Chapter 6

INTERSECTION SIGNED GRAPHS OF DOMINATING SIGNED GRAPHS

6.1 Introduction

Within the rapid growth of the Internet and the Web, and in the ease with which global communication now takes place, connectedness took an important place in modern society. Global phenomena, involving social networks, incentives and the behavior of people based on the links that connect us appear in a regular manner. Motivated by these developments, there is a growing multidisciplinary interest to understand how highly connected systems operate [31].

In social sciences we often deal with relations of opposite content, e.g., “love”-“hatred”, “likes”-“dislikes”, “tells truth to”-“lies to” etc. In common use opposite relations are termed positive and negative relations. A signed graph is one in which

\footnotesize
1This chapter is based on the Papers:
b). Note on Minimal Dominating Signed Graphs, Submitted.

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relations between entities may be of various types in contrast to an unsigned graph where all relations are of the same type. In signed graphs edge-coloring provides an elegant and uniform representation of the various types of relations where every type of relation is represented by a distinct color.

In the case where precisely one relation and its opposite are under consideration, then instead of two colors, the signs $+$ and $-$ are assigned to the edges of the corresponding graph in order to distinguish a relation from its opposite. In the case where precisely one relation and its opposite are under consideration, then instead of two colors, the signs $+$ and $-$ are assigned to the edges of the corresponding graph in order to distinguish a relation from its opposite. Formally, a signed graph $S = (G, \sigma)$ is a graph $G = (V, E)$ together with a function $\sigma : E \to \{+,-\}$, which associates each edge with the sign $+$ or $-$. In such a signed graph, a subset $A$ of $E(G)$ is said to be positive if it contains an even number of negative edges, otherwise is said to be negative. A signed graph $S$ is balanced if each cycle of $S$ is positive. Otherwise it is unbalanced.

The theory of balance goes back to Heider [50] who asserted that a social system is balanced if there is no tension and that unbalanced social structures exhibit a tension resulting in a tendency to change in the direction of balance. Since this first work of Heider, the notion of balance has been extensively studied by many mathematicians and psychologists.
In 1956, Cartwright and Harary [23] provided a mathematical model for balance through graphs. Their cornerstone result states that a signed graph is balanced if and only if in each cycle the number of negative edges is even. The following theorem of Harary gives an equivalent definition of a balanced signed graph.

**Theorem 6.1.1. (Harary [42])**

A signed graph is balanced if and only if its vertex set can be partitioned into two classes (one of the two classes may be empty) so that every edge joining vertices within a class is positive and every edge joining vertices between classes is negative.

Since then, mathematicians have written numerous papers on the topic of signed graphs. Many of these papers demonstrate the connection between signed graphs and different subjects: circuit design (Barahona [13]), coding theory (Solé and Zaslavsky [95]), physics (Toulouse [99]) and social psychology (Abelson and Rosenberg [2]). While these subjects seem unrelated, balance plays an important role in each of these fields.

Four years after Harary’s paper, Abelson and Rosenberg [2], wrote a paper in which they discuss algebraic methods to detect balance in a signed graphs. It was one of the first papers to propose a measure of imbalance, the “complexity” (which Harary called the “line index of balance”). Abelson and Rosenberg introduced an operation that changes a signed graph while preserving balance and they proved that this does not change the line index of imbalance. For more new notions on signed graphs refer the papers [85–94].
A marking of $S$ is a function $\mu : V(G) \to \{+,-\}$; A signed graph $S$ together with a marking $\mu$ is denoted by $S_\mu$. Given a signed graph $S$ one can easily define a marking $\mu$ of $S$ as follows: For any vertex $v \in V(S)$,

$$\mu(v) = \prod_{uv \in E(S)} \sigma(uv),$$

the marking $\mu$ of $S$ is called canonical marking of $S$. In a signed graph $S = (G, \sigma)$, for any $A \subseteq E(G)$ the sign $\sigma(A)$ is the product of the signs on the edges of $A$.

The following characterization of balanced signed graphs is well known.

**Theorem 6.1.2. (E. Sampathkumar [77])** A signed graph $S = (G, \sigma)$ is balanced if, and only if, there exists a marking $\mu$ of its vertices such that each edge $uv$ in $S$ satisfies $\sigma(uv) = \mu(u)\mu(v)$.

