda_n^2 + h_{s1} h_{s2}) \sinh[\lambda_n(\xi + h)] + \lambda_n (h_{s1} + h_{s2}) \cosh[\lambda_n(\xi + h)]} \tag{3.3.11}
\]

**MICHELL’S FUNCTION** \( M \)

A suitable form of \( M \) satisfying equation (3.2.9) is given by

\[
M = \left( \frac{2k}{a^2} \right) \sum_{n=1}^{\infty} \left[ \frac{\tilde{f} \lambda_n J_0(\lambda_n r)}{J_0^2(\lambda_n a)} \right] \\
\times \{ B_n [h_{s2} \cosh[\lambda_n(z + h)] + \lambda_n \sinh[\lambda_n(z + h)]] \\
\times C_n \lambda_n (z + h) [h_{s2} \sinh[\lambda_n(z + h)] + \lambda_n \cosh[\lambda_n(z + h)]] \} \tag{3.3.12}
\]

where \( B_n \) and \( C_n \) are arbitrary constants, which can be determined from the boundary condition (3.2.15).

**GOODIER’S THERMOELASTIC DISPLACEMENT POTENTIAL FUNCTION** \( \phi \)

Assuming that the displacements function \( \phi(r, z) \) has the form and satisfying the equation (3.2.1) as,

\[
\phi(r, z) = \sum_{n=1}^{\infty} D_n J_0(\lambda_n r) \\
\times \left[ \frac{(z + h) [h_{s2} \cosh[\lambda_n(z + h)] + \lambda_n \sinh[\lambda_n(z + h)]]}{(\lambda_n^2 + h_{s1} h_{s2}) \sinh[\lambda_n(\xi + h)] + \lambda_n (h_{s1} + h_{s2}) \cosh[\lambda_n(\xi + h)]} \right] \tag{3.3.13}
\]
and using $\phi$ in equation (3.2.1) one obtains

$$D_n = \frac{K \overline{f}(\lambda_n)}{a^2 \lambda_n J_0^2(\lambda_n a)}, \quad (3.3.14)$$

Thus the equation (3.3.12) becomes

$$\phi(r, z) = \left( \frac{K}{a^2} \right) \sum_{n=1}^{\infty} \left\{ \left[ \frac{\overline{f}(\lambda_n)J_0(\lambda_n r)}{\lambda_n J_0^2(\lambda_n a)} \right] \right.$$

$$\times \left[ \frac{(z + h)[h s_2 \cosh \lambda_n(z + h)] + \lambda_n \sinh \lambda_n(z + h)]}{(\lambda_n^2 + h s_1 h s_2) \sinh \lambda_n(\xi + h) + \lambda_n(h s_1 + h s_2) \cosh \lambda_n(\xi + h)]} \right\}.$$

**DETERMINATION OF DISPLACEMENT FUNCTION**

Now using equations (3.3.12) and (3.3.15) in (3.2.7) and (3.2.8), one obtains the expressions for displacement as,

$$u_r = \left( \frac{K}{a^2} \right) \sum_{n=1}^{\infty} \left\{ \left[ \frac{\overline{f}(\lambda_n)J_1(\lambda_n r)}{J_0^2(\lambda_n a)} \right] \right.$$

$$\times \left[ \frac{-(z + h)[h s_2 \cosh \lambda_n(z + h)] + \lambda_n \sinh \lambda_n(z + h)]}{(\lambda_n^2 + h s_1 h s_2) \sinh \lambda_n(\xi + h) + \lambda_n(h s_1 + h s_2) \cosh \lambda_n(\xi + h)]} \right\}.$$

$$u_z = \left( \frac{K}{a^2} \right) \sum_{n=1}^{\infty} \left\{ \left[ \frac{\overline{f}(\lambda_n)J_0(\lambda_n r)}{J_0^2(\lambda_n a)} \right] \right.$$

