CHAPTER - I
INTRODUCTION

Operations Research (O.R.) is a science which deals with problem, formulation, solution and finally appropriate decision making (D.M). It is most often used to analyze complex real life problems typically with the goal(s) of improving or optimizing performance. Today, it has become a professional discipline that deals with the application of scientific methods for D.M, and especially to the allocation of scarce resources.

The main aim of O.R. is to seek the best course of action of a decision-making problem under certain constraints. In recent years O.R. has successfully entered in many different areas of research in Defense, Industry, Agriculture, Economics, Health systems, Government, Service organizations and even in behavioral and Social Sciences.

O.R. is mainly concerned with the techniques of applying scientific knowledge, besides the development of science. It provides an understanding which gives the expert/manager new insights and capabilities to determine better solutions in his decision-making problems, with great speed, competence and confidence. O.R. improves the effectiveness and the efficiency of an institution, hence some of the
benefits offered by O.R. include:

- Decrease Cost or Investment
- Increase Revenue or Return on Investment
- Increase Market Share
- Manage and Reduce Risk
- Improve Quality
- Increase Throughput while Decreasing Delays
- Achieve Improved Utilization form Limited Resources
- Demonstrate Feasibility and Workability.

1.1 CONCEPT OF SEQUENCING AND SCHEDULING

Sequencing and scheduling is a form of decision-making that plays a crucial role in manufacturing and service industries. In the current competitive environment effective sequencing and scheduling has become a necessity for survival in the market place. Companies have to meet shipping dates; they have to schedule activities in such a way as to use the resources available in an efficient manner. Otherwise, it will impinge upon reputation of a business. As a result, good scheduling algorithms are needed.

Sequencing problems when more than one job is assigned to a machine or activity, the operator needs to know the order in which to process the jobs. The process of prioritizing jobs is called sequencing.
Thus job sequencing is the arrangement of the tasks required to be carried out sequentially. Hence the two techniques called priority rules and Jonson’s rule are adopted here. Priority rules provide guidelines for the sequence in which jobs should work, Yavuz et al. [90]. Priority rules try to minimize completion time, number of jobs in the system, and job lateness, while maximizing facility utilization.

Generally, scheduling problem can be approached in two steps; in the first step sequence is planned or decides how to choose the next task. In the second step, planning of start time and perhaps the completion time of each task is performed. Scheduling is the process of generating schedules. Schedule is a physical document and generally tells the happening of things and shows a plan for the timing of certain activities.

Basically, scheduling is the allocation of shared ‘resources’ over time to completing activities, Baker [11]. The terminology of scheduling theory usually takes place in the processing, manufacturing industries, production, transportation, distribution, and in information processing and communication. Scheduling problem is solved by using the mathematical techniques to allocate limited ‘resources’ to the processing of ‘tasks’. This allocation of resources is important since a proper allocation enables the companies to optimise their objectives and achieve their goals, Pinedo M [64]. The familiar scheduling problems are the bus schedules, the university timetable and the construction work.
Scheduling deals with the allocation of scarce resources to task over time. It is a decision-making process with the goal of optimization one or more objective functions. The resources and tasks in the organization can take many forms. The resources may be ‘Machines’ in a workshop, runways at an airport, crew at a construction site and so on. The tasks may be ‘Operations’ in a production process, take-offs and landings at an airport, stages in a construction project. Thus, resources are usually called machines and tasks called jobs. Sometimes, jobs may be consisting of several elementary tasks called ‘operations’. The environment of the scheduling problem is called the ‘job shop or simply the shop’.

The main objective of scheduling problem is to find the sequence for each operation on each machine that optimizes the objective function. The most common objective function that has been used in scheduling job-shop problem is “Minimization of make-span (C_{max}) i.e. Complete all jobs, to meet due dates, to minimize the lateness of the job”.

1.2 DEFINITIONS OF SCHEDULING

Different authors have given different definitions of ‘Scheduling’. Some of the important definitions of scheduling are as follows:

- “Scheduling is the allocation of resources over time to perform a collection of tasks….Scheduling is a decision-making function: it is the
process of determining a schedule…. Scheduling is a body of theory: it is a collection of principles, models, techniques, and logical conclusions that provide insight into the scheduling function”. - (Baker) [11]

- “Scheduling is the process of organizing, choosing, and timing resource usage to carry out all the activities necessary to produce the desired outputs at the desired times, while satisfying a large number of time and relationship constraints among the activities and the resources”.
  – (Morton & Pentico)

Therefore, from above two definitions, scheduling can be defined as a decision-making process that is concerned with the allocation of limited machines (resources) over time to perform a collection of jobs (tasks) in which one or several objectives have to be optimized.

- “A scheduling problem can be considered to be an exercise in finding an appropriate timetable for the processing of jobs, by machines, such that some performance measure achieves its optimal value”.
  - (V. Lowndes & J. M. Carter)

- “Scheduling has been defined as ‘the art of assigning resources to tasks in order to insure the termination of these tasks in reasonable amount of time”.
  - (Voss C, Tsikriktsis N)

- “Scheduling consists of planning and arranging jobs in an orderly
sequence of operations in order to meet customer’s requirements”.
– (D. R. Sule)

With these definitions, it can be seen that there are two aspects in scheduling to be considered concurrently, the satisfaction of constraints (e.g. availability of resources) and the optimization of objectives (e.g. processing times, Flow-times etc.).

The general definition of sequencing problem can be stated as follows:

There are \( m \) machines \( \{M_1, M_2, \ldots, M_m\} \) available and \( n \) jobs \( \{J_1, J_2, \ldots, J_n\} \) to be processed. A subset of these machines is required to complete the processing of each job. The flow pattern (process plan) for some or all jobs may or may not be fixed. Each job should be processed through machines in a particular order that satisfies the job’s technological constraints. The processing of job \( i \) on machine \( j \) is called an operation denoted by \( O_{ij} \). Associated with each operation is a processing time denoted by \( P_{ij} \) and a setup time denoted by \( S_{ij} \). Also associated with each job is weight, \( w_i \), a ready (release or arrival) time \( r_i \), and a due date \( d_i \). Finally, each job has an allowance time to be in the shop \( a_i \).