Let $S = (G, \sigma)$ be a signed graph. Complement of $S$ is a signed graph $\overline{S} = (\overline{G}, \sigma')$, where for any edge $e = uv \in \overline{G}$, $\sigma'(uv) = \mu(u)\mu(v)$. Clearly, $\overline{S}$ as defined here is a balanced signed graph due to Theorem 6.1.2.

The idea of switching a signed graph was introduced in [2] in connection with structural analysis of social behavior and also its deeper mathematical aspects, significance and connections may be found in [104].

Switching $S$ with respect to a marking $\mu$ is the operation of changing the sign of every edge of $S$ to its opposite whenever its end vertices are of opposite signs. The
signed graph obtained in this way is denoted by $S_\mu(S)$ and is called *\( \mu \)-switched signed graph* or just *switched signed graph*. Two signed graphs $S_1 = (G, \sigma)$ and $S_2 = (G', \sigma')$ are said to be *isomorphic*, written as $S_1 \cong S_2$ if there exists a graph isomorphism $f : G \rightarrow G'$ (that is a bijection $f : V(G) \rightarrow V(G')$ such that if $uv$ is an edge in $G$ then $f(u)f(v)$ is an edge in $G'$) such that for any edge $e \in E(G)$, $\sigma(e) = \sigma'(f(e))$. Further a signed graph $S_1 = (G, \sigma)$ *switches* to a signed graph $S_2 = (G', \sigma')$ (or that $S_1$ and $S_2$ are *switching equivalent*) written $S_1 \sim S_2$, whenever there exists a marking $\mu$ of $S_1$ such that $S_\mu(S_1) \cong S_2$. Note that $S_1 \sim S_2$ implies that $G \cong G'$, since the definition of switching does not involve change of adjacencies in the underlying graphs of the respective signed graphs.

Two signed graphs $S_1 = (G, \sigma)$ and $S_2 = (G', \sigma')$ are said to be *weakly isomorphic* ([96]) or *cycle isomorphic* ([102]) if there exists an isomorphism $\phi : G \rightarrow G'$ such that the sign of every cycle $Z$ in $S_1$ equals to the sign of $\phi(Z)$ in $S_2$. The following result is well known ([102]):

**Theorem 6.1.3.** (T. Zaslavsky [102]) *Two signed graphs $S_1$ and $S_2$ with the same underlying graph are switching equivalent if, and only if, they are cycle isomorphic.*

In [83], the authors also introduced the switching and cycle isomorphism for signed digraphs.
6.2 Domination in Graphs

Mathematical study of domination in graphs began around 1960, there are some references to domination-related problems about 100 years prior. In 1862, de Jaenisch [29] attempted to determine the minimum number of queens required to cover an $n \times n$ chess board. In 1892, W. W. Rouse Ball [76] reported three basic types of problems that chess players studied during that time.

The study of domination in graphs was further developed in the late 1950s and 1960s, beginning with Berge [15] in 1958. Berge wrote a book on graph theory, in which he introduced the “coefficient of external stability”, which is now known as the domination number of a graph. Oystein Ore [67] introduced the terms “dominating set” and “domination number” in his book on graph theory which was published in 1962. The problems described above were studied in more detail around 1964 by brothers Yaglom and Yaglom [101]. Their studies resulted in solutions to some of these problems for rooks, knights, kings, and bishops. A decade later, Cockayne and Hedetniemi [28] published a survey paper, in which the notation $\gamma(G)$ was first used for the domination number of a graph $G$. Since this paper was published, domination in graphs has been studied extensively and several additional research papers have been published on this topic.

Let $G = (V, E)$ be a graph. A set $D \subseteq V$ is a dominating set of $G$, if every vertex in $V - D$ is adjacent to some vertex in $D$. A dominating set $D$ of $G$ is minimal, if
for any vertex $v \in D$, $D - \{v\}$ is not a dominating set of $G$ (Ore [67]).

Let $S$ be a finite set and $F = \{S_1, S_2, ..., S_n\}$ be a partition of $S$. Then the intersection graph $\Omega(F)$ of $F$ is the graph whose vertices are the subsets in $F$ and in which two vertices $S_i$ and $S_j$ are adjacent if and only if $S_i \cap S_j \neq \emptyset$, $i \neq j$.

