CHAPTER 6

ELASTO-PLASTIC STRAIN MEASUREMENTS USING MOIRÉ INTERFEROMETRIC TECHNIQUE

6.1 PRINCIPLES OF THE TECHNIQUE

Moiré interferometry is a modern and attractive optical technique for full field strain measurements and it enables measurements of high sensitivity. It combines the concepts and techniques of geometrical moiré and optical interferometry. Moiré interferometry is capable of measuring in-plane displacements with sensitivity as high as 2.4 fringes/micrometre displacement. High sensitivity moiré interferometry is a useful experimental technique for elastic as well as elasto-plastic strain measurements as one gets abundant displacement fringes for making accurate strain analysis.

In moiré interferometry, a grating is applied to the surface of the specimen, and it deforms together with the specimen. This specimen grating is a reflection-type phase grating. It is formed on the specimen by the replication method. The specimen grating is viewed together with a superimposed reference grating, which is usually a virtual grating (Kobayashi 1987).

Two beams of Coherent light illuminate the specimen grating obliquely at equal angles. The two light beam generate constructive and destructive interference, i.e., a virtual grating. The deformed specimen grating and the virtual reference grating interact to form a moiré pattern (if having different frequencies), which is viewed and photographed by a Camera. In moiré interferometry, the initial frequency of the
specimen grating is generally fixed as half that of the virtual reference grating. The sensitivity of the measurement is controlled by the reference grating. The sensitivity of the displacement measurement is determined by the number of fringes generated per unit displacement and displacement sensitivity is equal to the frequency \( f \) of the reference grating. The number of fringes that cross an X-axis is \( N_x = f \) and for the case of \( f = 2400 \) lines/mm, the sensitivity is 2.4 fringes/µm displacement.

6.2 FEATURES OF MOIRÉ INTERFEROMETRY

Moiré interferometry is characterized by a number of excellent qualities, including the following:

I. real-time technique: the displacement fields can be viewed as loads are applied

II. high sensitivity to in-plane displacements: typically 2.4 fringes/µm displacement, virtually no sensitivity to out-of-plane displacement.

III. high spatial resolution: measurements can be made even in very small zones locally.

IV. high signal-to-noise ratio: the fringe patterns have high contrast, excellent visibility and practically free from noise.

V. large dynamic range: the method is compatible with a large range of displacements, strains and strain gradients.

6.3 EXPERIMENTAL INVESTIGATION

An experimental investigation was carried out to determine elasto-plastic strain and stress distributions on perforated aluminium plates using moiré interferometric technique. Two numbers of aluminium plates were investigated, having identical geometry and material characteristics. In this connection, two aluminium plates of size
300 mm length, 37.5 mm width and 3.175 mm thick were prepared (Specimen identification numbers - 1&2 and Material Group: II). A central circular hole of 7.5 mm diameter was then made on the two plates. The elastic modulus and Poisson's ratio for the aluminium specimen was obtained as 67.69 GPa and 0.32 respectively. A phase grating of 1200 lines/mm was replicated on the specimens near the hole (31x37.5 mm). The geometry of the plate with phase gratings is shown in Fig. 6.1

6.3.1 Preparation of Specimen Grating

Specimen gratings were prepared by a replication process from the epoxy submasters. To function as a mould for replication of the specimen grating, the epoxy submaster was coated with a reflective metal film. The film serves two purposes. It acts as a parting agent for separation of the grating and also provides the high reflectivity needed for good diffraction efficiency in the moiré interferometry system.

The metallic film used was aluminium and it was applied by a vacuum deposition process. The process is similar to that used for applying metallic mirror coatings, but in moiré interferometric experiments, weak adhesion is required instead of the strong adhesion desired for mirror coatings.

As it is required to measure both $u$ and $v$ components of displacements, a cross-line diffraction grating was used. The cross grating had peaks and troughs in two orthogonal directions. Before the grating was replicated on the specimen, it is highly essential to align the grating lines of the mould with respect to the specimen. For this purpose, a convenient alignment fixture was specially designed and fabricated.

