Interest in graph labeling began in mid-1960's with a conjecture of Ringel (1964) and a paper by Rosa (1967). Over the past four decades, the area of graph labeling developed very fast and over 500 papers have been published on graph labeling, refer Gallian(2003). The current study of this thesis is concerned with graceful, harmonious, cordial and elegant labelings of a few new families of cycle related and other graphs.

Graceful labeling was introduced by Rosa (1967) as a tool to decompose complete graph $K_{2n+1}$ into copies of a given tree with $n$ edges.

A graph $G(V,E)$ is called graceful if there exists an injection $f : V(G) \rightarrow \{0,1,2,\cdots,|E|\}$ such that the induced function $f' : E(G) \rightarrow \{1,2,\cdots,|E|\}$ defined by $f'(uv) = |f(u)-f(v)|$, $uv \in E(G)$ is a bijection.

Harmonious labeling was introduced by Graham and Sloane(1980) in connection with their study on additive bases problem stemming from error - correcting codes.

A graph $G(V,E)$ is called harmonious if there exists an injection $f$ from the vertices of $G$ to the group of integers modulo $|E|$ such that the induced function $f' : E(G) \rightarrow \{0,1,2,\cdots,|E|-1\}$ defined by $f'(uv) = f(u) + f(v) \pmod{|E|}$, $uv \in E(G)$ is a bijection.

The following elegant labeling was introduced by Chang et al (1981) as a variation of harmonious labeling.

Let $G$ be a graph with vertex set $V$ and edge set $E$. An injection $f$ from $V$ to the set $\{0,1,2,\cdots,|E|\}$ is called elegant if the induced function $f' : E(G) \rightarrow \{1,2,\cdots,|E|\}$ defined by $f'(uv) = f(u) + f(v) \pmod{|E|+1}$, $uv \in E(G)$ is a bijection.
Cahit (1987) defined the cordial labeling as a variation of both harmonious and graceful labeling.

Let \( f \) be a function from the vertices of \( G \) to \( \{0,1\} \) and for each edge \( uv \) assign the label \( |f(u) - f(v)| \). Call \( f \) a cordial labeling of \( G \) if the number of vertices labeled 0 and the number of vertices labeled 1 differ by at most 1 and the number of edges labeled 0 and the number of edges labeled 1 differ by at most 1.

Though cordial labeling is much simpler compared to graceful or harmonious labeling but deciding a graph is cordial is a NP-complete problem.

The Thesis comprises of six chapters.

In Chapter 1, a background of different labelings that are studied in this thesis is provided and the results contained in the remaining chapters are outlined.

Study on gracefulness of cycle related graphs was initiated from Rosa's result (1967) that cycle \( C_n \) is graceful iff \( n \equiv 0 \) or 3 (mod 4). A chord of a cycle is an edge joining two non-adjacent vertices of the cycle. A natural extension of the structure of a cycle with a chord is that of a cycle with a \( P_k \)-chord. A cycle with a \( P_k \)-chord \( (k \geq 2) \) is a graph obtained by joining a pair of non-adjacent vertices of a cycle of order \( n \) \( (n > 4) \) by a path of order \( k \). Koh and Yap (1985) have shown that cycles with \( P_3 \)-chords are graceful and conjectured that all cycles with \( P_k \)-chords are graceful. As an extension of the structure of a cycle with a chord or a cycle with a \( P_k \)-chord, we define three new families of graphs called cycle with parallel chords, cycle with parallel \( P_k \)-chords and cycle with parallel \( P_k \)-chords of increasing length.

