CHAPTER 3

TIME-DEPENDENT FLUCTUATIONS WITH VORTEX SHEDDING

3.1 INTRODUCTION

The phenomena of vortex shedding behind a circular cylinder in a flow field has been of great interest for a long time. Fromm and Harlow [9] have solved this basic time-dependent problem for a rectangular cylinder between parallel walls, which are then set in motion, by using DuFort Frankel scheme and using the condition of periodicity. They have shown the formation of the Karman vortex street. Later Hirota and Miyakoda [101] have succeeded in showing the appearance of Karman vortex street behind the cylinder at Reynolds number 100 with crude mesh size. Jain and Goel [69] in the absence of time-dependent fluctuations have studied the problem of heat transfer from the rear of a circular cylinder to the region of the separated flow in an infinite region of the flow field, in the presence of vortex shedding. A similar problem has been done for low to moderate Reynolds number bluff body heat transfer using spectral element code, NEKTON, by Karniadakis [102].

The purpose of this chapter is to study the combined effect of time-dependent fluctuations in the magnitude of the on-coming stream velocity together with the process of vortex shedding due to the instability of the wake, on the heat transfer characteristics of the unsteady Navier-Stokes and energy equations for laminar two-dimensional viscous flow past a circular cylinder.
3.2 ANALYSIS OF THE PROBLEM

Consider an unsteady flow of a viscous, incompressible fluid past a circular cylinder of radius 'a' when the flow starts impulsively at a time $t=0$ with a velocity $U(t)$, in the direction of $\theta=0$. The physical plane and the computational plane are shown in Figure 3.1 and Figure 3.2 respectively.

The origin for this study is at the front stagnation point. The governing equations in polar co-ordinates ($r,\theta$) are:

\[
\begin{align*}
1 \frac{\partial}{\partial r} \left( r \frac{\partial \phi}{\partial r} \right) + 1 \frac{\partial}{\partial \theta} \left( \frac{\partial \phi}{\partial \theta} \right) &= 0, \\
\frac{\partial \phi}{\partial t} + u \frac{\partial \phi}{\partial r} + \frac{v}{r} \frac{\partial \phi}{\partial \theta} - \frac{1}{\rho} \frac{\partial p}{\partial r} + \nu \left[ \frac{\partial^2 \phi}{\partial r^2} - \frac{2}{r} \frac{\partial \phi}{\partial r} - \frac{\partial^2 \phi}{\partial \theta^2} \right] &= 0, \\
\frac{\partial \psi}{\partial t} + u \frac{\partial \psi}{\partial r} + \frac{v}{r} \frac{\partial \psi}{\partial \theta} + \frac{1}{\rho} \frac{\partial \tau}{\partial r} + \nu \left[ \frac{\partial^2 \psi}{\partial r^2} - \frac{2}{r} \frac{\partial \psi}{\partial r} + \frac{\partial^2 \psi}{\partial \theta^2} \right] &= 0, \\
\frac{\partial \tau}{\partial t} + u \frac{\partial \tau}{\partial r} + \frac{v}{r} \frac{\partial \tau}{\partial \theta} &= \frac{K}{\rho c_p} \left( \frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r} \frac{\partial \psi}{\partial r} \right),
\end{align*}
\]

where the notations used here are the same as in chapter 2. Using the dimensionless quantities and transformation as mentioned in the previous chapter, the final form of the governing equations in ($\xi,\theta$) co-ordinates are as follows:

\[
\begin{align*}
\frac{\partial \zeta}{\partial t} + e^{-2\xi} \left( \frac{\partial (\psi,\xi)}{\partial (\theta,\xi)} \right) + e^{-\xi} \left( \cos \theta \frac{\partial \zeta}{\partial \xi} - \sin \theta \frac{\partial \zeta}{\partial \theta} \right) &= \frac{2e^{-2\xi}}{Re} \Delta^2 \zeta, \\
\zeta &= e^{-2\xi} \Delta^2 \psi, \\
\frac{\partial \theta_1}{\partial t} + e^{-2\xi} \left( \frac{\partial (\psi,\xi)}{\partial (\theta,\xi)} \right) + e^{-\xi} \left( \cos \theta_1 \frac{\partial \theta_1}{\partial \xi} - \sin \theta_1 \frac{\partial \theta_1}{\partial \theta} \right) &= \frac{2e^{-2\xi}}{Re Pr} \Delta^2 \theta_1,
\end{align*}
\]

where $\Delta^2 = \frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial \theta^2}$

