Abstract  A numerical search technique for designing a trajectory that transfers a spacecraft from a high inclination Earth orbit to a geo-stationary orbit using lunar gravity assist is presented. This technique generates precise transfer trajectory characteristics that achieve the specified geo-stationary targets. A modified genetic algorithm, genetic algorithm with adaptive bounds, is employed for numerical search. The better convergence efficiency is demonstrated. Guidelines to select search bounds for the unknown parameters for use in genetic algorithms are described. The impact of force models on precise solutions is studied and the importance of Earth's second zonal harmonic in the lunar gravity assist transfers is established.

8.1 Introduction

Conventionally, the transfer to geo-stationary orbit (GSO) is achieved by placing the spacecraft initially in a geo-stationary transfer orbit (GTO) with perigee altitude around 200 km and apogee altitude around 36000 km. The GTO orbital planes are inclined to the Earth equator because of launch station locations. Large amount of propellant is required to effect the plane change to attain zero inclination as well as to raise the perigee to 36000 km. These manoeuvres make the mission cost high, especially when the initial GTO inclinations are high. Alternate approaches [71-73] that reduce the fuel budget are discussed in the literature. They advocate the use of lunar gravity assist for such missions. The approach trajectory to the
moon, when it goes through the lunar gravity field, undergoes a plane change and gains or loses energy relative to the Earth. That is, after the close encounter, the speed of the space vehicle relative to the Earth increases or decreases depending on the geometry of the approach trajectory relative to the moon. The post encounter trajectory can have a wide spectrum of orbital characteristics. This phenomenon can be judiciously used to raise the perigee altitude of the return trajectory, rotate the apsidal line and change the orbital inclination by choosing appropriate initial transfer orbit characteristics relative to the Earth. Thus, the transfer of a spacecraft to geo-stationary orbit from a low earth parking orbit involves identification of appropriate initial transfer trajectory characteristics that results in a low inclination and GSO altitude as its perigee after encounter with the moon.

Circi et al. [73] obtained the initial trajectory design using Opik's method, and refined them by numerical integration and by Newton-Raphson algorithm. In this formulation, zero inclination for the return orbit is aimed at, which requires the moon at the node at the time of encounter. Based on this requirement, right ascension of ascending node of the transfer trajectory is fixed as that of the moon. Also, the initial apogee of the transfer trajectory is fixed because the Opik's method requires it. The time of departure and the argument of perigee are obtained using Opik’s method. Fixation of apogee determines the departure epoch. But, in reality, for a departure epoch many gravity assist opportunities could be obtained by varying the flight durations. The Circi’s formulation is not suitable to handle such scenarios. Also, it is not suitable to handle non-zero return orbit inclination. Further, Circi et al. observed that the effectiveness of the Newton-Raphson algorithm in the problem of determining the transfer trajectory design that involves unstable dynamics requires a good initial guess.

In this chapter, a numerical search technique that uses a modified version of genetic algorithm (GA) is proposed to arrive at the accurate initial transfer trajectory characteristics
that achieve the targets precisely. This formulation can handle all scenarios such as
generating transfer trajectory design for a given departure epoch and achieving a wide
spectrum of Earth return orbits. The process of finding minimum energy opportunity is not
included in the numerical search technique because the variation in energy requirements is
only marginal in wide regions of departure epochs that exist for this problem. So, the focus is
to generate trajectory parameters for a given departure epoch that achieve the targets
precisely. Guidelines to select search bounds for the unknown parameters for use in GA are
described. Because of the extreme sensitivity of the out-going trajectory to the initial
conditions, the performance of the regular genetic algorithm is found to be inadequate. A
modified version of GA: GA with adaptive bounds (GAAB) described in Chapter 7 has been
successfully employed to achieve fast convergence. Further, the effect of different force
models on GAAB solutions and on the achieved target parameters is studied. The contribution
of Earth's second zonal harmonic is established.

8.2 Techniques for Gravity Assist Transfer Trajectory Design

The techniques that are generally discussed in the literature to generate the design of
transfer trajectory using gravity assist are briefly described in the present context.

