CHAPTER 7

CONCLUSIONS

The conventional simplex procedure is a univariate search technique and suffers from the drawback of slow convergence. Researchers at one stage felt that one has to live with this algorithm since none was found better. Multivariate search procedures such as the gradient or conjugate gradient technique were used in seeking solution for nonlinear programming problems. The major obstacle in employing the multivariate search technique to solve linear programming problem was to select a set of linearly independent vectors to form a basis.

This problem was overcome recently by the development of a multiplex algorithm. A matrix of intercepts (referred as "g" matrix) of the promising variables is constructed to select more than one variable to enter the basis at a time. This paved the way to choose a set of linearly independent vectors and thus bring a large number of variables into the basis at a time. This algorithm led to saving in computational effort compared to the proven and well founded simplex algorithm. This algorithm was further explored and to reduce the computational effort, a modified multiplex algorithm (to induct a set of linearly independent vectors into the basis.) without constructing the intercept matrix at the beginning of each pass, has been successfully used.

Since the above two algorithms did not provide for the preservation of feasibility property, a revised multiplex algorithm to introduce the preservation of the feasibility property has also been attempted. Using this algorithm, it
has been further established that preservation of feasibility property introduces a drag on the rate of convergence. A significant conclusion arrived is that the acceptance of infeasible solutions in the course of searching for optimal solution make the convergence rapid.

With these investigations the author has been motivated in the following directions:

i. The first step is, since the feasibility is not preserved during the application of multiplex algorithm, it admits infeasible variables into the basis in the course of seeking optimality. It invokes the dual simplex algorithm to remove the infeasibility in the course of seeking optimal solution. This "one in and one out" procedure makes the convergence sluggish. Hence a dual multiplex algorithm has been developed to remove a set of infeasible variables at a time. In the course of investigation, a new range of basic solutions was hit.

The range of solution has been divided into two regions, in one of which the multiplex algorithm is employed to accelerate convergence and in the other, the dual multiplex algorithm has been used to remove infeasibility from the solution in as few passes as possible. These two algorithms together make a new approach to improve the rate of convergence.

ii. In this multiplex and dual multiplex algorithm the product of $\min\{ (z_j - c_j)x_j \}$ criterion is used for selecting the number of promising vectors in the case of multiplex algorithm and the product of $\min \{ (B^{-1}P_o)_i \}$ criterion is used for selecting the
number of variables to leave the basis in the dual multiplex algorithm. These criteria suppressed to a large extent the tendency of variables popping in and out of the basis which leads to considerable saving in computational effort.

iii. It has been observed that most of the real life linear programming problems are of high sparsity in nature. The developers of multiplex algorithm did not consider the sparsity of the constraint matrix. Checks were made for zero or non-zero entries before every multiplication or division which consumed lot of time. Hence using circularly linked list representation, only nonzero entries of the constraint coefficient matrix and other matrices are stored. In this representation the non-zero entries are stored in an efficient manner. It has more advantages over sequential list representation in terms of accessing and common operation involved on those entries. The above mentioned multiplex and dual multiplex algorithms are employed using data structure concept to solve linear programming problems. In addition to saving of memory requirement, this algorithm saves a lot of time due to this linked list representation.

iv. The multiplex and dual multiplex algorithms with data structure concept is applied to solve linear programming problems amenable to solution by the Decomposition Principle. This algorithm has worked efficiently and encouraging results have been obtained and the following observations are made:

a. When the size of the problem becomes large the percentage saving in computation time increases.
b. This algorithm will be very much suitable for large scale real life problems with high sparsity.

v. The multiplex and dual multiplex algorithm with data structure concept is used to study the effect of sensitivity analysis in the optimal solution of linear programming problems. Provisions are made to find the optimal solution during sensitivity for

1. changes in the coefficients of the objective function
2. changes in the right-hand-side vector $P^o$
3. changes in the constraint coefficient matrix
4. addition of a variable
5. addition of a constraint and
6. more than one type of changes (1-5) simultaneously.

It is found from computational experience, during post optimality analysis also the proposed algorithm works efficiently.

Computational efficiency of new algorithms can be established only when they are compared with existing algorithms. In order to compare the new algorithms, the programs are developed using the following methods.

i. Revised simplex procedure
ii. Revised simplex procedure with data structure and
iii. Multiplex and dual multiplex algorithms.

It has been observed that in the proposed algorithm convergence is rapid over the other three methods while
i. Solving Large scale Linear Programming problems 

ii. Solving problems amenable to solution by the decomposition principle and 

iii. Seeking optimal solution during sensitivity. 

All the packages are developed in Pascal, as it provides dynamic variables and also it supports linked list representation, and tested in VAX 11/780 system.

The following few problems are suggested for further investigation:

i. The multiplex and dual multiplex algorithms with data structure concepts were experimentally found to be efficient. An attempt may be made to formalize this with an abstract theoretical background. 

ii. The Multiplex and dual multiplex algorithms with data structure concept can be applied to solve integer programming problems.

iii. The "0" matrix enables to select a set of linearly independent vectors to construct a basis at the beginning of each pass. The author feels that if the selection criterion using "0" matrix is suitably modified, the solution to any linear programming problem can be obtained exactly in m number of iterations where m is the number of resource constraints.

iv. Redundant constraints, if any, in the problem will increase the computational effort. An attempt may be made to use the "0" matrix advantageously to detect such redundancies. Hence if redundant constraints are
removed at the beginning itself, it will increase the computational efficiency.

v. This multiplex and dual multiplex algorithms with data structure concept can also be extended to solve quadratic programming problems.

vi. It will be interesting to develop a graphics package to plot on a VDU showing the entering and leaving variables. Such a package may be used as a tool for computer aided instruction. This will be an excellent educational aid and may be attractive to novice programmers to solve linear programming problems.

vii. Parallel computational procedure may be thought of, wherever possible to improve the computational efficiency.