The initial and boundary conditions for the problem are:

On $\xi = 0$, $\psi = \frac{\partial \psi}{\partial \xi} = -\sin \theta$, $\theta_1 = 0$ at $t=0$ (3.8)

On $\xi = 0$, $\psi = \frac{\partial \psi}{\partial \xi} = -\sin \theta$, $\theta_1 = 1.0$ for $t>0$ (3.9)
Figure 3.1 Physical plane

Figure 3.2 Computational plane
For $t \geq 0$,

On $\theta = 0$, or $\theta = 2\pi$, $\psi = \zeta = \theta = 0$ \hspace{1cm} (3.10)

On $\xi = \Sigma (= \pi)$, $\zeta = \theta = 0$, $\psi = e^{\pi} e^{\sin \theta \cos \omega t}$ \hspace{1cm} (3.11)

The pressure distribution and the local Nusselt number on the surface of the cylinder are calculated using Equation (2.25) and Equation (2.28) respectively. Other parameters like drag coefficients due to pressure and friction and mean Nusselt number are also calculated using Equations (2.26), (2.27) and (2.29). The condition for vorticity on the surface of the cylinder for $t > 0$ is given by Equation (2.24). The numerical procedure adopted is the same as described in chapter 2 with the same mesh size, $\Delta t$ and relaxation factor. The Prandtl number, $Pr$, is chosen as 0.73 and the case of high frequency, $\omega = 0.75$, with amplitude $\epsilon = 0.1$ for Reynolds number, 200, is considered for the present study.

3.3 RESULTS AND DISCUSSION

A symmetric eddy pair and a symmetric flow pattern exist even for longer times, unless a perturbation is introduced in the wake. Hence some perturbation is introduced in the values of vorticity at a few points within the wake by adding 10% of the absolute maximum vorticity occurring in the flow (at one instant of time only), for accelerating the process of vortex shedding at time $t = 6$. The process of shedding starts from $t = 20$ onwards. The shedding processes are shown in Figures 3.3 to 3.7. The process of vortex shedding takes place at a rate slower than the rate at which the outer streamlines on the periphery of the wake move into the main stream. The fluid along the streamlines passing in the neighbourhood of the cylinder gets heated and these streamlines usually form S-shaped patterns in the wake which can be seen from Figures 3.3 to 3.7. Thus the fluid in the wake region gets heated faster than in the outer flow region. A similar pattern of streamlines are obtained when there are no fluctuations. Thus the time-dependent fluctuations do not seem to affect the pattern.
of streamlines. The drag co-efficient, \( C_D \), is plotted in Figure 3.8 as a function of time. When the fluctuations are present the drag oscillates with a constant wavelength (the value is approximately same as the case of no shedding, Figure 2.13) but the peak values of each oscillation do not decrease constantly or attain a steady value as in Figure 2.11, it also undergoes an oscillation though of larger wavelength.