Kulli and Janakiram [60] introduced a new class of intersection graphs in the field of domination theory. The common minimal dominating graph $CMD(G)$ of a graph $G$ is the graph having same vertex set as $G$ with two vertices adjacent in $CMD(G)$ if, and only if, there exists a minimal dominating set in $G$ containing them.

### 6.3 Common Minimal Dominating Signed Graphs

In this section, we introduce a natural extension of the notion of common minimal dominating graph to the realm of signed graphs since this appears to have interesting connections with complementary signed graph and neighborhood signed graph.

The common minimal dominating signed graph $CMD(S)$ of a signed graph $S = (G, \sigma)$ is a signed graph whose underlying graph is $CMD(G)$ and sign of any edge $uv$ in $CMD(S)$ is $\mu(u)\mu(v)$, where $\mu$ is the canonical marking of $S$. Further, a signed graph $S = (G, \sigma)$ is called common minimal dominating signed graph, if $S \cong CMD(S')$ for some signed graph $S'$. 
The following result indicates the limitations of the notion $CMD(S)$ introduced above, since the entire class of unbalanced signed graphs is forbidden to be common minimal dominating signed graphs.

**Proposition 6.3.1.** For any signed graph $S = (G, \sigma)$, its common minimal dominating signed graph $CMD(S)$ is balanced.

*Proof.* Since sign of any edge $uv$ in $CMD(S)$ is $\mu(u)\mu(v)$, where $\mu$ is the canonical marking of $S$, by Theorem 6.1.2, $CMD(S)$ is balanced. \qed

For any positive integer $k$, the $k^{th}$ iterated common minimal dominating signed graph $CMD(S)$ of $S$ is defined as follows:

$$CMD^0(S) = S, CMD^k(S) = CMD(CMD^{k-1}(S))$$

**Corollary 6.3.2.** For any signed graph $S = (G, \sigma)$ and any positive integer $k$, $CMD^k(S)$ is balanced.

In [60], the authors characterized graphs for which $CMD(G) \cong \overline{G}$.

**Theorem 6.3.3.** (Kulli and Janakiram [60])

For any graph $G = (V, E)$, $CMD(G) \cong \overline{G}$ if, and only if, every minimal dominating set of $G$ is independent.

We now characterize signed graphs whose common minimal dominating signed graphs and complementary signed graphs are switching equivalent.

**Theorem 6.3.4.** For any signed graph $S = (G, \sigma)$, $\overline{S} \sim CMD(S)$ if, and only if, every minimal dominating set of $G$ is independent.
Proof. Suppose $\overline{S} \sim CMD(S)$. This implies, $\overline{G} \cong CMD(G)$ and hence by Theorem 6.3.3, every minimal dominating set of $G$ is independent.

Conversely, suppose that every minimal dominating set of $G$ is independent. Then $\overline{G} \cong CMD(G)$ by Theorem 6.3.3. Now, if $S$ is a signed graph with every minimal dominating set of underlying graph $G$ is independent, by the definition of complementary signed graph and Proposition 6.3.1, $\overline{S}$ and $CMD(S)$ are balanced and hence, the result follows from Theorem 6.1.3. □

In [72], Rangarajan et al. introduced neighborhood signed graph of a signed graph as follows:

The neighborhood signed graph $N(S)$ of a signed graph $S = (G, \sigma)$ is a signed graph whose underlying graph is $N(G)$ and sign of any edge $uv$ is $N(S)$ is $\mu(u)\mu(v)$, where $\mu$ is the canonical marking of $S$. Further, a signed graph $S = (G, \sigma)$ is called neighborhood signed graph, if $S \cong N(S')$ for some signed graph $S'$. The following result restricts the class of neighborhood graphs.

Proposition 6.3.5. (Rangarajan et al. [72])

For any signed graph $S = (G, \sigma)$, its neighborhood signed graph $N(S)$ is balanced.

We now characterize signed graphs whose common minimal dominating signed graphs and neighborhood signed graphs are switching equivalent. In case of graphs the following result is due to Kulli and Janakiram [62].