$$\times \left[ \frac{[h s_2 \cosh \lambda_n(z + h)] + \lambda_n \sinh \lambda_n(z + h)]}{\lambda_n(\lambda_n^2 + h s_1 h s_2) \sinh \lambda_n(\xi + h) + \lambda_n(h s_1 + h s_2) \cosh \lambda_n(\xi + h)]} \right\}.$$
\[-B_n \lambda_n^2 [h_{s2} \cosh \{\lambda_n(z + h)\} + \lambda_n \sinh \{\lambda_n(z + h)\}] \]

\[+ C_n \lambda_n^2 \left[ 2(1 - 2\nu) [h_{s2} \cosh \{\lambda_n(z + h)\} + \lambda_n \sinh \{\lambda_n(z + h)\}] \right] \]

\[- \lambda_n(z + h) [h_{s2} \sinh \{\lambda_n(z + h)\} + \lambda_n \cosh \{\lambda_n(z + h)\}] \] .

\[(3.3.17)\]

**QUASI-STATIC THERMAL STRESSES**

Now using equations (3.3.10), (3.3.12) and (3.3.15) in equations (3.2.11) and (3.2.14), one obtains the expressions for stresses as,

\[
\sigma_{rr} = \left( \frac{2GK}{a^2} \right) \sum_{n=1}^{\infty} \left[ \frac{\bar{f}(\lambda_n) J_0(\lambda_n r)}{J_0^2(\lambda_n a)} \right] \left\{ \frac{\lambda_n J_0(\lambda_n r) - J_1(\lambda_n r)}{r} \right\} \]

\[\times \left[ \frac{- (z + h) [h_{s2} \cosh \{\lambda_n(z + h)\} + \lambda_n \sinh \{\lambda_n(z + h)\}] \right. \]

\[\left. \frac{\left( \lambda_n^2 + h_{s1} h_{s2} \right) \sinh \{\lambda_n(\xi + h)\} + \lambda_n(h_{s1} + h_{s2}) \cosh \{\lambda_n(\xi + h)\}}{\left( \lambda_n^2 + h_{s1} h_{s2} \right) \sinh \{\lambda_n(\xi + h)\} + \lambda_n(h_{s1} + h_{s2}) \cosh \{\lambda_n(\xi + h)\}} \right] \]

\[- B_n \lambda_n^2 \left( \lambda_n J_0(\lambda_n r) - \frac{J_1(\lambda_n r)}{r} \right) \]

\[\times [h_{s2} \sinh \{\lambda_n(z + h)\} + \lambda_n \cosh \{\lambda_n(z + h)\}] \]

\[+ C_n \lambda_n^2 \left[ 2\nu \lambda_n J_0(\lambda_n r) [h_{s2} \sinh \{\lambda_n(z + h)\} + \lambda_n \cosh \{\lambda_n(z + h)\}] \right] \]

\[+ \left( \lambda_n J_0(\lambda_n r) - \frac{J_1(\lambda_n r)}{r} \right) \]

\[\times [[h_{s2} \sinh \{\lambda_n(z + h)\} + \lambda_n \cosh \{\lambda_n(z + h)\}] \]

\[+ \lambda_n(z + h) [h_{s2} \cosh \{\lambda_n(z + h)\} + \lambda_n \sinh \{\lambda_n(z + h)\}] \] \}

\[(3.3.18)\]

\[
\sigma_{\theta \theta} = \left( \frac{2GK}{a^2} \right) \sum_{n=1}^{\infty} \left[ \frac{\bar{f}(\lambda_n) J_0(\lambda_n r)}{J_0^2(\lambda_n a)} \right] \left\{ - \frac{J_1(\lambda_n r)}{r} \right\} \]

\[\times \left[ \frac{(z + h) [h_{s2} \, \cosh \{\lambda_n(z + h)\} + \lambda_n \sinh \{\lambda_n(z + h)\}] \right. \]

\[\left. \frac{\left( \lambda_n^2 + h_{s1} h_{s2} \right) \sinh \{\lambda_n(\xi + h)\} + \lambda_n(h_{s1} + h_{s2}) \cosh \{\lambda_n(\xi + h)\}}{\left( \lambda_n^2 + h_{s1} h_{s2} \right) \sinh \{\lambda_n(\xi + h)\} + \lambda_n(h_{s1} + h_{s2}) \cosh \{\lambda_n(\xi + h)\}} \right] \]
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Now in order to satisfy the boundary conditions given in the equation