The variations of the local Nusselt number around the surface of the cylinder at different times are also plotted in Figure 3.9. In Figure 3.10 the behaviour of the Nusselt number distribution in the physical plane at different times are shown. Before disturbing the wake the Nusselt number near the rear stagnation point attains the peak value and is symmetric about the central line within the wake but after the wake has been disturbed the peak value in the wake region starts decreasing and continues to do so upto \( t=16 \); after which it oscillates in the wake region. This oscillation in the magnitude is due to the presence of time-dependent fluctuations in the oncoming stream velocity. Also from Figure 3.9 it can be noted that there is no symmetry of Nusselt number distribution about the centreline within the wake region, while outside the wake region it is almost symmetric. Further it is found that in the wake region the peak values are oscillating to the left and right of the centreline. These oscillations are due to the shedding of vortices. A similar behaviour has been observed when no time-dependent fluctuations are introduced. This view is confirmed in Eckert and Soehngen [99] and also in Jain and Goel [69]. The mean Nusselt number, \( \text{Nu}_m \), i.e., the total heat flowing from the cylinder when no time-dependent fluctuations are introduced but shedding is included, decreases steeply in the initial stages of the flow and at later stages, the curve shows a gradual decrease with time as shown in Figure 3.11. When the time-dependent fluctuations are introduced the effect is seen clearly that the mean Nusselt number does not vary gradually with time, but oscillates
with a constant wavelength with time. It can also be seen that the mid values of these oscillations approximately coincide with those values obtained without fluctuations. As in the case of total drag, the mean Nusselt number also has a similar behaviour, viz., the peak values of these oscillations themselves undergo another oscillation of larger wavelength, due to the combined effect of time-dependent fluctuations and shedding of vortices. When no fluctuations are present, the comparison of the mean Nusselt number values obtained by other authors is given in Table 3.1.

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<thead>
<tr>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>McAdams</td>
<td>7.2172</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kramers</td>
<td>7.6034</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Eckert and Drake [103]</td>
<td>8.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jain and Goel</td>
<td>7.63 (extrapolated)</td>
<td></td>
<td></td>
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<tr>
<td>Karniadakis</td>
<td>8.375</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Present analysis</td>
<td>7.83 at t=30</td>
<td></td>
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</table>

McAdams [104] has given the empirical formula
\[ \text{Nu}_m = 0.43 + 0.48 \sqrt{\text{Re}} \]
while Kramers [105] has given
\[ \text{Nu}_m = 0.39 + 0.51 \sqrt{\text{Re}} \]

Figure 3.12 shows the pressure distribution on the surface of the cylinder. At t=6.0 the pressure distribution is symmetric but as the process of shedding starts the symmetry in the wake region is distorted. Figures 3.13, 3.14 and 3.15 depicts the isotherm pattern when fluctuations are included at different times. It has been observed that the isotherms have similar behaviour with and without fluctuations in the magnitude of the oncoming stream velocity. The vorticity distribution at various times is shown in Figure 3.16.
The vorticity at the rear stagnation point at different times are given in Table 3.2

<table>
<thead>
<tr>
<th>$t$</th>
<th>$\zeta_0$</th>
</tr>
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<tbody>
<tr>
<td>6</td>
<td>-2.472x10^{-3}</td>
</tr>
<tr>
<td>16</td>
<td>0.7238</td>
</tr>
<tr>
<td>20</td>
<td>-1.693</td>
</tr>
<tr>
<td>24</td>
<td>-0.8963</td>
</tr>
<tr>
<td>28</td>
<td>2.112</td>
</tr>
<tr>
<td>32</td>
<td>1.465</td>
</tr>
<tr>
<td>36</td>
<td>-3.163</td>
</tr>
<tr>
<td>40</td>
<td>0.3958</td>
</tr>
<tr>
<td>44</td>
<td>1.280</td>
</tr>
<tr>
<td>48</td>
<td>-3.967</td>
</tr>
<tr>
<td>52</td>
<td>4.442</td>
</tr>
</tbody>
</table>
Figure 3.3 Streamlines pattern at t=20

-y = -2*0
-1.0
0.2
0.1
-0.5
0.2
0.1
0.0
-0.5
-1.0
-2.0

Figure 3.3 Streamlines pattern at t=20
Figure 3.4 Streamlines pattern at t=24
Figure 3.5 Streamlines pattern at $t=28$
Figure 3.7 Streamlines pattern at t=36
Figure 2.12 Total drag co-efficient with time at $Re=100$, $\varepsilon=0.1$, $\omega=0.1$
Figure 3.9 Local Nusselt number on the surface of the cylinder
Figure 3.10 Local Nusselt number around the circular cylinder
Figure 3.11 Mean Nusselt number with and without fluctuations.
Figure 3.12 Pressure distribution on the surface of the cylinder
Figure 3.13 Isotherm pattern at t=20
Figure 3.14 Isotherm pattern at $t=28$
Figure 3.15 Isotherm pattern at $t=32$
Figure 3.16 Vorticity distribution on the surface of the cylinder