Thus, the general scheduling problem is to generate a sequence
that satisfies the following conditions:

i. All jobs are processed

ii. All technological constraints are meet for all jobs (feasibility condition)

iii. All criteria that were selected are optimized.

1.3 SCHEDULING TERMINOLOGIES

• Job (n)

A job can be made up of any number of tasks. It is easy to think of a job as making a product and each task as an activity that contributes to making that product, such as a painting task, assembling task and so on.

• Number of Machines (m)

A machine is available to execute jobs and tasks. Different machine environments exist, such as single machine and parallel machines.

It means the activity stages/service facilities/operations through which a job must be pass before it is completed.

For example, a book to be printed has to be processed through composing, printing, binding, finishing etc. In this case book constitutes the job and the various activities like composing, printing etc. constitute the number of machines.
• **Processing Time (P_j)**

   It is defined as the length of time required for processing of a job or task. In other words, it is the time required to process job (j) on any machine. The processing time (P_j), will normally include both actual processing time and set-up time.

• **Ready Time (r_j)**

   It is the time at which the processing of job (j) is available for processing.

• **Due Date (d_j)**

   The latest time by which a job should be completed so that is not late. The time, at which the processing of job (j) is due to be completed, Due Date (D.D) is also known as ‘promised delivery dates’ based upon an individual agreement with customer.

   Commonly, it is generated as results of scheduling decisions represent output from the scheduling function. In deterministic case, scheduling decisions will determine the most fundamental piece of data to be used in evaluating schedules.

• **Completion Time (C_j)**

   It is defined as “the time at which the processing of job (j) is finished”. In other words, the time at which the job (j) is completed in a sequence. Performance measure like flow time, lateness and tardiness for evaluating schedules are usually functions of job completion time.
• **Flow Time** \((F_j)\)

It is the amount of time job \((j)\) spends in the system. Flow time is a measure of actual time spent by a job in the system. This in turn gives some idea about in-process inventory of the shop floor.

• **Makespan** \((C_{\text{max}})\)

The Makespan of a schedule in the flow shop scheduling problem is the total time taken to complete processing of all the jobs.

• **Lateness** \((L_j)\)

It is the amount of time by which the completion time of job \((j)\) differs from the due date \((C_j - d_j)\). Lateness measure the conformity of the schedule to a given due date. Lateness quantity takes on negative values whenever a job is completed early. Negative lateness represents better than requested, while positive lateness represents poorer service than requested.

• **Tardiness** \((T_j)\)

The amount of time by which a job \((j)\) is late. For operations that are early or on time, the tardiness is considered to be zero.

Thus, \(T_j = \begin{cases} 0 & , \quad L_j \end{cases}\)

### 1.4 CHARACTERISTICS OF A GOOD SCHEDULE

In scheduling problem, finding a good objective to minimize or maximize is difficult and it may be a crucial step for several reasons.
First, important objectives like “Customer satisfaction” for “quality or Promptness” are difficult to quantify and do not appear among the accounting numbers, these enhance the quality of service. A shop usually deals with three types of objectives, such as:

- a. Maximize shop throughput over time period;
- b. Satisfy customer desire for quality and promptness; and

Scheduling can be difficult from both technical and implementation point of view. There are three characteristics which causes scheduling to be extremely difficult to automate.

- a. Its combinatorial complexity;
- b. The conflicting nature of the requirements of a “good” schedule;
- c. The uncertainty of execution caused by the stochastic and dynamic nature of most scheduling environments.

The most prominent feature of scheduling problem is its combinatorial explosiveness. Most combinatorial optimization problems, expect a few very simplistic ones are the NP class, and are therefore difficult to solve. A second difficulty with scheduling involves assessing the value of a scheduling decision within a specific domain. Often it is unclear how one decision will influence the global satisfaction of organizational goals such as machine utilization, due date satisfaction or work in progress levels. In optimization, both
‘quantitative’ as well as ‘qualitative’ objectives such as minimization of costs along with customer satisfaction with quality or promptness this makes the problem more complex. Thirdly, unforeseen events like machine breakdown, power failure, and personal absences in a machine shop may invalidate a schedule. An effective schedule should be able to adapt to these environmental and executional uncertainties. Hence it is very complex problem far from being completely solved to optimality in most of the cases.

1.5 SCHEDULING ENVIRONMENT

Scheduling involves decisions of how to allocate and utilize limited resources to accomplish a set of tasks. The role of scheduling in manufacturing is depicted within the flow of information and products. Scheduling activities interface with many aspects of an organization and have a direct influence on the company’s performance. Job requirements are generated as a result of sales activities as orders flow into an organization. A standard lead time is often quoted when accepting an order and is used when estimating delivery dates. In practice, delivery dates are in fact affected by the processing times of the jobs, available capacity, work already scheduled and the efficiency of the operation.

Operating efficiencies are directly impacted by the performance of the scheduling method in reducing the amount of idle time and increasing the throughput of work per unit time. Poor scheduling
decisions can strain material requirements planning, shop floor control, capacity utilization, and working capital resources. Ultimately poor performance of a scheduling methodology can lead to decision that adversely affect a company’s sales and profitability.

In a manufacturing environment, several operations may be required to transform components and fabricate items into finished products. These transformations can occur as a result of processing operations on one or more resources. To accomplish these transformations, manufacturing job orders are often released to perform certain tasks on these resources in a specified sequence. Depending upon the type of operating environment, the sequences for each job may perhaps be determined by the scheduler.

In a job-shop production scheduling, there are jobs with various levels of importance and due dates that are to be processed by many types of machines (grinding, drilling, etc.). Each job requires a sequence of operations for completion, and each operation requires a specific type(s) of machine for a specified duration of time. An operation may begin only when all its preceding operations have been completed. See Figure 1.1, for sample job with operations shown according to its process plan. The process plans are usually different for different jobs. The capacity of each type of machine is finite and may be time varying.
Production function encompasses the activities of procurement, allocation and utilization of resources. The main “Objective of production function is to produce the goods and services demanded by the customers in the most efficient and economical way. Therefore efficient management of the production function is of most importance in order to achieve this objective”.