\[
\sigma_{zz} = \left( \frac{2GK}{a^2} \right) \sum_{n=1}^{\infty} \left[ \frac{\tilde{f}(\lambda_n) J_0(\lambda_n r)}{J_0^2(\lambda_n a)} \right] \left( \lambda_n^2 \right) \sum_{n=1}^{\infty} \left[ \frac{\tilde{f}(\lambda_n) J_1(\lambda_n r)}{J_1^2(\lambda_n a)} \right]
\]

\[
- \left[ \frac{2J_0(\lambda_n r)(z + h)[h_{s2} \sinh \lambda_n(z + h)] + \lambda_n \cosh \lambda_n(z + h)]}{(\lambda_n^2 + h_{s1} h_{s2}) \sinh \lambda_n(\xi + h) + \lambda_n(h_{s1} + h_{s2}) \cosh \lambda_n(\xi + h)} \right] + B_n \lambda_n^2 \left( \frac{J_1(\lambda_n r)}{r} \right) [h_{s2}. \sinh \lambda_n(z + h)] + \lambda_n \cosh \lambda_n(z + h)]
\]

\[
+ C_n \lambda_n^2 \left[ h_{s2} \sinh \lambda_n(z + h) + \lambda_n \cosh \lambda_n(z + h) \right] + \left( \frac{J_1(\lambda_n r)}{r} \right) \times \left[ [h_{s2} \sinh \lambda_n(z + h)] + \lambda_n \cosh \lambda_n(z + h)] \right]
\]

\[
+ \lambda_n(z + h) [h_{s2} \cosh \lambda_n(z + h)] + \lambda_n \sinh \lambda_n(z + h)]] \right]
\]

\[
(3.3.19)
\]

\[
\sigma_{rz} = \left( \frac{2GK}{a^2} \right) \sum_{n=1}^{\infty} \left[ \frac{\tilde{f}(\lambda_n) J_1(\lambda_n r)}{J_1^2(\lambda_n a)} \right] \left( \lambda_n^2 \right) \sum_{n=1}^{\infty} \left[ \frac{\tilde{f}(\lambda_n) J_2(\lambda_n r)}{J_2^2(\lambda_n a)} \right]
\]

\[
- \left[ \frac{h_{s2} \cosh \lambda_n(z + h)] + \lambda_n \sinh \lambda_n(z + h)]}{(\lambda_n^2 + h_{s1} h_{s2}) \sinh \lambda_n(\xi + h) + \lambda_n(h_{s1} + h_{s2}) \cosh \lambda_n(\xi + h)} \right] - B_n \lambda_n^2 \left[ h_{s2} \cosh \lambda_n(z + h)] + \lambda_n \sinh \lambda_n(z + h)] \right]
\]

\[
+ C_n \lambda_n^2 \left[ 1 - 2\nu \right] \left[ h_{s2} \sinh \lambda_n(z + h)] + \lambda_n \cosh \lambda_n(z + h)] \right] - \lambda_n(z + h) [h_{s2} \cosh \lambda_n(z + h)] + \lambda_n \sinh \lambda_n(z + h)]] \right]
\]

\[
(3.3.20)
\]

Now in order to satisfy the boundary conditions given in the equation

\[
\]
(3.2.15), we use equations (3.3.18), (3.3.20) and (3.3.21) for $B_n$ and $C_n$, one obtains

$$B_n = \frac{(1 - 2\nu)}{\lambda_n^3 \left[ (\lambda_n^2 + h_s h_l) \sinh [\lambda_n (\xi + h)] + \lambda_n (h_s + h_l) \cosh [\lambda_n (\xi + h)] \right]}$$

(3.3.22)

and

$$C_n = \frac{1}{\lambda_n^3 \left[ (\lambda_n^2 + h_s h_l) \sinh [\lambda_n (\xi + h)] + \lambda_n (h_s + h_l) \cosh [\lambda_n (\xi + h)] \right]}.$$  

