APPENDIX 4

LAWS OF SIMILITUDE FOR MODELS

A 4.1 ONE-TENTH MODEL IN ALUMINIUM

To obtain the Young's modulus of the material, a tension test is carried out on three specimens cut from the aluminium sheets. The average value of the modulus of elasticity from these tests is found to be $0.723 \times 10^5$ N/mm$^2$ (Fig A4.1). The modulus of elasticity of the prototype material is found to be $2.047 \times 10^5$ N/mm$^2$.

A 4.2. DERIVATION OF SCALE FACTORS FOR LOADS AND DEFLECTION OF THE MODEL

\[
K = \frac{L_p}{L_m} = 10
\]  

...(A4.1)

Strain in a member of the model = Strain in the corresponding member of the prototype

\[
\varepsilon_m = \varepsilon_p
\]  

...(A4.2)

\[
\frac{\sigma_p}{\sigma_m} = \frac{E_p}{E_m} = \lambda = \frac{2.047 \times 10^5}{0.723 \times 10^5} = 2.8313
\]  

...(A4.3)

\[
\frac{P_p}{P_m} = \lambda K^2 = 2.8313 \times 10^2 = 283.13
\]  

...(A4.4)

Also, \[
\frac{\sigma_p}{\sigma_m} = \frac{F_p/A_p}{F_m/A_m} = \frac{F_p}{F_m} \cdot \frac{A_m}{A_p} = \frac{F_p}{F_m} \cdot 100F_m
\]

\[
\frac{F_p}{F_m} = 283.13
\]  

...(A4.6)
FIG. A4-1 STRESS-STRAIN BEHAVIOUR OF ALUMINIUM STRIP IN TENSION

\[ \sigma = \begin{align*}
0.746 \times 10^5 \text{ N/mm}^2 \\
0.704 \times 10^5 \text{ N/mm}^2 \\
0.718 \times 10^5 \text{ N/mm}^2 \\
0.723 \times 10^5 \text{ N/mm}^2
\end{align*} \]
Where $F_p$ and $F_m$ are the forces in the corresponding member of the prototype and the model.

Thus the loads to be applied on the model should be $1/283.13$ times the actual loads on the prototype. The forces in members of the model will be $1/283.13$ times the forces in the corresponding members of the prototype while the stresses in them will be in the ratio of 2.8313.

By similar argument weight of the model should be equal to

$$\frac{\text{weight of the prototype}}{283.13}$$

Weight of the prototype is found to be 7757.75 kg.
Thus the model should have self weight equal to

$$\frac{7757.75}{283.13} = 27.4 \text{ kg}.$$

But the actual self weight is found to be equal to 3.45 kg.

So the simulate the necessary self weight, weights equalling 23.95 kg have to be hung at different levels distributed over large number of nodes.

A 4.2.2 Derivation of deflection

$$\delta_p = \frac{P_p L^3}{\alpha E_p I_p} \quad \ldots (A4.8)$$

$$\delta_m = \frac{P_m L^3}{\alpha E_m I_m} \quad \ldots (A4.9)$$

where $\alpha$ is some constant depending on the base fixity. \ldots (A4.10)

\[ \therefore \quad \frac{\delta_p}{\delta_m} = K \]
Since the loads calculated using equation (A4.4) are high, a load slicing factor of 10 is used.

Thus the governing relationships are

\[ P_m = \frac{P}{283.13 \times 10^2} = \frac{P}{283.13} \]  
\[ \delta_m = \frac{\delta_p}{100} \]  

...(A4.12)

...(A4.13)

Self weight simulated is also sliced by 10, i.e., to 2.395 kg.

Sand bags totalling 2.395 kg in weight are hung at various levels at nodes to simulate the self weight of the tower. With this slicing factor the stress relation reduces to

\[ \frac{\sigma_p}{\sigma_m} = 2.8313 \times 10^2 = 28.313 \]  

...(A4.14)

A 4.3 ONE-TENTH MODEL IN STEEL

Tension tests are conducted on two strips and the average value of the modulus of elasticity is found to be \( 1.919035 \times 10^5 \) N/mm\(^2\). Fig A4.2 shows the stress-strain behaviour of the steel strip in tension.

A 4.4 DERIVATION OF SCALE FACTORS FOR LOADS AND DEFLECTION OF THE MODEL

\[ \frac{E_p}{E_m} = \frac{2.047 \times 10^5}{1.919035 \times 10^5} = 1.06668 = \lambda \]  
\[ \frac{P_p}{P_m} = \lambda K^2 = 1.06668 \times 10^2 = 106.668 \]  
\[ \frac{F_p}{F_m} = 106.668 \]  

...(A4.15)

...(A4.16)

...(A4.17)
FIG. A4.2 STRESS-STRAIN BEHAVIOUR OF STEEL STRIP IN TENSION

\[ \sigma = \begin{cases} \varepsilon \times 1 \times 10^5 \text{ N/mm}^2 \\ \varepsilon_{\text{mod}} \times 1 \times 10^5 \text{ N/mm}^2 \\ \varepsilon_{\text{max}} \times 1 \times 10^5 \text{ N/mm}^2 \end{cases} \]
Thus the loads to be applied on the model should be \( \frac{1}{106.668} \) times the actual load on the prototype. Similarly the forces in members of the model will be \( \frac{1}{106.668} \) times the forces in the corresponding member of the prototype while the stresses will be in the ratio of 1.0668. Weight of the model should be equal to

\[
\text{Weight of the prototype} \quad \frac{\sigma_p}{\sigma_m} = 1.0668 \quad \text{(A4.18)}
\]

\[
\frac{\delta_p}{\delta_m} = 10 \quad \text{(A4.19)}
\]

Since the loads calculated using equation (A4.16) are high, a load slicing factor of 10 is used.

Thus

\[
P_m = \frac{P_p}{106.668 \times 10} = \frac{P_p}{1066.68} \quad \text{(A4.20)}
\]

\[
\frac{\delta_m}{\delta_p} = \frac{6.65}{100} = 0.0665
\]

Self weight simulated is also sliced by 10 i.e., to 6.608 kg.
Sand bags totalling 6.608 kg in weight are hung at various levels at nodes to simulate the self weight of the tower. With this slicing factor, the stress relation reduces to

\[
\frac{\sigma_p}{\sigma_m} = 1.0668 \times 10 = 10.668
\]

...(A4.21)