8.2.1 Tisserand criterion

The underlying assumptions in the usage of this criterion are:

(i) The bodies (primary and secondary, e.g. Earth and Moon) are in motion around
their common center of mass in circular coplanar orbits.

(ii) The secondary body's gravity acceleration is discarded when the spacecraft is not
closer to the secondary body.
Under the above assumptions, Tisserand set a quantity (T), which is invariant for the incoming (approach) and outgoing (return) legs of gravity assist trajectories relative to the primary body, given by

\[ T = \frac{p}{a} + 2\left(\frac{a}{p}\right)\left(1 - e^2\right)^{\frac{1}{2}} \cos i \]

where \( p \) is the distance between the primary and secondary body, and \( a, e, i \) are the semi major axis, eccentricity and inclination of the incoming or outgoing trajectories. Using this criterion some preliminary analysis can be carried out.

8.2.2 Point conic / Patched conic gravity assist trajectories

In point conic technique, the approach (from an earth orbit to the moon) and the return orbit phases (from the moon to an earth orbit) of the geocentric trajectory are analytically obtained by treating each of the phases as a two-body Lambert problem. In both the phases, the center of the moon is taken as one of the two points of the Lambert problem. The geocentric velocities of these phases at the center of the moon are assumed as the geocentric velocities at the imaginary boundary of the mean sphere of influence (MSI). The selenocentric incoming and outgoing asymptotic velocity magnitudes are matched by varying the flight duration of return leg of the geocentric transfer trajectory for a given departure epoch and for an approach flight duration. On propagation under a force model that includes the gravity fields of the Earth and the moon, the transfer trajectory design obtained by the point conic process impacts the moon. On further propagation, the geocentric trajectory becomes hyperbolic after the notional flyby.
Patched conic technique determines three conics: (i) a geocentric conic up to the boundary of MSI of the moon considering the Earth gravity field only (ii) a selenocentric conic along which the spacecraft travels from the boundary of the MSI to the boundary of the MSI considering the moon's gravity field only (iii) a geocentric conic from the boundary of MSI of the moon considering the Earth's gravity filed only. The geocentric and selenocentric conics are patched at the boundaries of MSI by iteration.

In these techniques, grid search is used to find the minimum energy opportunity and its characteristics. The resulting transfer trajectories involve enormous errors in achieving the targets. However, they provide a good knowledge about the trajectory design parameters and the minimum energy opportunities.

8.2.3 Gravity assist trajectories by numerical search

In the absence of accurate analytical algorithms, search using numerical integration is the only efficient technique for the design of the lunar gravity assist trajectory to GSO from a low earth parking orbit. In this technique, the complete trajectory, encompassing the approach leg and return leg after encounter, is simulated for several sets of initial parameters and an appropriate set that achieves the targets is chosen.

8.3 Transfer Trajectory Parameters

The transfer trajectory characteristics are described by six parameters: semi-major axis \(a\), eccentricity \(e\), inclination \(i\), right ascension of ascending node \(\Omega\), argument of perigee \(\omega\) and true anomaly \(\nu\). To describe the motion, all these parameters must be obtained at the time of departure. Because the transfer is from an Earth
parking orbit (say, GTO), the initial perigee and inclination of the transfer trajectory are fixed as that of the Earth parking orbit. Also, tangential injection from the Earth parking orbit is possible only when the departure is from the perigee of the transfer trajectory thus fixing the true anomaly to zero. Furthermore, the equation that relates the position of the moon in terms of right ascension and declination (\( \alpha_M, \delta_M \)) and \( \Omega_r \), as given in Chapter 4,

\[
\sin(\alpha_M - \Omega_r) = \frac{\tan \delta_M}{\tan i_r}
\]

points out that the right ascension of ascending node fixes the approach flight duration of the trajectory for a departure epoch. It is well known that for a departure epoch, point conic technique produces transfer trajectories for various combinations of approach and return flight durations. This is illustrated in a plot and discussed later. In other words, different \( \Omega_r \)'s result in different combinations of approach and return flight durations. So, the right ascension of ascending node can be fixed at a feasible value, which is equivalent to fixing the approach flight duration in the point conic solution procedure. The requirement of near zero inclination for the return trajectory necessitates the moon's presence near the node at the time of spacecraft's encounter with the moon. Also to avoid out-of-plane maneuvers, it is required that the transfer trajectory plane must contain the moon at the encounter time. These requirements fix the ranges for right ascension of ascending node (\( \Omega_r \)) in the neighborhood of the moon's right ascension of ascending node at the encounter time. Thus, the unknown parameters which are to be determined reduce to two \( (h_a, \omega) \), where \( h_a \) is the apogee altitude.