Theorem 6.3.6. (Kulli and Janakiram [62])

If $G$ is a $(p - 2)$-regular graph with $p \geq 6$, then $CMD(G) \cong N(G)$. 
**Theorem 6.3.7.** For any signed graph $S = (G, \sigma)$, $CMD(S) \sim N(S)$ if, and only if, $G$ is a $(p - 2)$-regular graph with $p \geq 6$.

**Proof.** Suppose $CMD(S) \sim N(S)$. This implies, $CMD(G) \cong N(G)$ and hence by Theorem 6.3.6, we see that the graph $G$ must be $(p - 2)$-regular graph with $p \geq 6$.

Conversely, suppose that $G$ is $(p - 2)$-regular graph with $p \geq 6$. Then $CMD(G) \cong N(G)$ by Theorem 6.3.6. Now, if $S$ is a signed graph with underlying graph as $(p - 2)$-regular graph with $p \geq 6$, by Propositions 6.3.1 and 6.3.5, $CMD(S)$ and $N(S)$ are balanced and hence, the result follows from Theorem 6.1.3. \qed

### 6.3.1 Characterization of $CMD(S)$

The following result characterize signed graphs which are common minimal dominating signed graphs.

**Theorem 6.3.8.** A signed graph $S = (G, \sigma)$ is a common minimal dominating signed graph if, and only if, $S$ is balanced signed graph and its underlying graph $G$ is a $CMD(G)$.

**Proof.** Suppose that $S$ is balanced and its underlying graph $G$ is a common minimal dominating graph. Then there exists a graph $H$ such that $CMD(H) \cong G$. Since $S$ is balanced, by Theorem 6.1.2, there exists a marking $\mu$ of $G$ such that each edge $uv$ in $S$ satisfies $\sigma(uv) = \mu(u)\mu(v)$. Now consider the signed graph $S' = (H, \sigma')$, where for any edge $e$ in $H$, $\sigma'(e)$ is the marking of the corresponding vertex in $G$. Then clearly,
\(CMD(S') \cong S\). Hence \(S\) is a common minimal dominating signed graph.

Conversely, suppose that \(S = (G, \sigma)\) is a common minimal dominating signed graph. Then there exists a signed graph \(S' = (H, \sigma')\) such that \(CMD(S') \cong S\). Hence by Proposition 6.3.1, \(S\) is balanced. \(\square\)

### 6.4 Minimal Dominating Signed Graphs

Kulli and Janakiram [61] introduced a new class of intersection graphs in the field of domination theory. The \(\text{minimal dominating graph } MD(G)\) of a graph \(G\) is the intersection graph defined on the family of all minimal dominating sets of vertices in \(G\).

We now extend the notion of \(MD(G)\) to the realm of signed graphs. The \(\text{minimal dominating signed graph } MD(S)\) of a signed graph \(S = (S^u, \sigma)\) is a signed graph whose underlying graph is \(MD(G)\) and sign of any edge \(PQ\) in \(MD(S)\) is \(\mu(P)\mu(Q)\), where \(\mu\) is the canonical marking of \(S\), \(P\) and \(Q\) are any two minimal dominating sets of vertices in \(S^u\). Further, a signed graph \(S = (G, \sigma)\) is called minimal dominating signed graph, if \(S \cong MD(S')\) for some signed graph \(S'\). In the following subsection, we present a structural characterization of common minimal dominating signed graphs. The following result indicates the limitations of the notion \(CMD(S)\) introduced above, since the entire class of unbalanced signed graphs is forbidden to be common minimal dominating signed graphs.
Proposition 6.4.1. For any signed graph $S = (G, \sigma)$, its minimal dominating signed graph $MD(S)$ is balanced.

Proof. Since sign of any edge $PQ$ in $MD(S)$ is $\mu(P)\mu(Q)$, where $\mu$ is the canonical marking of $S$, by Theorem 6.1.2, $MD(S)$ is balanced.

For any positive integer $k$, the $k^{th}$ iterated minimal dominating signed graph $MD(S)$ of $S$ is defined as follows:

$$MD^0(S) = S, MD^k(S) = MD(MD^{k-1}(S))$$

Corollary 6.4.2. For any signed graph $S = (G, \sigma)$ and any positive integer $k$, $MD^k(S)$ is balanced.

