ABSTRACT

The fundamental equations that govern fluid flows are inherently a nonlinear set of coupled partial differential equations. Exact solutions for these equations are found possible only for a limited number of problems involving simple geometries and simple conditions. Therefore, we have to resort to approximate analytical methods or fully numerical procedures or a suitable combination of both, in order to obtain relevant information about solutions of relatively more complex problems.

The method of perturbation (asymptotic) expansion stands foremost among the approximation methods. This method is implicitly based on the assumption that an exact solution of a physical problem has an asymptotic expansion of certain form. The present thesis employs both regular and singular perturbation techniques, to solve some problems in viscous flows including convective heat transfer. The study undertaken in this work is the result of motivation derived from the applications of fluidmechanical problems to viscometry, suspension rheology, human circulatory system, industrial design and manufacture of products involving improved transfer of heat and mass such as power generators, chemical reactors, superheaters / coolers etc. The thesis consists of six chapters.
Chapter 1 gives a brief introduction to perturbation analysis, followed by pertinent literature in respect of the specific problems studied.

The information concerning the hydrodynamical interaction between two or more rigid bodies in low Reynolds number flows, is needed in theoretical investigations of the behaviour of suspensions of submicron particles. In this context, the problem of slow and steady rotation of two identical spheres rotating about their line of centres is studied. This forms the subject matter of Chapter 2. The flow pattern is no longer symmetrical with regard to the equatorial planes of the two spheres. Consequently the spheres must experience drag along the axis in addition to the torque. The available expressions for torque and drag in the literature are improved. Interestingly, it is seen that the present result predicts a reduction in torque on both bodies with increasing speed, thereby showing the clear influence of second body in the medium. The drag on either body is also found to be considerably reduced.

Chapter 3 deals with a similar two-body problem with the spheres replaced by two nearly spherical solids of revolution (not necessarily identical), \( r = 1 + \eta f_1(\theta) \) and \( r' = 1 + \eta f_2(\theta) \). Assuming that \( R_0 \) (Reynolds number due to rotation) and \( \eta \) (roughness parameter) are of same order of smallness, general expressions for torque and drag are obtained up to \( O(R_0^m \eta^n)_{m+n \geq 2} \). Corresponding particular computations for typical solids characterised by \( f(\theta) = \cos \theta, \ \ P_3(\cos \theta) \) and \( \sin 2\theta \) are derived. The dependence of the shape of the body etc. on the drag, are discussed through graphs.
Chapter 4 concerns with fluid transportation in helical pipes. The curvature $\epsilon$ and torsion $\alpha$ of the generic spatial curve spanning the pipe are assumed to be small, such that $\delta = \tan \xi = \epsilon / \alpha < 1$, where $\xi$ is the helical angle. Pursuing a regular perturbation analysis in terms of $\delta$, the expression for flow rate is obtained up to $O(\delta^m \alpha^n)_{m+n \leq 6}$. This expression is numerically computed and compared with the prediction of previous studies, to show the influence of torsion.

Chapter 5 deals with convective heat transfer from a spinning sphere in a viscous incompressible flow. The physical parameters of interest here, namely, $Re$ (Reynolds number based on uniform streaming), $R_0$ (Reynolds number based on axial spin), $Pr$ (Prandtl number) are such that $Re < 1$, $R_0 < 1$, $R_0^2 / Re = O(1)$ and $Pr = O(1)$. The problem reduces to solving a coupled system of equations for the balance of momentum and energy, with velocity being the coupling factor. Using the method of matched asymptotic expansions, locally valid expansions for the temperature field close to and far away from the sphere are obtained up to $O(Re^2 Pr^2)$. Analytical expression for the mean Nusselt number $Nu$, describing its functional dependence on the parameters cited above, is obtained. It is found that the contribution of spin to $Nu$ is of $O(R_0^4)$. When $Re Pr < 1$, the spin tends to increase the rate of heat transfer while the effect gets reversed when $Re Pr > 1$.

Chapter 6 deals with heat transfer in parallel flow between corrugated plates. It consists of three sections. In the first section, exact solution of the steady energy equation in the case of classical Couette flow is extended, when the plates are no longer planar but are characterised by sinusoidal corrugations and
phase difference. The expression for the mean Nusselt number \( \text{Nu} \) describing the rate of heat transfer from the warmer (moving) plate is obtained. It is found that \( \text{Nu} \) increases with phase difference \( \beta \) and reaches the maximum when \( \beta = \pi \). This feature is physically consistent with the drag behaviour observed by earlier authors. In the second section, a similar analysis is pursued when both the plates are at equal temperature. In the third section, heat transfer in Poiseuille flow between fixed corrugated plates is considered. The effect of flow rate enhancement due to corrugations, towards convective rate of heat transfer and viscous generation of heat conduction is brought out. It is found that, by designing the warmer plate with a suitable phase difference, maximum heating or cooling can be achieved depending on the values of the parameter \( EcPr \), where \( Ec \) is the Eckert number and \( Pr \) is the Prandtl number.