Fig. 1.1 - Production Scheduling Interface Cycle

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13
1.6 PURPOSE OF SCHEDULING

Production control is the task of ‘Predicting’, ‘Planning’ and ‘Scheduling’ of the work, taking into account ‘Manpower’, ‘Materials Availability’ and ‘Capacity restrictions’ so as to achieve proper ‘quality and quantity at the time it is needed and then following up the schedule to see that the plan is carried out’.

There are number of reasons why a manufacturing organization wants to develop a production schedule for some time period into the future. Proper scheduling helps in achieving high efficiency of the system by the optimal utilization of the machines and maintaining low work- in-process inventory. Shop floor personal use the schedule to guide their action, positioning work in a way that will smooth the execution of the schedule. Another potential use of scheduling is to provide a yardstick by which the performance of the shop floor personal is measured.

1.7 OBJECTIVES OF SCHEDULING

Some of the most common objectives of scheduling are:

• To meet customer’s due dates (Promised delivery dates)
• To maximize customer satisfaction
• To minimize average flow time through the system
• To minimize total penalty cost
• To provide for high machine utilization
• To minimize production cost.

These objectives are aimed primarily to provide a ‘High Level of Customer Service’ and ‘High Level of Plant Efficiency’. The objectives mentioned above are often conflicting. One can do a better job of meeting due dates if more capacity is provided and if the work center capacity is less utilized. Similarly, more capacity will typically reduce flow time, but at reduced capacity utilization.

1.8 CLASSIFICATION OF SCHEDULING PROBLEM

Today’s technology is developing with incredible changes in global marketplace where all companies are under increasing pressure to reduce lead times and to maintain a high on-time delivery performance. Proper scheduling (determine the sequence in which operations are to be performed) of jobs is indispensable for the successful operation of a shop. Therefore “Effective scheduling of jobs is the key of achieving the goal(s)”. In production manufacturing scheduling system there are many different types of problems, Ghorbanali Mohammadi [39]. They are broadly classified in to four groups as follows:

1. Single Machine Scheduling Problem (SMSP)
2. Parallel Machine Scheduling Problem (PMSP)
3. Flow Shop Scheduling Problem (FSSP)
4. Job Shop Scheduling Problem (JSSP)
Each of these problems is unique and each has its own constraints and objectives. A more detailed description of each problem is given as:

1.8.1 SINGLE MACHINE SCHEDULING PROBLEM (SMSP)

Definition: - “A system with one machine and n independent jobs to be processed, each job has one operation to be performed is called **SMSP**”.

![Fig.1.2 - Single Machine Scheduling Problem](Machine-1)

Only one machine and n jobs to be processed

SMS models are important for various reasons. The SMS model environment is simple and it is a special case of all other environments. The results that can be obtained for SM models not only provide insights into the SM environment, but they also provide a basis for heuristics that are applicable to more complicated machine environments. In practice, scheduling problems in more complicated machine environments are often decomposed into sub problems that deal with single machines.

The basic SMS problem is characterized by: Panneerselvam R, [62].

- A set of independent, single operation jobs are available for processing at time zero.
• Set-up time of each of the jobs is independent of its position in sequence of jobs. So, the set-up time of each job can be included in its processing time.

• One machine is continuously available and is never kept idle when work is waiting.

• Each job is processed till its completion without break.

1.8.2 PARALLEL MACHINE SCHEDULING PROBLEM (PMSP)

Definition: - “Parallel machine scheduling involves scheduling a set of tasks on two or more machines that work in parallel with each other”. The machines perform identical operations and may or may not operate at the same pace. Parallel Machine layout is shown in Figure 1.3

![Parallel Machines Scheduling Problem with n-jobs](image-url)
1.8.3 FLOW SHOP SCHEDULING PROBLEM (FSSP)

Typical manufacturing system that produce a variety of products, each requiring a specified set of operations on specific machines in a predetermined order. If each product requires the same machine in the same sequence, the system is called a Flow - Shop Scheduling Problem (FSSP). In other words, “A flow shop scheduling problem consists of two or more machines and a set of tasks that must be processed on each of these machines. This arrangement is called a FSSP”.

In general flow shop model, there are series of machines numbered 1, 2, 3…m, and each job has exactly m tasks. The first task of every job is done on machine 1, second task on machine 2 and so on. Every job goes through all m machines in a unidirectional order (path). The processing times for each job can vary from machine to machine and the processing times on each machine can vary from job to job.

![Diagram of Flow-Shop Scheduling Problem]

**Fig. 1.4 - Arrangement of Flow-Shop Scheduling Problem**

Figure 1.4 shows a pure flow-shop in which jobs must be processed on each machine in exactly the same order. A general flow-
shop is somewhat different. In that a job may skip a particular machine. For instance, although every job must be processed from left to right in figure1.4, some jobs may go from machine 1 to say, machine 3 and then machine 4.

1.8.4 JOB SHOP SCHEDULING PROBLEM (JSSP)

The classical job-shop scheduling problem is defined as “There are n- Jobs to be processed through m- machines. Each job must pass through each machine exactly once. The processing of a job on a machine is called an ‘operation’ and requires a duration called the processing time”. The flow of the tasks in a job does not have to be unidirectional. Each job may use a machine more than once [7].

1.9 SIGNIFICANCE OF SCHEDULING

- Efficient Utilization
  Capacity Utilization of …Staff, Labor, Space, Equipment, Facilities etc
- Minimization
  Minimization of…Customer Waiting Time, Processing Time, Completion time, Penalty cost, Inventories etc.