(3.3.23)

Using these values of $B_n$ and $C_n$ in equations (3.3.16)–(3.3.21), one obtain the expressions for displacement and stresses as,

$$u_r = \left( \frac{2K(1 - 2\nu)}{a^2} \right) \sum_{n=1}^{\infty} \left[ \frac{\bar{f}(\lambda_n) J_1(\lambda_n r)}{J_0^2(\lambda_n a)} \right]$$

$$\times \left[ \frac{h_s h_l \sinh [\lambda_n (z + h)] + \lambda_n \cosh [\lambda_n (z + h)]}{(\lambda_n^2 + h_s h_l) \sinh [\lambda_n (\xi + h)] + \lambda_n (h_s + h_l) \cosh [\lambda_n (\xi + h)]} \right]$$

(3.3.24)

$$u_z = \left( \frac{2K(1 - 2\nu)}{a^2} \right) \sum_{n=1}^{\infty} \left[ \frac{\bar{f}(\lambda_n) J_0(\lambda_n r)}{J_0^2(\lambda_n a)} \right]$$

$$\times \left[ \frac{h_s h_l \cosh [\lambda_n (z + h)] + \lambda_n \sinh [\lambda_n (z + h)]}{(\lambda_n^2 + h_s h_l) \sinh [\lambda_n (\xi + h)] + \lambda_n (h_s + h_l) \cosh [\lambda_n (\xi + h)]} \right]$$

(3.3.25)

$$\sigma_{rr} = \left( \frac{4KG(1 - \nu)}{a^2} \right) \sum_{n=1}^{\infty} \left[ \frac{\bar{f}(\lambda_n) J_1(\lambda_n r)}{r J_0^2(\lambda_n a)} \right]$$

$$\times \left[ \frac{h_s h_l \sinh [\lambda_n (z + h)] + \lambda_n \cosh [\lambda_n (z + h)]}{(\lambda_n^2 + h_s h_l) \sinh [\lambda_n (\xi + h)] + \lambda_n (h_s + h_l) \cosh [\lambda_n (\xi + h)]} \right]$$

(3.3.26)
\[
\sigma_{\theta\theta} = \left( \frac{-4KG(1-\nu)}{a^2} \right) \sum_{n=1}^{\infty} \left[ \frac{f(\lambda_n)}{J_p^2(\lambda_n a)} \right] \left( \lambda_n J_0(\lambda_n r) - \frac{J_1(\lambda_n r)}{r} \right) x \left[ \frac{h_{s2} \sinh[\lambda_n(z+h)] + \lambda_n \cosh[\lambda_n(z+h)]}{(\lambda_n^2 + h_{s1}h_{s2}) \sinh[\lambda_n(\xi+h)] + \lambda_n(h_{s1} + h_{s2}) \cosh[\lambda_n(\xi+h)]} \right]
\]

\[
\sigma_{zz} = 0 \quad (3.3.27)
\]

and

\[
\sigma_{rz} = 0. \quad (3.3.29)
\]

### 3.4 Special Case and Numerical Calculations

Setting

\[
f(r) = (r^2 - a^2)^2 \quad (3.4.1)
\]

Using equation (3.4.1) in equation (3.3.8), one obtains

\[
\overline{f}(\lambda_n) = \int_0^a r(r^2 - a^2)^2 J_0(\lambda_n r)dr,
\]

\[
\overline{f}(\lambda_n) = \frac{8a \{(8 - a^2\lambda_n^2)J_1(\lambda_n a) - 4a\lambda_n J_0(\lambda_n a)\}}{\lambda_n^5}. \quad (3.4.2)
\]

Numerical calculations have been carried out for a steel (SN 50C) plate with parameters chosen \(a = 1m, h = 0.2m\). The thermal diffusivity is given by \(k = 15.9 \times 10^6 (m^2s^{-1})\) and the Poisson ratio by \(\nu = 0.281\). The relative heat transfer coefficients \(h_{s1}=10\) and \(h_{s2}=5\).
TRANSCENDENTAL ROOTS

