Chapter 2

Review of Literature

2.1 Lambert Problem

Determination of an orbit connecting two positions in a desired time of flight under a central force field is the famous Lambert's problem. Analytical techniques for lunar transfer trajectory design use the Lambert problem solutions in the transfer trajectory design process. Lambert problem solutions are the lifeline of the design process. So, we start with review of literature in the Lambert problem area. Lambert conjectured that the orbital transfer time depends only upon the semi-major axis, the sum of the distances of the initial and final points of the arc from the center of force, and the length of the chord joining these points. The conjecture was referred to as Lambert's theorem. Euler developed the special case of Lambert's theorem for the parabola and did not extend it for ellipse and for hyperbola. Lagrange was the first to provide analytical proof of Lambert's theorem. Lambert's problem solution is obtained using the functional relationship provided by Lambert's theorem. Many great mathematicians like Euler and Gauss solved this classical orbit determination problem. Escobal [7] consolidates six methodologies in his book. He discusses about advantages and disadvantages of the methodologies. These methods have different formulations for different types of conics: circular, elliptic, parabolic or hyperbolic. They also encounter mathematical singularities. Lancaster [8] discusses a unified form of Lambert's theorem. Battin [9, 10] and Vaughan [10] develop a unified algorithm, which is uniformly valid for all types of orbits and overcomes the singularity difficulties. This algorithm generates conic characteristics, which are essentially described in the plane of motion. But, the initial state that comprises of three
position and three velocity components must be determined to describe the orientation of the plane also. Der [11] presents a procedure that determines the complete state for all types of conics. Ramanan [12] records the experiences with the Battin’s algorithm for the Lambert’s problem and establishes the supremacy of the algorithm over the other algorithms in terms of convergence.

2.2 Techniques for Interplanetary Transfer Trajectory

Using Lambert problem solutions, many analytical techniques that substitute the propagation-by-numerical-integration were formulated for generating interplanetary transfer trajectories. Later, they were modified for the lunar transfer trajectory design problems. Use of Lambert problem solution for the design of transfer trajectory is often referred to as point conic technique. In point conic technique the target planets are assumed to be point masses. Patched conic technique includes the gravity of the bodies one at a time basis and improves the point conic design. In most of the interplanetary missions, the trajectory design is obtained using patched conic technique. The patched conic design parameters were refined using brute force methods based on numerical integration. Many authors describe patched conic technique concepts. Escobal [13] and Charles Brown [14] give a detailed account of this technique in their books. It must be noted that these techniques provide ways of accounting the forces acting on the spacecraft only and they do not generate the transfer trajectory design. The interplanetary transfer trajectory when generated using analytical techniques consists of three phases: (i) departure hyperbolic trajectory phase relative to the departure planet where only the gravity field of the departure planet is assumed, (ii) interplanetary transfer trajectory phase relative to the central body where the only the gravity field of the central body is assumed and (iii) approach hyperbolic trajectory phase relative to the target
planet where only the gravity field of the target planet is assumed. These three phases must be synthesized to realize a mission.

Victor Clarke[15] in his trend setting paper describes methodologies to obtain transfer trajectories to the moon and other planets. Breakwell and Lawrence [16] present methodologies based on patched conic technique to obtain interplanetary transfer trajectories. They analyze the trajectories and their characteristics extensively and demonstrate the use of patched conic technique for generating interplanetary transfer trajectories. Wilson and Miller [17] discuss trajectory simulation methods used in the interplanetary trajectory design problem. Stump and Weiss [18] solve the trajectory design problem as an application of solution to n-body problem. Cornelisse [19] and Walberg [20] give a review of techniques of transfer trajectory design. Bell [21] presents methods to obtain transplanetary injection condition that puts the spacecraft in hyperbolic trajectory relative to the departure planet. He searches all through the Earth parking orbit that minimizes the impulse at departure. Sauer [22] analyzes the effect of target planet parking orbit constraints on the transfer trajectory design.