1.10 LITERATURE REVIEW

- Background on Multicriteria Flow Shop Scheduling
  In the recent literature on multi-objective optimization is plenty. However, the multi-objective Permutation Flow-shop Problem or PFSP field is relatively scarce, [11, 62, 64] especially when compared against
the number of papers published for this problem that consider one single objective. The few proposed multi-objective methods for the PFSP are mainly based on evolutionary optimization and some in local search methods like simulated annealing or tabu search. It could be argued that many reviews have been published about multi-objective scheduling. Review by Nagar et al. [58] is mostly centered on single machine problems. As a matter of fact there are only four survey papers related with flow-shop. In another review by T’kindt et al. [76] reviewed 15 flow-shop papers where most of them are about the specific two machine case. Another review is given by Jones et al. [48]. However, this is more a quantification of papers in multi-objective optimization. Finally, the more recent review of Hoogeveen, H [41] contains mainly results for one machine and parallel machines scheduling problems. The papers reviewed about flow-shop scheduling are all restricted to the two machine case. For all these reasons, this paper provides a complete and comprehensive review about multi-objective flow-shop and job-shop. In the following, the notation of T’kindt et al. [77] will be used to specify the technique and objectives studied by each reviewed paper. For example, a weighted makespan and total tardiness bi-criteria flow-shop problem is denoted as F//F1 (Cmax, T). In numerical experiments, the gradual-priority weighting approach is shown superior. Framinan et al.
proposed several heuristics along with a comprehensive computational evaluation for the $m$ machine makespan and flowtime flow shop problem. Allahverdi, A [3] also studies the same objectives. A total of 10 heuristics are comprehensively studied in a computational experiment. Among the studied methods, three proposed heuristics from the author outperform the others. Several dominance relations for special cases are proposed as well. A different set of objectives, namely makespan and maximum tardiness, are studied by Allahverdi, A [4]. Two variations are tested, in the first one; a weighted combination of the two objectives subject to a maximum tardiness value is studied. In the second, the weighted combination of criteria is examined.

In a more recent paper Arroyo[8] & Armentano et al. [9] carry out a similar study but in this case using genetic algorithms as solution tools. Although shown to be better than other approaches. A very similar study is also presented by the same author in Ravindran et al. [67] present three heuristics aimed at minimizing makespan and flowtime. Allouche et al. [5] proposes an aggregation procedure that integrates three different criteria to find the best sequence in a flow-shop production environment. Dugardin et al. [31] focuses on the multi-objective resolution of a reentrant hybrid flow-shop scheduling problem (RHFS) and objectives are: the maximization of the utilization rate of the bottleneck and the minimization of the maximum completion time.
• **Background on Multi-Criteria Job-Shop Scheduling**

  Thiagarajan *et al.* [80] addresses the problem of scheduling in dynamic assembly job-shops with the consideration of jobs having different earliness, tardiness and holding costs. Cheng *et al.* [22] study the problem of scheduling $n$ deteriorating jobs on $m$ identical parallel machines. Each job's processing time is a non-decreasing function of its start time. Tavakkoli – Moghaddam *et al.* [78] presents a fuzzy-neural approach for constraint satisfaction of a generalized job-shop scheduling problem (GJSSP) fuzzy processing times. It was an extension of recently developed research in a GJSSP where the processing time of operations was constant. But they assume that the processing time of jobs is uncertain. Adibi, Huang *et al.* [1,42] is used to solve the job-shop scheduling problem using ant colony optimization (ACO) algorithm and compared with the solution obtained by LINGO is studied.

  More recently, TOPSIS method is a popular approach in multiple criterion decision making (MCDM) and has been widely used in the literature. Muley A.A & V.H.Bajaj [56] used it to solve for the selection of alternatives where the weights of each criterion are described. TOPSIS method is also discussed in Ying - Ming Wang *et al.* [91], Fabio J. J. Santos *et al.* [33], Jiang Jiang *et al.* [47], Ali Mohammadi *et al.* [2], & Mir.B. Aryanezhad *et al.* [54].
Scheduling problems with more than two objectives also have been studied. Daniels et al. [25] noted that two factors prohibit the application of multi-objective solution approaches to scheduling problems. First, multiple criteria scheduling problems are invariably more difficult to solve than single objective problems and second, multi-objective scheduling approaches often develop a large number of efficient schedules. This complicates identifying the best solution for the decision maker. Chen & Hawng et al. [20] presented results on the complexity of multi-criteria scheduling problems. They noted that considering more than one objective does not simplify the scheduling problem and that multi objective scheduling problems are at least as difficult as the corresponding single objective scheduling problems.

Rajendran et al. [67] proposed a heuristic algorithm based on heuristic preference relation to address the problem of developing a schedule to minimize Makespan and total flow time. The proposed algorithm was then extended to cover the problem of scheduling to minimize Makespan, total flow time and machine idle time. Employed GA to solve real-world Multi-Criteria scheduling problem in a flow shop. GA used a Utility Function to handle multiple objectives.

Nagar et al. [57] provided a detailed literature survey of multiple and bi-criteria scheduling. The survey showed that conventional techniques have not been very successful for solving multiple criteria
scheduling problems. They suggested use of techniques such as simulated annealing, tabu search, and genetic algorithms in conjunction with problem specific heuristics to develop hybrid algorithms. It was noted that such hybrid algorithms could be better equipped to solve multiple criteria scheduling problems. At the same time, Nagar also stressed the need for development of interactive models.

Recently, T’kindt & Billaut et al. [77] presented an extensive survey of multi-criteria scheduling problems. The survey showed that three kinds of multi-criteria scheduling problems have been tackled. The first one dealt with minimizing lexicographical order of criteria. The second type of problems considered a convex combination of criteria and the third class dealt with finding all-strict Pareto optima. It was noted that when criteria are conflicting, usually a solution that optimizes all the objectives does not exist. This requires using a new definition of optimality, the Pareto optimality definition. The Pareto optimality principle states that the community improves if any individual improves and none gets worse. Thus the objective function improves if any of the individual objectives are improved upon, with none other getting worse.