### 8.4 Problem Formulation

Determine at an initial epoch for departure:
apogee altitude of the transfer trajectory
argument of perigee of the transfer trajectory

Objective:

Achieve GSO altitude and inclination after lunar encounter

8.5 Numerical Search Technique using GAAB

A search for the appropriate values of the unknown parameters of the lunar gravity assist trajectory is carried out to achieve the terminal conditions on GSO altitude and inclination. This numerical search is streamlined using genetic algorithm (GA). A brief description about the genetic algorithm and a brief account of the modified version of GA, referred to as ‘Genetic algorithm with adaptive bounds (GAAB)’, are given in the previous chapter.

8.5.1 Bounds for unknown parameters

The bounds for the argument of perigee are fixed based on the observation that the transfer angle is around 180 deg and the perigee of the transfer orbit must be near the earth equator. There are two possible ranges for argument of perigee (i) around zero deg (ii) around 180 deg. The ranges for apogee are fixed in the neighborhood of geocentric radial distance of the moon at the encounter time.

8.5.2 Fitness function

The fitness value for each set of input parameters must be ascertained. The fitness function is computed as
\[ f = \frac{1}{1 + \text{obj}} \]

where

\[ \text{obj} = \left[ \frac{h_{\text{RET}}(t_f) - h_{\text{GSO}}}{w_h} \right]^2 + \left[ \frac{i_{\text{RET}}(t_f) - i_{\text{GSO}}}{w_i} \right]^2 \]

The required terminal conditions of GSO altitude and inclination are represented by \( h_{\text{GSO}} \) and \( i_{\text{GSO}} \) respectively. The parameters \( h_{\text{RET}}(t_f) \) and \( i_{\text{RET}}(t_f) \) represent the values of altitude and inclination of a current simulation at the time of termination. The weight factors and \( w_h \) and \( w_i \) are chosen to scale the magnitudes of the parameters appropriately. The proper choice of the weight factors enables uniform convergence on both altitude and inclination and forces the solution to meet the constraints. The objective function does not include the terms for minimizing energy.

### 8.5.3 Termination criteria

The trajectory simulation is terminated on satisfying one of the following two criteria:

1) The time of propagation is equal to a prefixed time,
2) The radial distance is equal to GSO altitude.

The terminal time \( t_f \) is set to equal the prefixed time or the time at which criterion 2 is satisfied. The first criterion is necessary to restrict the propagation time to a reasonable value. Otherwise, if the GSO altitude is not reached the propagation will continue indefinitely. The second criterion is necessary to handle the situations where the radial distance equals the GSO altitude before the pre-fixed time. The simulation model and the solution process are as
described in the previous chapter. However, this formulation differs from the formulation of direct one-way transfer presented in Chapter 6. The differences are: (i) the number of unknown parameters in this formulation is less (ii) the flight duration is not fixed (iii) objective function is different (iv) termination criteria is different (v) guidelines for fixing the bounds are different.