Wilson [23], in his path breaking work, develops a new pseudo state theory that helps generate near accurate transfer trajectory design. This technique accounts for the central body gravity field also in the neighborhood of target planets. The force model of this technique includes the gravity fields of the target bodies and the central body involved. Pseudo state theory is an effective substitute for numerical integration. Byrnes [24] applies this technique to design the interplanetary gravity assist transfer trajectory. He develops required state transition matrices that help transform a state related to one body to a state related to another body. Goodyear [25] derives the partial derivatives of the state. These partial derivatives are used by Byrnes to develop his state transition matrices. Byrnes and Hooper [26] use pseudo
state technique as a multi step propagation technique for trajectory propagation and establish
the accuracy of the trajectory design parameters compared to other techniques. Sergeyevsky,
Byrnes, and D'Amario [27] use pseudo state technique to develop a one step algorithm for the
transfer trajectory design. They assume the center of the planets as their targets and avoid
the need of state transition matrices. In this case, the departure and approach trajectories are
rectilinear hyperbolas. Sergeyevsky et al. [28] analyze the trajectories to Mars using the
above-mentioned one step algorithm. The concepts involved are discussed in Chapter 4. The
details of Galileo mission are worked out using pseudo state technique and the design
analysis is provided by D'Amario et al. [29]. Further D'Amario [30] gives the details of the
software that generates optimal planetary mission design. Sweetser [31] compiles the
analytical techniques for the transfer trajectory design with special emphasis on the
contribution of size of the pseudosphere (similar to the mean sphere of influence used in the
patched conic technique) and derives an empirical relation to find the size of the
pseudosphere. Krystyna Kledron and Sweetser [32] compare various techniques of transfer
trajectory design. Ramanan and Subba Rao [33] discuss the strategies to achieve the
asymptotic conditions of the transfer trajectory from an Earth parking orbit. They present
analytical strategies to generate the geocentric hyperbolic phase of the transfer trajectory
design. Ramanan [34] records the experiences in the implementation of the pseudo state
technique as suggested by Sergeyevsky et al. for interplanetary transfer trajectory problem.
The design is verified using the software ‘GOPS (General Orbit Prediction Software)'
developed by Ramanan et al. [35]. Pieter Kallemeyn et al. [36] and Thurman and Pollmeier
[37] discuss the navigation and guidance requirements in the context of Mars Path finder
mission.
In general, the papers in this area discuss about the planetary mission design based on some techniques such as patched conic technique and pseudo state technique, which are substitutes for numerical integration. Although the resulting trajectories involve errors, they provide broad insight into the design options. An important contribution in this area is pseudo state theory as an effective substitute for numerical integration. The trajectory design using pseudo state theory reduces the errors in the resulting trajectories occurring in the designs of other techniques by 90 to 95%. Most of the papers do not discuss methodologies for generating planetary orbiter mission. Design and analysis for planetary orbiter missions also is not discussed.

2.3 Techniques for Lunar Transfer Trajectory

Lunar transfer trajectory consists of only two phases in the context of application of analytical techniques: (i) geocentric phase where the transfer trajectory is elliptic relative to the Earth (ii) selenocentric phase wherein the trajectory is hyperbolic relative to the moon. So, the transfer trajectory design problem to the moon, although follows same baseline techniques such as patched conic and pseudo state techniques for accounting forces, must have a different approach from the one that is followed in interplanetary trajectory design. Escobal [38] presents an algorithm for the design of lunar transfer trajectory based on patched conic technique. Kevorkian and Brachet [39] numerically analyze nature of the asymptotic lunar trajectory conditions. Battin [40] develops an algorithm based on patched conic technique to arrive at Earth parking orbit requirements (translunar state). He considers circular orbits for the Earth parking orbits. Cook [41] refers to a newer method, called multi conic propagation, based on pseudo state technique and uses it to obtain trajectory design. It uses pseudo state concept a number of times. Each step begins with a conic propagation with respect to the
Earth and the end point is corrected for the effect of moon's gravity. Propagation is continued till the desired arrival conditions are achieved. This method was used in all Apollo programs. Gottlieb et al. [42] present an algorithm based on pseudo state technique for the trajectory design. The method iterates over the time of flight for the hyperbolic phase.