1.11 FUZZY SET THEORY (Preliminary Idea)

The fuzzy set theory was developed to define and solve the complex system with sources of uncertainty or imprecision’s which are non-statistical in nature. Fuzzy set theory is a theory of graded concept
(a matter of degree) but not a theory of chance or probability the term ‘Fuzzy’ was proposed by Prof. L.A. Zadeh [92] in 1962. The fuzzy set theory is primarily concerned with quantifying the vagueness in human thoughts and perceptions. The transition from vagueness to quantification is performed by applying fuzzy set theory as depicted in the given figure.

A short description of the fuzzy set theory is given below.

<table>
<thead>
<tr>
<th>Fuzzy Set Theory</th>
</tr>
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<tbody>
<tr>
<td>Vagueness</td>
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<tr>
<td>Quantification</td>
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</table>

- **Crisp Set**

A classical set is defined by crisp boundaries i.e. there is no uncertainty in the prescription of the elements of the set.

**Def.** “A classical or crisp set is normally defined as a well-defined collection of elements or objects \( x \in X \) which can be finite, countable or over countable”.

- **Fuzzy Set**

A fuzzy set is a class of objects in which there is no sharp boundary between those objects that belong to the class and those that do not.

**Def.** “Let \( X \) be a collection of objects and \( x \) be an element of \( X \), then a fuzzy subset \( \bar{A} \) in \( X \) (or a fuzzy set \( \bar{A} \) in \( X \) is a set of ordered pairs
\( \hat{A} = \{ (x, \mu_{A}(x)) : x \in X \} \), Where \( \mu_{A}(x) \) is called the **membership function** or grade of membership of \( x \) in \( \hat{A} \) which maps \( X \) to the membership space \( M \) which is considered as the closed interval \([0, 1]\).

**Note:** When \( M \) consists of only two points 0 and 1, \( \hat{A} \) becomes a non-fuzzy set (or Crisp set) and \( \mu_{A}(x) \) reduces to the characteristic function of the non-fuzzy set (or crisp set).

- **Equality**

  Two fuzzy sets \( \hat{A}, \hat{B} \) are said to equal if and only if
  \[
  \mu_{\hat{A}}(x) = \mu_{\hat{B}}(x) \quad \forall x \in X.
  \]

- **Containment**

  A fuzzy set \( \hat{A} \) is contained in or is a subset of a fuzzy set \( \hat{B} \), written as \( \hat{A} \subseteq \hat{B} \) if and only if \( \mu_{\hat{A}}(x) \leq \mu_{\hat{B}}(x) \), \( \forall x \in X \).

- **Support**

  The support of a fuzzy set \( \hat{A} \) is a crisp set \( S(\hat{A}) \) such that \( x \in S(\hat{A}) \Longleftrightarrow \mu_{\hat{A}}(x) > 0 \)

- **Normality**

  A fuzzy set \( \hat{A} \) is normal if and only if
  \[
  \max_{x \in X} \mu_{\hat{A}}(x) = 1.
  \]
• \(\alpha\)-Level Set

The set of elements which belongs to the fuzzy set \(A\) at least to the degree \(\alpha\) is called the \(\alpha\)-level set:

\[
A_{\alpha} = \{x \in X \mid \mu_A(x) \geq \alpha\} \quad \text{and} \quad A_{\alpha} = \{x \in X \mid \mu_A(x) > \alpha\}
\]

is called 'strong \(\alpha\)-level set'.

• Convexity

Let \(A\) be a fuzzy set in \(X\). Then \(A\) is convex if and only if for any \(x_1, x_2, x \in X\), the membership function of \(A\) satisfies the inequality

\[
\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_A(x_1), (\mu_A(x_2))) \quad \text{for} \quad 0 \leq \lambda \leq 1
\]

1.12 FUZZY ARITHMETIC OPERATIONS

Let \(A\) and \(B\) be two fuzzy numbers of the universe of discourse \(U\) with the membership functions \(f_A\) and \(f_B\), respectively, where \(f_A : U \rightarrow [0,1] \) and \(f_B : U \rightarrow [0,1] \). Let \(X\) and \(Y\) be two real numbers in \(U\), then the fuzzy number arithmetic operations are:

• Fuzzy Number Addition:

\[
f_{A+B}(Z) = \bigvee_{x+y \leq Z} \left[ f_A(x) \wedge f_B(y) \right]
\]

• Fuzzy Number Subtraction:

\[
f_{A-B}(Z) = \bigvee_{x+y \leq Z} \left[ f_A(x) \wedge f_B(y) \right]
\]
• Fuzzy Number Multiplication:

\[ f_{\tilde{A} \cdot \tilde{B}}(Z) = \bigvee_{Z=x+y} \left[ f_{\tilde{A}}(x) \cdot f_{\tilde{B}}(y) \right] \]

• Fuzzy Number Division:

\[ f_{\tilde{A}/\tilde{B}}(Z) = \bigvee_{Z=x+y} \left[ f_{\tilde{A}}(x) \cdot \bigwedge_{x+y} f_{\tilde{B}}(y) \right] \]

Let \( \tilde{A} \) and \( \tilde{B} \) be two trapezoidal fuzzy numbers parameterized by \((a_1, b_1, c_1, d_1)\) and \((a_2, b_2, c_2, d_2)\), respectively, where,

1. \( a_1 \leq a_2 , b_1 \leq b_2 , c_1 \leq c_2 \) and \( d_1 \leq d_2 \)
2. \( d_1/d_2 \geq a_2/a_1 \)

The fuzzy arithmetic operations of trapezoidal fuzzy number \( \tilde{A} \) and \( \tilde{B} \) can be expressed as follows.