Theorem 6.4.3. For any two signed graphs $S_1$ and $S_2$ with the same underlying graph, their minimal dominating signed graphs are switching equivalent.

Proof. Suppose $S_1 = (S_1^u, \sigma)$ and $S_2 = (S_2^u, \sigma')$ be two signed graphs with $S_1^u \cong S_2^u$. By Proposition 6.4.1, $MD(S_1)$ and $MD(S_2)$ are balanced and hence, the result follows from Theorem 6.1.3.

We now characterize the signed whose minimal dominating signed graphs and common minimal dominating signed graphs are switching equivalent. In case of graphs the following result is due to Kulli and Janakiram [62]:

Theorem 6.4.4. (Kulli and Janakiram [62])

If $G$ is a $(p - 3)$-regular graph and every minimal dominating set of $G$ is independent, then $MD(G) \cong CMD(G)$. 
Proposition 6.4.5. For any signed graph \( S = (G, \sigma) \), \( MD(S) \sim CMD(S) \) if, and only if, \( G \) is a \((p - 3)\)-regular graph and every minimal dominating set of \( G \) is independent.

Proof. Suppose \( MD(S) \sim CMD(S) \). This implies, \( MD(G) \cong CMD(G) \) and hence by Theorem 6.4.4, we see that the graph \( G \) must be \((p - 3)\)-regular graph and every minimal dominating set of \( G \) is independent.

Conversely, suppose that \( G \) is \((p - 3)\)-regular graph and every minimal dominating set of \( G \) is independent. Then \( MD(G) \cong CMD(G) \) by Theorem 6.4.4. Now, if \( S \) is a signed graph with underlying graph as \((p - 3)\)-regular graph and every minimal dominating set of \( G \) is independent, by Propositions 6.3.1 and 6.4.1, \( CMD(S) \) and \( MD(S) \) are balanced and hence, the result follows from Theorem 6.1.3.

6.4.1 Characterization of \( MD(S) \)

The following result characterize signed graphs which are minimal dominating signed graphs.

Theorem 6.4.6. A signed graph \( S = (G, \sigma) \) is a minimal dominating signed graph if, and only if, \( S \) is balanced signed graph and its underlying graph \( G \) is a \( MD(G) \).

Proof. Suppose that \( S \) is balanced and its underlying graph \( G \) is a minimal dominating graph. Then there exists a graph \( H \) such that \( MD(H) \cong G \). Since \( S \) is balanced, by Theorem 6.1.2, there exists a marking \( \mu \) of \( G \) such that each edge \( uv \) in \( S \) satisfies \( \sigma(uv) = \mu(u)\mu(v) \). Now consider the signed graph \( S' = (H, \sigma') \), where for
any edge $e$ in $H$, $\sigma'(e)$ is the marking of the corresponding vertex in $G$. Then clearly, $MD(S') \cong S$. Hence $S$ is a minimal dominating signed graph.

Conversely, suppose that $S = (G, \sigma)$ is a minimal dominating signed graph. Then there exists a signed graph $S' = (H, \sigma')$ such that $MD(S') \cong S$. Hence by Proposition 6.4.1, $S$ is balanced. \qed

6.5 \textbf{Negation of CMD}(S) \& \textbf{MD}(S)

The notion of negation $\eta(S)$ of a given signed graph $S$ defined in [41] as follows: $\eta(S)$ has the same underlying graph as that of $S$ with the sign of each edge opposite to that given to it in $S$. However, this definition does not say anything about what to do with nonadjacent pairs of vertices in $S$ while applying the unary operator $\eta(.)$ of taking the negation of $S$.

Theorems 6.3.4, 6.3.7 & 6.4.5 provides easy solutions to other signed graph switching equivalence relations, which are given in the following results.

\textbf{Corollary 6.5.1.} For any signed graph $S = (G, \sigma)$, $\overline{\eta(S)} \sim CMD(S)$ (or $\overline{S} \sim CMD(\eta(S))$) if, and only if, every minimal dominating set of $G$ is independent.

\textbf{Corollary 6.5.2.} For any signed graph $S = (G, \sigma)$, $\overline{\eta(S)} \sim CMD(\eta(S))$ if, and only if, every minimal dominating set of $G$ is independent.
Corollary 6.5.3. For any signed graph \( S = (G, \sigma) \), \( CMD(S) \sim N(\eta(S)) \) (or \( CMD(\eta(S)) \sim N(S) \)) if, and only if, \( G \) is a \((p - 2)\)-regular graph with \( p \geq 6 \).