A graph \( G \) is called a cycle with parallel chords if \( G \) is obtained from the cycle \( C_n : v_0 v_1 v_2 \cdots v_{n-1} v_0 \) \( (n \geq 6) \) by adding the chords \( v_1 v_{n-1}, v_2 v_{n-2}, \cdots, v_\alpha v_\beta \), where \( \alpha = \lfloor \frac{n}{2} \rfloor - 1 \) and \( \beta = \lfloor \frac{n}{2} \rfloor + 2 \), if \( n \) is odd or \( \beta = \lfloor \frac{n}{2} \rfloor + 1 \), if \( n \) is even.
A graph $G$ is called a *cycle with parallel $P_k$-chords* if $G$ is obtained from the cycle $C_n : u_0u_1 \cdots u_{n-1}u_0 (n \geq 6)$ by adding disjoint paths $P_k$'s ($k \geq 3$ and $k$ fixed) between the pair of vertices $(u_1, u_{n-1}), (u_2, u_{n-2}), \cdots, (u_{\alpha}, u_{\beta})$ of $C_n$, where $\alpha = \left[\frac{n}{2}\right] - 1$ and $\beta = \left[\frac{n}{2}\right] + 2$, if $n$ is odd or $\beta = \left[\frac{n}{2}\right] + 1$, if $n$ is even.

A cycle with parallel $P_k$-chords is said to be a *cycle with parallel $P_k$-chords of increasing length*, if the pair of vertices $(u_k, u_{n-k})$ is joined by respectively a $P_{k+1}$-chord, for $1 \leq k \leq \left[\frac{n}{2}\right] - 1$.

In Chapter 2 it is shown that every $n$-cycle ($n \geq 6$) with parallel chords and every $n$-cycle ($n \geq 6$) with parallel $P_k$-chords are graceful for $k = 3$ and for $k = 2r$, where $2 \leq r \leq 5$. Further, it is shown that for $n \equiv 2 \pmod{4}$, an $n$-cycle with parallel $P_k$-chords of increasing length is graceful, for $1 \leq k \leq \left[\frac{n}{2}\right] - 1$.

In Chapter 3 it is shown that union of cycle with parallel chords and complete bipartite graph is graceful. Also, it is shown that union of cycle with parallel chords and certain paths is graceful.

In Chapter 4 it is shown that a cycle $C_n$ with parallel chords is cordial and a cycle $C_n$ with parallel $P_k$-chords is cordial, for all $n \geq 7$ except for $n = 4r + 2$, $r \geq 1$, and for any odd positive integer $k \geq 3$. Further, it is shown that every even-multiple subdivision of any graph is cordial (a graph $H$ is called *even-multiple subdivision of a graph $G$* if $H$ is obtained from $G$ by replacing every edge $e$ of $G$ by a set of pairs of paths $(P, Q)$'s where the lengths of both the paths $P$ and $Q$ are either of the form $0 \equiv (\pmod{4})$ or of the form $2 \equiv (\pmod{4})$ to even, by merging the origin and terminus of all the pairs of paths $(P, Q)$'s with the ends of the edge $e$).
In Chapter 5 it is proved that any given set of graphs can be packed into a graceful, harmonious and elegant graph (a sequence of graphs $G_1, G_2, \cdots, G_t$ is said to be packed into a graph $G$ if $G$ has edge disjoint subgraphs $F_1, F_2, \cdots, F_t$ such that $F_j \cong G_j$, for $j = 1, 2, \cdots, t$). Also it is proved that every graph is a subgraph of a cordial graph.

In Chapter 6 it is shown that the graph $P_n^k$ (the $k$th power of path $P_n$), is the graph obtained from $P_n$ by adding edges between all vertices $u$ and $v$ of $P_n$ with $d(u, v) = k$ and the graphs $P_m^2 + \overline{K}_n$, $S_m + S_n$ and $S_m + \overline{K}_n$ are elegant, for all $m, n \geq 1$. Further, it is proved that every even cycle $C_{2n} : a_0a_1a_2 \cdots a_{2n-2}a_0$ with $2n - 3$ chords incident at a common vertex is elegant, for $n \geq 2$ and also it is shown that the graph $C_3 \times P_m$ is elegant, for all $m \geq 1$. 