• Fuzzy Number Addition:

\( (a_1, b_1, c_1, d_1) + (a_2, b_2, c_2, d_2) = (a_1 + a_2 , b_1 + b_2 , c_1 + c_2 , d_1 + d_2) \)

• Fuzzy Number Subtraction:

\( (a_1, b_1, c_1, d_1) - (a_2, b_2, c_2, d_2) = (a_1 - a_2 , b_1 - b_2 , c_1 - c_2 , d_1 - d_2) \)

• Fuzzy Number Multiplication:

\( (a_1, b_1, c_1, d_1) \cdot (a_2, b_2, c_2, d_2) = (a_1 \cdot a_2 , b_1 \cdot b_2 , c_1 \cdot c_2 , d_1 \cdot d_2) \)

• Fuzzy Number Division:

\( (a_1, b_1, c_1, d_1) / (a_2, b_2, c_2, d_2) = (a_1/a_2 , b_1/b_2 , c_1/c_2 , d_1/d_2) \)
Types and Sources of Fuzziness

- **Types of Fuzziness**

  One of the essential problems related to information processing is the presence of imprecision or fuzziness in the available data. Ideally, the information should be precise and certain, but in reality it is very often necessary to use information which does not have those characteristics. Concerning to the imprecision or fuzziness inherent in the information, it can generally be differentiated into the following types.

  - **Intrinsic fuzziness**

    One of the basic features of human knowledge is its capability to process complex information and to communicate with others using natural language. However, the perception of certain information or a situation may be varied, depending on the environment and preference inherent in human subjectivity. Unfortunately, many cases of decision-making are undertaken in such circumstances. Expressions such as “tall man”, “high profit”, “reasonable economic growth” and “conducive environment” are examples of intrinsic fuzziness. These expressions are ill defined and the meaning perceived in these notions will depend on the decision situation and the subjective judgment based on human experience. In this case, there is no exact definition due to context dependence and subjectivity.
• **Informational fuzziness**

  This type of fuzziness is due to the asymmetry between the availability of abundant information on the one side and the limitation of the human capability to process all information simultaneously into a single perceived criterion on the other side. In a complex decision situation, judgment on the decision, alternative may be determined not only by a single criterion, but commonly by a large number of criteria representing the intended goals. The abundant information available for the assessment of criteria will certainly increase the complexity of the problem. With the increasing complexity, the ability of a human being to make a precise and significant decision will drop until a threshold is reached beyond which precision and significant become almost mutually exclusive characteristics Zimmermann [97-99]. Notions such as “prospective investment”, “comfortable hotel”, “creditworthiness”, “country risk” and “key sector” are examples of informational fuzziness. Although these notions are clearly defined, the assessment of the existing alternative cannot be judged only on the basis of one single criterion, but as a result of the evaluation of different related criteria simultaneously. For example, to define, if an economy is conducive for investment, the quantitative criteria, such as growth rate, inflation, interest rate, tax rate and market size together with qualitative measures
such as government regulation, the judiciary system and the stage of
development of the country, should all be taken into account.

• **Relational fuzziness**

  Relational fuzziness occurs when certain phenomena or
relationships are vague, due to non-dichotomy characteristics, which are
very frequently involved in the formulation of implicit expressions.
Expressions such as “not so much higher than average value”,
“approximately the same”, “above limit level” or the expression “if the
net present value remains positively high, then the project is very
feasible” are examples of relational fuzziness frequently used in
everyday life. To deal with this kind of imprecision, fuzzy set theory is a
perfect means for modeling imprecision arising from mental phenomena
in the human judgment, which are heavily involved in the process of
decision analysis.

➤ **Sources of Fuzziness**

  Regarding the causes of imprecision or fuzziness, there are
varieties of sources [20, 97] that, in general, can be outlined as follows:

• **Lack of information**

  The main source of imprecision can probably be attributed to a
lack of information. If the decision maker does not have any information
about which of the possible states of nature will occur, it will cause a
quantitative lack of information. The imprecision may also stem from
the inability to obtain the exact information, due to difficulty in measuring the respective characteristics. This situation is complicated by the fact, that the term “measurement” itself has also very different interpretations in different areas [97]. Sometimes, crisp data is technically obtainable but the cost is too high or the time required is too long. In other cases, the decision maker may be simply not interested in obtaining the exact value, particularly when the data is very sensitive and thus uses an approximation of the crisp data or even employs linguistic descriptions. The other source of uncertainty, which can also be classified under lack of information, is ambiguity, referring to a situation in which certain linguistic information, for instance, has entirely different meanings in different contexts, due to subjectivity. Even when the same linguistic term is used, the intended meaning may be quite different. This phenomenon reflects the fact that the same term may possess different meanings for different situations. A human observer can normally easily interpret a word or term correctly if he knows the context of the word. As long as the exact characteristic cannot yet be measured perfectly, there are some uncertainties about the real measurement and we only know the indicated measurement. In summary, the obtained information is fuzzy because the information is unavailable, non-obtainable, or some of the characteristic of the alternatives are context-dependent resulting from subjective judgment. It
is, thus, not quantifiable and can only be expressed qualitatively using linguistic terms.

- **Abundance of information**

  Macro-economic data of a country, presented in a statistical report, may contain information about the production levels, import and export, the inflation rate, the interest rate, the per capita income, the labor force, the unemployment level, the customer price index and other related data. However, it is still very difficult to obtain a straightforward conclusion about whether the economy is conducive to investment, due to its hierarchical nature. In this case, despite a large amount of information being available, the ability of human beings to perceive and process simultaneously a large number of data is limited and thus produces a certain degree of uncertainty. To deal with such a situation and to reduce the complexity of the decision process, the available data are generally transformed into perceivable information in a single criterion. Aggregating the available information into perceived information or focusing attention on the criteria that seem to be the most relevant through deletion of redundant information is very useful to facilitate the overall judgment. Confronted with this kind of uncertainty, it is very clear that the higher degree of certainty cannot be achieved by gathering even more data, but by reducing the complexity through
transforming the available data into appropriate perceived or condensed information.

- **Conflicting evidence**

  Uncertainty might also be due to conflicting evidence concerning a certain situation. If the two classes of existing information are in conflict, an increase of the information might not reduce uncertainty at all, but rather increase the conflict and complexity. The reasons for this conflict of evidence can certainly be varied. To reduce uncertainty due to conflicting evidence, it is better to check the correctness of the available information again rather than gathering more information. In some cases, however, deleting some pieces of information might reduce the conflict and move the situation closer in the direction of a higher degree of certainty.