8.6 Results and Discussion

To illustrate, an approach trajectory (Earth-Moon leg of the trajectory) with a perigee altitude of 300 km and an inclination of 50 deg is considered. After encounter with the moon, a return trajectory (Moon-Earth leg of the trajectory) with a perigee altitude of 35900 km and an inclination of near zero deg relative to the Earth equator must be achieved. The task is to find the parameters apogee altitude \( h_a \) and argument of perigee \( \omega_1 \) of the approach phase of the transfer orbit, which after a lunar encounter achieve the target perigee and inclination without any additional propulsion. A target orbit inclination (GSO) of 1 deg is considered for demonstration purposes. The departure is assumed in January 2007. In this period, feasible regions for departure that require minimum energy for transfer must be identified. The departure is assumed in January 2007. To achieve near zero GSO inclination after lunar encounter, the declination of the moon during spacecraft's close approach must be less than the GSO inclination. The Moon's declination is between ±1 deg on the 9th and 23rd of January 2007 (refer Figure 8.1). The close approach must take place around these dates. In this example, we consider 23rd January as the lunar encounter day and we fix the departure epoch at 18th January, 20 hrs. The corresponding right ascension of the moon is in the range of -3 to 0 deg (refer Figure 8.2). The parameter \( \Omega_1 \) is varied in this region.
8.6.1 Precise solution using GAAB

The bounds were chosen as [360,000km 420,000 km] for the apogee altitude and
[170deg 190deg] for the argument of perigee. The crossover and mutation probabilities are
fixed as 0.8 and 0.01 respectively. The population size is fixed at 40. For illustrations, a
propagation force model consisting of non-spherical gravity models of the Earth (10x 0 field)
and the moon (9x0 field) and the Sun’s point mass effect is considered. The proper choice of
the weight factors enables uniform convergence ensuring desired error levels on both altitude
and inclination. For this study, a value of 500 is used for the ratio of the weights. Other values
will also work. With this value, and a convergence that achieves a fitness value of .999999,
the error on the achieved altitude is less than 0.5 km and the error on the achieved inclination
is less than 0.001 deg. The errors are different with other values.

Figures 8.3 and 8.4 provide comparison of the performances of GA and GAAB. The
population size is set as 40 for this comparison. The targets achieved are plotted with respect
to number of generations. GA achieves 35600 km of return orbit perigee altitude and 6 deg of
return orbit inclination at the end of 10th generation. But, later on, the convergence is very slow
and even after 100th generation the best solution corresponds to achieved targets of 35618
km and 5.8 deg. To achieve the targets exactly with GA, the bounds have to be changed and
the search must be repeated. Two types of changes must be done (i) reducing the width of the
bounds (ii) moving the boundaries of the bounds. GAAB incorporates this phenomenon and
achieves good convergence. GAAB achieves a maximum fitness value of one that
guarantees 35900 km and 1 deg for the return orbit perigee altitude and inclination
respectively.

Figure 8.5 depicts the adaptation process of GAAB. The width of the bounds, initially
wide, narrows as the solution process progresses. The GA process involves selection of
discrete values from the bounds for the initial parameters. In a wide bound, the values are sparsely selected and the trajectories are generated. These trajectories are highly sensitive to the initial parameters. The adaptation process enables dense discretization in the reduced bounds and overcomes the high sensitivity problem.

The initial conditions of lunar gravity assist trajectories for a right ascension of ascending node range of $-0.75$ to $-2.25$ deg are given in Table 8.1. Approach flight duration ranges from 4.44 to 4.69 days. In all the cases, the desired return orbit perigee altitude and inclination are precisely achieved. If one considers an initial transfer orbit of $300 \times 35,900$ km at an inclination of 50 deg, transfer to GSO by usual methods requires a velocity addition of $2,350$ m/s. Using the lunar gravity assist as described above, the total velocity requirement would be $1,767$ m/s, a reduction of $583$ m/s. A typical gravity assist trajectory is depicted in Figure 8.6. The change in inclination from 50 deg to 1 deg after lunar encounter is clearly seen.

### 8.6.2 Effect of Earth's oblateness

To assess the contribution of various components of the force model GAAB initial conditions are generated using the following force models:

- **Case 1** - spherical gravity fields for the Earth and the moon
- **Case 2** - Case 1 + Earth's second zonal harmonic
- **Case 3** - non spherical gravity models for the Earth and the moon + point mass Sun.