The alignment fixture consisted of a flat base, a bar with a narrow groove parallel to the base and an adjustable fixture. The mould was attached temporarily to the fixture with double-sided adhesive tape. An unexpanded low power laser beam was passed through a hole in the bar and it interrupted the grating mould. The adjustable
FIG. 6.1 DETAILS OF SPECIMEN GEOMETRY WITH GRATINGS FOR MOIRÉ INTERFEROMETRIC EXPERIMENTS

NOTE:
ALL DIMENSIONS IN mm

7.5 Ø HOLE
PHASE GRATINGS
1200 l/mm

37.5
fixture screws were operated so that the +1 and -1 diffraction orders strike the parallel groove. This ensures the grating lines accurately parallel to the base. Next, a thin parallel bar (1 mm thick and 13 mm wide) was cemented to the mould, while the bar was resting on the base at the alignment position, as a reference surface.

The procedure adopted in producing the specimen grating by a replication process is briefly explained. A pool of liquid adhesive (room temperature cure special epoxy adhesive) was poured on the epoxy submaster mould and the adhesive was squeezed into a thin film by uniformly pressing the test specimen against the mould. As the excess liquid adhesive was squeezed out, it was cleared away carefully from all the specimen boundaries with cotton swabs. Cleaning was continued during the entire time that the liquid was flowing out. After polymerization (after 24 hours), the mould was separated from the test specimen. Only a small force was required to be applied to separate them as the weakest interface occurred between the mould and the metal film. By this procedure, high frequency phase type diffraction grating was replicated on the specimen.

6.3.2 Requirements of Specimen gratings

The features that enhance the performance of specimen gratings for moiré interferometry are given below:

a) The grating must adhere to the specimen and deform together with the specimen, without delamination or slippage relative to the specimen surface. It should not reinforce the specimen, i.e., the specimen displacements should not be significantly altered.

b) The grating should be thin to minimize shear lag in regions of large strain gradients.

c) The grating surface should be specular or mirror like. This is the single most important reason for the excellent clarity of moiré interferometry fringes.
d) The grating should have a symmetrical furrow profile

e) The grating should have high diffraction efficiency in the diffraction orders that are utilised. This helps to use the lower power lasers and/or shorter exposure times.

6.3.3 Optical System

The optical system used for the experiments comprised of the following three sub-systems:

a) The illumination system consisting of a laser, an optical fibre system, a plane mirror and a parabolic mirror.

b) The moiré interferometric arrangement, which divides the input beam into four separate beams and direct them onto the specimen grating.

c) The camera system with a collecting lens to record the fringes.

Both u & v displacement fields are required to be recorded to determine the full state of strain at the surface of the test specimen. The two-beam optical arrangement is shown in Fig. 6.2. The incident collimated beam was formed by a parabolic mirror and an optical fiber, with the fiber end located at the focal plane of the mirror. Light input from a He-Ne laser operating at 632.8 nm wavelength and 50 mw power. The initial frequency of the specimen grating was \( f/2 = 1200 \) lines/mm. Half of the collimated laser light strikes the specimen grating directly, while the other half impinges indirectly in a symmetrical direction after reflection from a plane mirror. The two beams intersect at the specimen plane with included angle \( 2\alpha \) and form a virtual reference grating of frequency \( f = 2400 \) lines/mm. The two beams of light are diffracted by the specimen grating (+1 and -1 diffraction orders) to emerge essentially perpendicular to its surface. They interfere with each other to produce the fringe pattern of moiré interferometry. The two beams were collected by a lens and projected...
FIG. 6.2 TWO BEAM OPTICAL ARRANGEMENT
onto a camera screen, where the interference patterns were viewed and photographed. The two-beam optical system gives the fringe patterns corresponding to one of the in-plane displacement components only. Hence a four-beam optical arrangement was used to record both the in-plane displacement components. The four-beam optical system was formed by duplicating the two-beam optical arrangement in the horizontal plane and in the vertical plane. Two adjustable mirrors (A & B) were added to the basic system. The four-beam optical arrangement is shown in Fig. 6.3. This optical arrangement was built to record u & v displacement fringe patterns separately by blocking alternate pairs of incoming beams. Fig. 6.4 shows the photograph of test set up for moire interferometric experiments.