### 1.13 GOALS OF FUZZY SET THEORY

The primary goals of fuzzy set theory are as follows:

- **Modeling of Uncertainty**: Certainly, fuzzy set theory has been used to model linguistic “uncertainty”. On the other hand, probabilistic or random “uncertainty” can be modeled by probability theory.

- **Relaxation**: Most of the classical optimization and modeling methods are dichotomous structure i.e. they distinguish between feasibility and non-feasibility, between optimality and non-optimality, between identity and non-identity.
• **Complexity reduction:** The human ability to perceive and process different information simultaneously is limited, when the data is too large. Hence, fuzzy set theory via linguistic variables tries to reduce the complexity of data to such extent that they become perceivable and meaningful to human observers and human being are able to recognize the important structures in the data.

• **Meaning preserving reasoning:** In knowledge based systems such as expert systems the help pf human expertise is often sought which is then processed in the inference engines to obtain acceptable solutions such as diagnosis, proposals for decisions etc. The knowledge is very often stored in form of if-then rules. When these rules are processed in the computer, only their values of the words or statements are processed and not their real meaning. This amounts to symbol processing and not the processing of knowledge. Fuzzy set theory has now been used to attach meaning via linguistic variables to linguistic statements and then design interface engines which not only perform symbol processing but also meaning preserving reasoning.

• **Efficient approximation:** In many real world problems, it is neither necessary nor desirable to obtain exact or exactly optimal solutions to existing problems. It is often important to obtain good approximate solutions efficiently i.e. quickly or with small computational effort.
1.14 FUZZY NUMBERS AND MEMBERSHIP FUNCTIONS

A fuzzy number is a special class of a fuzzy set. Different definitions and properties of fuzzy numbers are encountered in the literature but they all agree on that a fuzzy number represents the conception of “a set of real numbers close to a” where A is the number being fuzzified.

A fuzzy number is a fuzzy set in the universe of discourse X that is both convex and normal. Fig-1.5 shows a fuzzy number $\tilde{A}$ of the universe of discourse X that is both convex and normal. The term fuzzy number is used to handle imprecise numerical quantities. A general definition of a fuzzy number according to Dubois & Prade et al [28] is a real fuzzy number $\tilde{A}$ described as a fuzzy subset on the real line $\mathbb{R}$ whose membership function $\mu_{\tilde{A}}(x)$ is

1. A continuous mapping from $\mathbb{R}$ to the closed interval [0,1],
2. Constant on $(-\infty, a_1]$, $\mu_{\tilde{A}}(x) = 0, \forall x \in (-\infty, a_1]$.
3. Strictly increasing on $[a_1, a_2]$; e.g., $\mu_{\tilde{A}}(x) = f(x) \in [a_1, a_2]$ where $f(x)$ is a strictly increasing function of $x$.
4. Constant on $[a_2, a_3]$: e.g., $\mu_{\tilde{A}}(x) = 1, \forall x \in [a_2, a_3]$.
5. Strictly decreasing on $[a_3, a_4]$: e.g., $\mu_{\tilde{A}}(x) = g(x), \forall x \in [a_3, a_4]$ where $g(x)$ is a strictly decreasing function of $x$. 

36
6. Constant one \((a_i, \infty]\): e.g., \(\mu(x) = 0, \forall x \in (a_i, \infty]\)

![Membership Function of a General Fuzzy Number](image)

Fig-1.5 Membership Function of a General Fuzzy Number \(\tilde{A} = (a_1, a_2, a_3, a_4)\)

A general shape of a fuzzy number following the above definition may be shown pictorially as in Fig 1.5. Here, \(a_1, a_2, a_3\), and \(a_4\) are real numbers. A fuzzy number \(\tilde{A}\) in \(X\) is said to be discrete or continuous according as its membership function \(\mu_{\tilde{A}}(x)\) is discrete or continuous.

Triangular Fuzzy Number, Trapezoidal Fuzzy Number, Parabolic Fuzzy Number and Parabolic Flat Fuzzy Number are a special class of continuous fuzzy numbers.

- **Linear Fuzzy Number (LFN)**

A LFN \(\tilde{A}\) is specified by two parameters \((a_i, a_e)\) and is defined by its continuous membership function follows \(\mu_{\tilde{A}}(x): X \rightarrow [0, 1]\) as follows:
• **Triangular Fuzzy Number (TFN)**

A TFN $\tilde{A}$ is specified by the triplet $(a_1, a_2, a_3)$ and is defined by its continuous membership function $\mu_{\tilde{A}}(x): X \rightarrow [0,1]$ as follows:
A PFN $\tilde{A}$ is also specified by the triplet $(a_1, a_2, a_3)$ and is defined by its continuous membership function $\mu_A(x): X \to [0,1]$ as follows:

\[
\mu_A(x) = \begin{cases} 
\frac{x-a_1}{a_2-a_1} & \text{if } a_1 \leq x \leq a_2 \\
\frac{a_3-x}{a_3-a_2} & \text{if } a_2 \leq x \leq a_3 \\
0 & \text{otherwise}
\end{cases}
\]

- **Parabolic Fuzzy Number (PFN)**

Fig -1.8- Membership function of a PFN
A Trapezoidal Fuzzy Number (TrFN) $A$ is specified by four parameters $(a_1, a_2, a_3, a_4)$ and is defined by its continuous membership function $\mu_A(x) : X \rightarrow [0,1]$ as follows:

$$
\mu_A(x) = \begin{cases} 
\frac{x-a_2}{a_3-a_2} & \text{if } a_2 \leq x \leq a_3 \\
\frac{a_3-x}{a_4-a_3} & \text{if } a_3 \leq x \leq a_4 \\
0 & \text{otherwise}
\end{cases}
$$