Corollary 6.5.4. For any signed graph \( S = (G, \sigma) \), \( CMD(\eta(S)) \sim N(\eta(S)) \) if, and only if, \( G \) is a \((p - 2)\)-regular graph with \( p \geq 6 \).

Corollary 6.5.5. For any signed graph \( S = (G, \sigma) \), \( MD(\eta(S)) \sim CMD(S) \) (or \( MD(S) \sim CMD(\eta(S)) \) or \( MD(\eta(S)) \sim CMD(\eta(S)) \)) if, and only if, \( G \) is a \((p - 3)\)-regular graph and every minimal dominating set of \( G \) is independent.

For a signed graph \( S = (G, \sigma) \), we proved \( CMD(S) \) and \( MD(S) \) are balanced (as per the Propositions 6.3.1 & 6.4.1). We now examine, the conditions under which negation of \( CMD(S) \) (\( MD(S) \)) is balanced.

Theorem 6.5.6. Let \( S = (G, \sigma) \) be a signed graph. If \( CMD(G) \) (\( MD(G) \)) is bipartite then \( \eta(CMD(S)) \) (\( \eta(MD(S)) \)) is balanced.

Proof. Since, by Proposition 6.3.1 (Proposition 6.4.1), \( CMD(S) \) (\( MD(S) \)) is balanced, each cycle \( C \) in \( CMD(S) \) contains even number of negative edges. Also, since \( CMD(G) \) (\( MD(G) \)) is bipartite, all cycles have even length; thus, the number of positive edges on any cycle \( C \) in \( CMD(S) \) (\( MD(S) \)) is also even. Hence \( \eta(CMD(S)) \) (\( \eta(MD(S)) \)) is balanced. \( \square \)
Conclusions and Scope for Future Work

6.6 Conclusions

In conclusion, we made contributions to the classical model of signed graph theory as well as generalizations of signed graphs. In particular, we introduced many new notions in signed graphs and studied their structural characterizations. Singleton (1968) introduced the concept of the antipodal graph of a graph $G$, denoted by $A(G)$, is the graph on the same vertices as of $G$, two vertices being adjacent if the distance between them is equal to the diameter of $G$. Analogously in chapter 2, we defined the antipodal signed graph $A(S)$ of a signed graph $S = (G, \sigma)$ as a signed graph, $A(S) = (A(G), \sigma')$, where $A(G)$ is the underlying graph of $A(S)$, and for any edge $e = uv$ in $A(S)$, $\sigma'(e) = \mu(u)\mu(v)$, where for any $v \in V$, $\mu(v) = \prod_{u \in N(v)} \sigma(uv)$. It is shown that for any signed graph $S$, its $A(S)$ is balanced and we offered a structural characterization of antipodal signed graphs. Further, we characterized signed graphs $S$ for which $S \sim A(S)$ and $\overline{S} \sim A(S)$ where $\sim$ denotes switching equivalence and $A(S)$ and $\overline{S}$ are denotes the antipodal signed graph and complementary signed graph of $S$ respectively.
In chapter 3, we introduced the notion called $S$-Antipodal Signed Graphs and studied its properties and nature of the notion. We note that there are many open problems related to the above said notion. These open problems, are enlisted in the following section ‘Scope for Future Work’.

In chapter 4, we introduced the notion the $t$-Path Step Signed Graph $(S)_t = ((G)_t, \sigma')$ of a given signed graph $S = (G, \sigma)$ and offer a structural characterization of $t$-path step signed graphs. Further, we characterized signed graphs which are switching equivalent to their 3-path step signed graphs.