Belborno and Miller [43,44] describe a methodology conceptually similar to weak stability boundary transfer to generate lunar transfer trajectory design. Yamakawa et al. [45] explain the underlying theory of orbital mechanics, in the trajectory design problem. Some authors treated this transfer trajectory problem as restricted 3-body problem and solve through Jacobi integral. Pemicka et al.[46] and Sweetser et al. [47] have attempted the design of trajectory using Jacobi integral. Though these designs cannot be put into practical use, they give an overall insight into the problem. In his Breakwell memorial lecture, Battin [48] recapitulates the moments of discoveries of many methodologies related to Apollo program.

Many research papers have appeared presenting the trajectory design for various missions. Bernard Kaufmann et al. [49] discuss the trajectory design of Clementine mission, Wagner [50] presents Common Lunar Lander (CLL) mission trajectory design, David Lozier et al.[51] discuss the trajectory design of Lunar Prospector mission and Yasuihiro Kawakatsu [52] describes the Japanese Selene mission trajectory design. Jay Middour et al. [53] present the trajectory design execution procedures for the Clementine mission. Ramanan [54, 55] analyzes translunar injection conditions / parking orbit conditions from a circular parking orbit and from an elliptic parking orbit. A special type of transfer using electric propulsion is discussed by Kluever [56].

The papers, in this area, discuss the lunar trajectory design for various missions. Some of them discuss methodologies for generating the trajectory. However, they consider circular Earth parking orbits from which translunar injection is executed. They are not valid for
elliptic Earth parking orbits. Also, as in the case of literature of interplanetary trajectory design, mission design and analysis for orbiter missions to the moon is not discussed.

2.4 Stable Lunar Parking orbits

Appropriate choice for the lunar parking orbit must be made for the transfer trajectory design. Suitable lunar parking orbit parameters are chosen to meet certain mission objectives such as long orbital life and less maintenance cost. Cook [57] develops closed form expressions to find the eccentricity for a given inclination and this combination provides stability in the orbit evolution of the lunar crafts. Orbits with these parameters are known as frozen orbits. These closed form solutions are the functions of the lunar gravity field, which consider only zonal terms. He identifies three inclinations, which provide frozen orbits for 150 km circular orbits. Cook and Sweetser [58] discuss the limitations of the frozen orbit concept for circular orbits. They envisage frequent orbit correction maneuvers for tighter mission specifications on eccentricity and argument of periapsis. Park and Junkins [59] use the same closed form expressions and arrive at frozen orbit parameters using updated gravity models. They identify an inclination of 101.5° as the candidate for 100 km circular orbits. Also, Konopoliv et al. [60] obtain frozen orbit parameters using the same formulation with the zonal terms of the LUN60D gravity model and they conclude that the determination of a polar circular frozen orbit is inconclusive. A large variation in the predicted life times for different orbit inclinations is reported in the literature. Literature also records variation in the predicted life times with different gravity models for same orbit inclination. Frank [61] and Konopoliv [62] present lunar gravity models based on Clementine and Lunar prospector missions respectively. Bills and Ferrari [63] also present a 16 x 16 lunar gravity model. Analysis of lunar parking orbits helps fixing the arrival orbit characteristics.
Literature discusses the orbit characteristics that ensure orbit stability, which results in less maintenance cost. It gives analytical frozen orbit solutions based on zonal harmonic terms of the lunar gravity field. The difficulty encountered in the solution determination process when the number zonal terms are more is described. The tesseral harmonic terms that cause medium periodic (less than moon's orbital period and more than spacecraft orbital period) variations in the orbit evolution are not considered in the frozen orbit identification.