The angles of diffraction of the grating are determined by the equation

\[ \sin \beta_m = \sin \alpha + m \lambda f \]  

(6.1)

where \( m \) is the diffraction order, \( f \) is the grating frequency, \( \alpha \) is the angle of incidence and \( \beta_m \) is the angle of the \( m^{th} \) diffraction order. The angle between neighbouring diffraction orders is small for a coarse grating and it is large for a fine grating. A special case of interest is where the Zeroth diffraction order and its neighbour, the -1 order, are symmetrical with respect to the grating normal. If the angle of incidence is \( \alpha \), we have \( \beta_0 = \alpha \) and \( \beta_{-1} = -\alpha \). Equation (6.1) reduces to

\[ \sin \alpha = \frac{\lambda}{2f} \]  

(6.2)

This defines the angle of incidence \( \alpha \) to achieve the special condition of symmetry. When \( \alpha \) is large enough, only two diffraction orders can exist. The other orders cannot exist. The lower limit of \( \alpha \) is 19.5° and upper limit is approached as \( \alpha \) approaches 90°. The theoretical limit is \( f = 2/\lambda \). We can get a grating frequency of 2400 lines/mm when \( \alpha = 49.4^\circ \) with \( \lambda = 632.8 \) nm.
FIG. 6.3  FOUR BEAM OPTICAL ARRANGEMENT
FIG. 6.4 TEST SET-UP FOR MOIRÉ INTERFEROMETRIC EXPERIMENTS
6.3.4 Loading and Recording of Fringes

In order to apply an uniaxial tension to the test specimens, a loading fixture was designed and fabricated. Incremental tensile loads were applied in steps. In order to measure the loads applied to the test specimens, electrical resistance strain gages were fixed along the loading direction on both surfaces of the specimens and strains were recorded using a portable strain gage data logger. Fig. 6.4 shows the test set-up for moiré interferometric experiments. For Specimen - 1, 2.26 kN, 5.0 kN, 8.0 kN, 10.20 kN, 14.72 kN and 19.82 kN were the magnitudes of tensile load applied in six stages. For Specimen - 2, 4.24 kN, 8.25 kN, 12.27 kN, 16.38 kN and 21.30 kN were the magnitudes of tensile load applied in five stages. In all the load steps for both the tested specimens, the fringe patterns for the in-plane displacement fields were photographically recorded.

The beam of collimated laser light can be assumed to consist of four parts \( A', B', C' \) & \( D' \). The light from portions of \( C' \) & \( D' \) of the collimated laser beam was blocked by an opaque screen and thereby allowing the portion of \( A' \) & \( B' \) only to start with. Light from portion \( A' \) after striking the mirror \( A \), was reflected upward to lie in a plane parallel to the YZ plane. They reached the specimen at an angle \(+\alpha\) in the vertical plane. At the same time, light from portion \( B' \) of the incident beam after striking the mirror \( B \), was reflected downward at an angle \(-\alpha\). These two beams, from mirrors \( A \) and \( B \), combined to form a virtual reference grating with its grating lines perpendicular to the \( y \) axis. The \( v \) field (\( N_y \) fringes) was formed by interaction of the \( y \) family of grating lines with the virtual reference grating lines formed by \( A' \) & \( B' \) light beams. Next, portions \( A' \) & \( B' \) of the collimated laser light beam were blocked and the portion of \( C' \) & \( D' \) were allowed to strike the specimen. After one part was reflected from mirror \( C \), the two beams reached the specimens with angles \(+\alpha\) and \(-\alpha\) in the XZ plane. They combined to form a virtual reference grating with its lines perpendicular to the \( X \)-axis and produced \( u \) displacement (\( N_x \) fringes) field. For each load steps, the two
moire fringe patterns were photographed separately by blocking alternate pairs of incoming u & v beams.

6.4 STRAIN ANALYSIS

The location of the zero-order fringe in a moiré interferometric pattern is arbitrary since rigid body motions are inconsequential for strain analysis (Daniel Post et al 1994). Any fringe-black or white can be assigned as the zero-order fringe. The rules of topography of continuous surfaces govern the order of fringes. Adjacent fringes differ by plus or minus one fringe order, except in zones of local maxima or minima, where they can have equal fringe orders. Local maxima or minima are usually distinguished by closed loops or intersecting fringes. Fringes of unequal orders cannot intersect. The fringe order at any point is unique, independent of the path of the fringe count used to reach the point. The relationship between the fringe order and displacement is given by

\[ u = \frac{1}{f} N_x, \quad v = \frac{1}{f} N_y \]  

(6.3)