The results are given in Table 8.2. These initial conditions are propagated with the Case 3 force model, referred to as the reference force model. Note the change in the initial value of apogee. The initial argument of perigee of the transfer trajectory does not change much due to
force models. The resulting return orbits are also given in Table 8.2. The GAAB initial conditions of Case 1 when propagated with the reference force model, results in a very large return orbit perigee altitude of 342,259.7 kms. But, the GAAB solution of Case 2, achieves a target perigee altitude of 33,822 kms. Further, GAAB solution of Case 3 is propagated without the effect of Earth’s second zonal harmonic. The spacecraft experiences a velocity change of about 2 m/s after a flight of 30 minutes. At this time the spacecraft reaches a radial distance of about 14,000 kms. The velocity change due to other forces is less than $10^{-3}$ m/s. After this distance the influence of asphericity is negligible. However, the initial velocity change of 2 m/s leads to large deviations in the return orbit (see Table 8.2). It is clear that the spherical gravity models are not sufficient for precise targeting and the major contribution is from Earth’s second zonal harmonic. Thus the orbit propagation model of numerical search technique must include at least the second zonal harmonic of the Earth.

8.7 Conclusions

The design characteristics of lunar gravity assist trajectories are obtained using a numerical search technique employing a modified version of the genetic algorithm – genetic algorithm with adaptive bounds. The functioning of the search technique is demonstrated. The modified version of genetic algorithm improves the convergence and helps achieve the targets accurately. The effect of different force models used in the design determination process is evaluated. The second zonal harmonic of Earth’s gravity plays an important role on the transfer trajectory and its achieved targets. It should be considered in the propagation force model of any numerical search process.
Table 8.1
Initial conditions of precise GAAB solution for lunar gravity assist trajectory

<table>
<thead>
<tr>
<th>$\Omega$ (deg)</th>
<th>$h_a$ (km)</th>
<th>$\omega$ (deg)</th>
<th>Approach/Total Flight Duration (days)</th>
<th>Impulses at dep./ for GSO (m/s)</th>
<th>Flyby Altitude (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.75</td>
<td>387,032.2</td>
<td>184.551</td>
<td>4.694 / 8.057</td>
<td>678.4 / 1090.1</td>
<td>8845.4</td>
</tr>
<tr>
<td>-1.0</td>
<td>386,724.6</td>
<td>184.134</td>
<td>4.646 / 7.980</td>
<td>678.3 / 1091.4</td>
<td>8496.7</td>
</tr>
<tr>
<td>-1.5</td>
<td>386,484.8</td>
<td>183.732</td>
<td>4.576 / 7.941</td>
<td>678.3 / 1089.3</td>
<td>8411.8</td>
</tr>
<tr>
<td>-2.0</td>
<td>386,238.2</td>
<td>183.148</td>
<td>4.493 / 7.844</td>
<td>678.2 / 1089.7</td>
<td>8047.1</td>
</tr>
<tr>
<td>-2.25</td>
<td>386,151.6</td>
<td>182.828</td>
<td>4.444 / 7.786</td>
<td>678.2 / 1090.3</td>
<td>7814.5</td>
</tr>
</tbody>
</table>
Table 8.2
Transfer trajectory parameters under different force models and the resulting targets on propagation

<table>
<thead>
<tr>
<th>Parameters</th>
<th>GAAB Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Case 1</td>
</tr>
<tr>
<td><strong>At departure</strong></td>
<td></td>
</tr>
<tr>
<td>$h_a$ (km)</td>
<td>375,697.1</td>
</tr>
<tr>
<td>$\omega_i$ (deg)</td>
<td>184.3360</td>
</tr>
<tr>
<td><strong>Return orbit on propagation with reference force model</strong></td>
<td></td>
</tr>
<tr>
<td>Perigee altitude (km)</td>
<td>342,259.7</td>
</tr>
<tr>
<td>Inclination (deg)</td>
<td>15.54</td>
</tr>
</tbody>
</table>
Figure 8.1 Declination profile of the moon during January 2007
Figure 8.2 Right ascension profile of the moon during January 2007
Figure 8.3 Comparison of convergence pattern for GSO altitude
Figure 8.4 Comparison of convergence pattern for GSO altitude
Figure 8.5  Adaptation process of GAAB
Figure 8.6 Typical lunar gravity assist trajectory