In chapter 5, an extension of antipodal signed graphs is given for signed digraphs; namely, for a signed digraph $S = (D, \sigma)$, where $D = (V, A)$ is a digraph called underlying digraph of $S$, the antipodal signed digraph $\overrightarrow{A}(S) = (\overrightarrow{A}(D), \sigma')$ of a signed digraph $S = (D, \sigma)$ is a signed digraph whose underlying digraph is $\overrightarrow{A}(D)$ called antipodal digraph and sign of any arc $e = \overrightarrow{uv}$ in $\overrightarrow{A}(S)$, $\sigma'(e) = \mu(u)\mu(v)$, where for any $v \in V$, $\mu(v) = \prod_{u \in N(v)} \sigma(\overrightarrow{uv})$. Further, we characterized signed digraphs $S$ for which $S \sim \overrightarrow{A}(S)$ and $\overline{S} \sim \overrightarrow{A}(S)$ where $\sim$ denotes switching equivalence and $\overrightarrow{A}(S)$ and $\overline{S}$ are denotes the antipodal signed digraph and complementary signed digraph of $S$ respectively. Also, we offered a structural characterization of antipodal signed digraphs.
In chapter 6, we introduced two new class of intersection signed graphs in the field of signed graph theory. The intersection signed graphs are: common minimal dominating signed graph $CMD(S)$ and minimal dominating signed graph $MD(S)$. Also, we obtained structural characterizations of $CMD(S)$ and $MD(S)$. In the sequel, we also obtained switching equivalence characterizations: $\overline{S} \sim CMD(S), CMD(S) \sim N(S)$ and $MD(S) \sim CMD(S)$, where $\overline{S}$, $CMD(S)$, $N(S)$ and $MD(S)$ are complementary signed graph, common minimal signed graph, neighborhood signed graph and minimal dominating signed graph of $S$ respectively.

Finally in this thesis, we have developed a new approach to overcome the limitation of traditional parametric notions in initialization and topological changes.
6.7 Scope for Future Work

Problems that arose in the mainstreams of thought on the themes pursued in each of the foregoing chapters have been explicitly mentioned in the respective chapters. In this section, we have put together some problems that looked stray in the main course of my research work reported so far, for, on second thought we felt they might be of independent interest to investigate and perhaps may get linked up eventually to some of the directions of research reported in this thesis. We have preferred presenting them with some illustrative examples, just to attract attention of an inquisitive researcher.

1. Characterize signed graphs $S$ such that:
   a). $S \cong A(S)$
   b). $\overline{S} \cong A(S)$
   c). $S \cong A(\eta(S))$
   d). $\overline{S} \cong A(\eta(S))$
   e). $S \cong \eta(A(S))$
   f). $\overline{S} \cong \eta(A(S))$.

2. Characterize signed graphs $S$ such that:
   a). $S \cong A^*(S)$
   b). $\overline{S} \cong A^*(S)$
   c). $A^*(S) \cong A(S)$
   d). $A^*(S) \cong A^*(\overline{S})$
   e). $S \cong A^*(\eta(S))$
   f). $A^*(S) \cong A(\eta(S))$
g). \( A^*(S) \cong A^*(\eta(S)) \)

h). \( A^*(\eta(S)) \cong A^*(\overline{S}) \).

3. Characterize signed graphs \( S \) such that:
   a). \( S \cong (S)_3 \)
   b). \( S \cong \eta((S)_3) \)
   c). \( \eta(S) \cong (S)_3. \)

4. Characterize signed digraphs \( S \) such that:
   a). \( S \cong \overrightarrow{A}(S) \)
   b). \( S \cong \eta(\overrightarrow{A}(S)) \)
   c). \( S \cong \overrightarrow{A}(\eta(S)) \)
   d). \( \overline{S} \sim \overrightarrow{A}(\eta(S)). \)

5. Characterize signed graphs \( S \) such that:
   a). \( \overline{S} \cong CMD(S) \)
   b). \( CMD(S) \cong N(S) \)
   c). \( MD(S) \cong CMD(S) \)
   d). \( \overline{\eta(S)} \cong CMD(S) \)
   e). \( \overline{S} \cong CMD(\eta(S)) \)
   f). \( \overline{\eta(S)} \cong CMD(\eta(S)) \)
   g). \( CMD(S) \cong N(\eta(S)) \)
   h). \( CMD(S) \cong N(\eta(S)) \)
i). $CMD(\eta(S)) \sim N(\eta(S))$

j). $MD(\eta(S)) \sim CMD(S)$

k). $MD(S) \sim CMD(\eta(S))$

l). $MD(\eta(S)) \sim CMD(\eta(S))$. 