Once the zero-order fringe is selected, we have to then determine whether the fringe order of a neighbouring fringe is greater or less. This means that we have to ascertain the sign of the fringe gradient and the following sign convention is adopted. As the \( N_x \) pattern is scanned in the x direction, the sign \( \partial N_x / \partial x \) of \( \partial N_x / \partial x \) is positive if the fringe order is increasing. Similarly \( \partial N_y / \partial y \) is positive if \( N_y \) increases with \( y \) (i.e. along a path of increasing y values). The gradient \( \partial N_y / \partial x \) is positive when \( N_y \) increases in the positive x direction. The gradient \( \partial N_x / \partial y \) is positive when \( N_x \) increases with positive \( y \). This convention gives tensile strains correspond to positive fringe gradients and compressive strains to negative gradients. Hence, the problem of fringe ordering reduces to a determination of the sign of the fringe gradient.
For the present investigation, the sign can be easily inferred from the geometry and loading conditions. Circumstances could arise in some general cases where one need to carry out some simple experimental technique to determine the sign of fringe gradient. If, during the experiment the specimen is moved gently in the \( \pm x \) direction the fringe order \( N_x \) at every point increases. This means that the fringes all move toward the direction of lower order fringes. If the fringe movement is in the \( \pm x \) direction in any region, the gradient \( \frac{\partial N_x}{\partial x} \) is negative in that region. If the fringe moves in the \( \pm x \) direction, the gradient is positive. The technique and inference is the same for the \( y \) direction also.

The high sensitivity of moiré interferometry makes it amenable to analysis in the small strain domain. The inverse of displacement sensitivity defines the contour interval \( (1/f) \). The contour interval is the displacement per fringe order. When \( f = 2400 \) lines/mm, the displacement per fringe order is 0.417 micrometre. Because of this dense fringe density makes it possible to get abundance of displacement data and hence the strains were extracted from the displacement fields using the standard strain-displacement relationships. The moiré fringes were closely spaced and it is practical to approximate the derivative by its finite increment. If the incremental displacement \( \Delta u \) between two points \( \Delta x \) apart is shown in the fringe pattern by a small change in fringe order \( \Delta N_x \), then

\[
\Delta u = \frac{1}{f} \Delta N_x \quad \text{and} \quad \varepsilon_x = \frac{1}{f} \frac{\Delta N_x}{\Delta x}
\]

Similarly the other normal strain and shear strain relationships are shown below:

\[
\varepsilon_y = \frac{1}{f} \frac{\Delta N_y}{\Delta y}
\]

\[
\gamma_{xy} = \frac{1}{f^2} \left( \frac{\Delta N_x}{\Delta x} - \frac{\Delta N_y}{\Delta y} \right)
\]
Thus, the fringe gradient, measured in a prescribed direction, is directly proportional to the strain in that same direction. In the finite increment form given above, the increments are good approximations of the differentials when $\Delta x$ and $\Delta y$ are small compared to the dimensions of the specimen and are also small compared to the widths of zones of high strain gradients. An enlarged photographic print was used for the strain analysis. Hence the distances measured on the fringe patterns were divided by the image magnification factor to obtain the corresponding distances on the specimen.

The displacement values along the critical section (Line A-A) for all the investigated cases were converted into strains. Being the principal axis, shear strain is zero along this line. Strain analysis from the fringe measurements taken on both sides of the hole along the critical section were carried out and the strain distribution along Line A-A were obtained by averaging the strains at locations symmetrically placed with respect to the centre of the hole.

6.5 MEASUREMENTS ALONG LINE B-B

Apart from the fringe measurements taken on the critical section (Line A-A) for all the investigated cases, fringe measurements were also taken along Line B-B for three load steps of Specimen-2. Line B-B is parallel to Line A-A and is situated at 1.875 mm above Line A-A. 8.25 kN, 12.27 kN and 16.38 kN are the three load steps of Specimen-2 wherein moiré interferometric fringe analysis were carried out for this section. The displacement values along the Line B-B for the three load steps of Specimen-2 were then converted into strains. Strain analysis from the fringe measurements taken on both sides of the hole were carried out and the strain distribution along Line B-B were obtained by averaging the strains at locations symmetrically placed with respect to the centre of the hole. The two normal strains ($\varepsilon_x$ and $\varepsilon_y$) and the shear strain ($\gamma_{xy}$) along Line B-B were obtained.
6.6 RESIDUAL PLASTIC STRAIN MEASUREMENTS

For Specimen-1, after loading up to 14.72 kN and 19.82 kN, the loads were removed and residual plastic deformation developed under the 'No-load' condition were photographically recorded. In a similar fashion, for Specimen-2, after loading up to 2130 kN, the load was removed and the residual plastic deformation was recorded. By using the strain-displacement relationships, the residual permanent strain distributions along the critical sections were obtained. Strain analysis from the fringe measurements taken on both sides of the hole were carried out and the average permanent strain distribution along the Line A-A were obtained. Results and discussions on moire interferometric experiments are given in Chapter 10.