The transcendental roots of \( J_1(\lambda_n a) \) as in [10] are \( \lambda_1 = 3.8317, \lambda_2 = 7.0156, \lambda_3 = 10.1735, \lambda_4 = 13.3237, \lambda_5 = 16.470, \lambda_6 = 19.6159, \lambda_7 = 22.7601, \lambda_8 = 25.9037, \lambda_9 = 29.0468, \lambda_{10} = 32.18. \)

For convenience, we set

\[
\alpha = \left( \frac{-16}{10^2a} \right), \quad \beta = \left( \frac{16K}{10^2a} \right) \quad \text{and} \quad \gamma = \left( \frac{32GK}{10^2a} \right)
\]

in equations (3.3.11) and (3.3.24)–(3.3.27).

In order to examine the influence of an unknown temperature on the upper surface of the circular plate, we performed numerical calculations for \( z = \frac{h}{2}, r = 0, 0.2, 0.4, 0.6, 0.8, 1m \) and \( \xi = -0.2, -0.1, 0, 0.1, 0.2m. \)

Numerical variations in radial directions are shown in the figures with help of a computer programme.

3.5 Concluding Remarks

In this chapter, we discussed the inverse quasi-static thermoelastic problem in a thick circular plate which is free from traction subjected to an arbitrary known interior temperature and determined
the expressions for unknown temperature, displacement and stress functions.

As a special case a mathematical model is constructed for

\[ f(r) = (r^2 - a^2)^2 \]

and numerical calculations were performed.

The thermoelastic behavior such as an unknown temperature, displacement and stresses with the help of arbitrary known interior temperature is examined.

**Figure 3.1**, the unknown temperature \( g(r) \) develops tensile stress within annular region \( 0.6 \leq r \leq 1 \) and compressive stress in circular region \( 0 \leq r \leq 0.6 \) in the radial direction.

**Figure 3.2**, the radial displacement \( u_r \) shows the normal curve and it is zero at \( r = 0 \), the circular boundary of the circular plate. Also it develops the tensile stress in the radial direction.

**Figure 3.3**, the axial displacement \( u_z \) develops tensile stress within circular region \( 0 \leq r \leq 0.6 \) and compressive stress in annular region \( 0.6 \leq r \leq 1 \).

**Figure 3.4**, the radial stress \( \sigma_{rr} \) shows the normal curve and it is zero at \( r = 0 \), the circular boundary of the circular plate. Also it
develops the compressive stresses in the radial direction.

**Figure 3.5**, the angular stress $\sigma_{\theta\theta}$ develops tensile stress within annular region $0.7 \leq r \leq 1$ and compressive stress in circular region $0 \leq r \leq 0.7$ in the radial direction.

We can summaries that, the displacement and stress components which occur near heat source. With an increases the temperature in the circular plate will tend to expand in the radial direction as well as in the axial direction. In the plane state of stress the stress components $\sigma_{zz}$ and $\sigma_{rz}$ are zero. Also from the figures of displacements, it can be observed that the displacement occurs around the center towards in the downward direction. we concluded that, due to unknown temperature, the circular plate expands in the axial direction and bends concavely at the center. This expansion is inversely proportional to the thickness of the circular plate.

The results obtained here are more useful in engineering problems particularly in the determination of the state of strain in a thick circular plate.

Any particular case of special interest can be derived by assigning suitable values to the parameters and functions in the expressions (3.3.11) and (3.3.24)–(3.3.27).
Figure 3.1: Unknown temperature \( \frac{g(r)}{\alpha} \) in radial direction.

Figure 3.2: The radial displacement function \( \frac{u_r}{\beta} \) in radial direction.
Figure 3.3: The axial displacement function $\frac{u_z}{\beta}$ in radial direction.

Figure 3.4: The radial stress function $\frac{\sigma_{rr}}{\gamma}$ in radial direction.
Figure 3.5: The angular stress function $\frac{\sigma_{\theta \theta}}{\gamma}$ in radial direction.

References


4. Roy Choudhary, S. K.: A Note of Quasi-Static Stress in a Thin Circular Plate due to Transient Temperature Applied along the Circumference of a