Fig.1.9 Membership function of a TrFN
1.15 APPLICATIONS OF FUZZY SET THEORY

There have been numerous applications of fuzzy set theory that are continually reported in the literature. They are briefly mentioned as follows: First, applications to mathematics include generalizations of traditional mathematics such as topology, graph theory, algebra and logic and so on. Then follow the model based or algorithmic applications such as fuzzy optimization (Fuzzy linear programming, fuzzy nonlinear programming, fuzzy dynamic programming etc.), fuzzy clustering (both hierarchical and objective function type), fuzzy petri nets and fuzzy multiple criteria decision analysis. Knowledge based applications are expert systems, fuzzy control, fuzzy rule based classification methods and fuzzy data analysis. Information processing applications are such as fuzzy data banks, query languages, fuzzy programming languages and fuzzy library systems. Also, the traditional operational research models viz. transportation model, economic lot sizing or inventory control models, scheduling and maintenance models, reliability engineering problems have all found their fuzzy counter parts. Further traditional statistical techniques like control charts, time series forecasting have also been fuzzified and applied to various situations. The area such as, fuzzy set theory has made inroads into all branches of engineering, intelligence, all branches of sciences, economics, social sciences, management, medicine, metrology etc.
1.16 ORGANIZATION OF THE THESIS

In this proposed study, some models of Job Scheduling Problems (JSP) are formulated and solved under fuzzy approach. This thesis consists of Six Chapters.

Chapter I – Introduction

This Chapter gives the introduction of Scheduling Problem and classification of different scheduling models. Basic definition of JSP, terminology used and objective of the study are presented. A brief literature review of previous research efforts in Classical Job Scheduling Problems and Fuzzy Job Scheduling Problems are discussed. Introduction to fuzzy set, basic definitions and goals of fuzzy set theory are also discussed. The application of fuzzy set theory in Job Scheduling problem is presented. At the end an overview of this dissertation work is also presented.

Chapter II - Single Machine Scheduling Problem to Minimize Total Penalty Cost: Genetic Algorithmic Approach

In this Chapter, we present Single Machine Scheduling Problem (SMSP) approach to the modern industrial manufacturing areas. Scheduling deals with the process of assigning limited machine (Resources) to a set of n jobs (Tasks) over a period of time with multiple due dates. Penalty is imposed if jobs are not completed within their due dates. Our objective is to minimize the total penalty cost for all jobs. By
using some properties of single machine scheduling problem, it is solved by Genetic Algorithm (GA) approach. Finally, proposed method is illustrated by a numerical example.

Chapter III - Single Machine Scheduling Problem under Fuzzy Processing Time and Fuzzy Due Dates

In this Chapter, we consider $n$-jobs to be processed on Single Machine Scheduling Problem (SMSP) involving fuzzy processing time and fuzzy due dates. The different due dates for each job be considered which meet the demand of customer with more satisfaction level. The main objective of this chapter is, the total penalty cost to be minimum in the schedule of the jobs on the single machine. This cost is composed of the total earliness and the total tardiness cost.

Here, an algorithm is developed using Average High Ranking Method (AHRM) which minimizes the total penalty cost due to earliness (lateness) of jobs in fuzzy environment. Finally, numerical example is given to illustrate proposed method.

Chapter IV- Flow-Shop Scheduling Problem Using Fuzzy Approach

In this Chapter, we present fuzzy scheduling approach to the modern engineering and industrial manufacturing units. These units are facing lots of problems in many aspects such as processing time of jobs, setup time of machines, raw material, man power, electricity and customer’s constraints. In a real production scheduling environment,
there exist various internal or external uncertainties. These are due to incomplete knowledge or uncertain production environment. It is often difficult to model all parameters in the scheduling system with crisp values. When a new job starts processing, such a complex problem of vagueness and uncertainties can be handled by the theory of fuzzy logic.

Here, we propose fuzzy model with flexible constraints imposed on the processing capacity. The task is to find a schedule which achieves maximum level of satisfaction as well as minimal setup and holding cost. Fuzzy flow shop scheduling problem (FSSP) is solved by using Fuzzy Triangular Membership Function (FTMF).

Chapter V- Fuzzy TOPSIS Method for Solving Job-Shop Scheduling Problem

In this Chapter, we focused on the scheduling and sequencing which is a form of Decision Making (D.M) that plays a crucial role in production manufacturing and service industries. Job- scheduling problem arise when there is a choice as to the order in which number of tasks can be performed. In such problems, we determine an appropriate order or schedule for a series of jobs to be done on a finite number of service facilities.

Chapter presents, the way of making schedule of a finite number of jobs on a finite number of machines of unequal efficiencies by using the Technique for Order Preference by Similarity to Ideal Solution
(TOPSIS) Method in fuzzy environment. The order of machines is random. The time taken by the machines for conducting jobs is assumed as imprecise processing time or fuzzy numbers. The fuzzy evaluation values are given by Triangular Fuzzy Numbers (TFNs). Weights are given to each machine according to their efficiency. A new distance is defined using which the distance of each job from the positive and negative ideal solutions are calculated. Also a closeness coefficient is defined to determine the ranking order of the jobs so that the complete schedule or sequence may finish at minimum time-span. Finally, a numerical example is illustrated for proposed method.

Chapter VI - Selection of Job shop Scheduling Problem Using Linguistic Variables

This Chapter Concentrates on JSS Problem under fuzzy approach to solve real life Tailor job –shop problems. The scheduling problem is a very common problem of a tailor shop; here we try to find out an optimal scheduling sequence to perform the jobs that arrive at the shop. In this chapter, customer priority is expressed in linguistic description like Bad, Low, Medium, High, Very High etc. Fuzzy sets are used for modeling uncertainty due to vagueness. These sets are used primarily when the uncertainty is due to semantic definitions rather than due to
randomness. Fuzzy membership functions are used to define how well a value “fits” into a fuzzy set.

Finally, the scope of the further study is discussed. At the end of the thesis comprehensive Bibliography on Fuzzy Job Scheduling Problem (FJSP) is listed.