2.5 Numerical Search Technique

Trajectory design methods based on patched / pseudo state techniques generate trajectory design subject to certain assumptions on modeling of forces of the bodies involved. For example, the oblateness of the Earth cannot be included in the analytical trajectory design process. Penzo [64] studies the effect of oblateness of the Earth on lunar and interplanetary trajectories. The only way to generate precise trajectory design by including all perturbations is by using numerical integration for trajectory propagation and by conducting a numerical search using some optimization technique. Hanson and Krogh [65] present a new algorithm to account for these perturbations. Initial conditions obtained by analytical techniques are perturbed in sequential order, partial derivatives are constructed and a set of initial conditions is produced. These conditions are propagated and a new cycle of partial derivatives using the errors are obtained. This process is continued until the final conditions are sufficiently accurate. The process is similar to a differential correction method. However, the search for precise trajectory can be conducted using some optimization technique also. Wang [66] and Betts [67] provide a survey of various optimization techniques that can be used in non-linear trajectory search problems. Kalyanmoy Deb [68] describes classical version of genetic algorithm, an evolutionary search technique. Adimurthy et al. [69] modify genetic algorithm for
fast convergence by adapting bounds as the search progresses. Gaze et al. [70] apply genetic algorithm to interplanetary trajectory optimization. However, in this algorithm, the base line technique for modeling the forces is patched conic technique and not the numerical integration.

The gravity assist from the moon is used judiciously for many purposes: (i) for transferring a spacecraft in an Earth parking orbit to a geo-stationary orbit (GSO) (ii) for free return to Earth after encounter with the moon (iii) for travel to other outer planets. Achieving this trajectory design is again based on either analytical methods or numerical search using some optimization technique. Graziani et al.[71] obtain approximate solution for the trajectory design that transfer a spacecraft to GSO using Opik’s method based on Tisserand criteria for a given flight duration and departure date. They identify appropriate flight duration and departure date, for which this approximate Opik’s solution becomes exact, by varying them and by numerically propagating the approximate initial conditions. They demonstrate the superiority of this transfer over other transfers for launches from mid latitude launch stations. Jah et al. [72] obtain initial transfer trajectory conditions using patched conic approximations. They used these approximations to establish the usefulness of lunar gravity assist trajectories. Circi et al. [73] obtain the initial trajectory design using Opik’s method, and refine them by numerical integration and by Newton-Raphson algorithm. Morskoi et al. [74] discuss the design of lunar gravity assist transfer and compares this design with other scenarios of transfer in terms of energy requirements. Ramanan [75] generates a typical case of transfer trajectory that achieves the transfer to GSO. Mezzedimi [76] analyzes the gravity assist trajectories and their initial conditions. Miele et al. [77] present mission design for free return to Earth after encounter with Mars. Miele and Mancuso [78] design a mission that return to the Earth after flying by the moon. Hantzche [79] also analyzes free-return-to-the-Earth
trajectories by treating the problem as restricted 3-body problem. Kawaguchi et al. [80] demonstrate the use of lunar gravity assist for Martian missions. The Japanese Planet-B mission to Mars is planned using this concept. Kluever [81] designs a return trajectory using nuclear electric propulsion.

The papers in this area discuss the numerical search procedures. Differential correction procedure and Newton Rapson algorithm are adapted to produce precise trajectory design. It is pointed out therein that a good initial guess is required for fast convergence. Some papers use genetic algorithm for the search of the initial state. But they do not produce precise design because they use patched conic technique for trajectory propagation.

2.6 Literature Based on this Research

The research work presented herein carried out is published in archival journals. A brief account of these publications and their contents is given. The new algorithm that generates initial departure state for transfer trajectory design together with the parking orbit characteristics and its performance is consolidated in [82]. The algorithm is uniformly valid for circular as well as elliptic Earth parking orbits. Because the main purpose of this development is to generate initial departure state, for simplicity, arrival is assumed to be at the centre of the moon. However, for orbiter missions arrival must not be at the centre of the moon. So, methods are developed to achieve non impact arrival conditions, and they are used in the trajectory design of orbiter missions to the moon. These methods together with the algorithm are presented in [83]. Analysis carried out for fixing appropriate choice of lunar parking orbit parameters is presented in [84]. The frozen orbit characteristics that provide stability in orbit evolution are obtained by including zonal as well as tesseral harmonic terms of the lunar gravity field. A numerical procedure using genetic algorithm to achieve precise transfer
trajectory design is given in [85]. Numerical integration is used for trajectory propagation in this procedure. Further, this numerical procedure using genetic algorithm is applied for lunar gravity assist transfer to GSO and the details are given in [85, 86, 87]. The seminar papers [88] and [89] reports the analysis of the design of transfer trajectories using the methodologies reported in the above.