CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

Linear programming problems in general are concerned with the allocation of scarce resources - labour, materials, and capital - in the “best” possible manner so that costs are minimized or profits maximized. In using the term “best” it is implied that some choice or a set of alternate courses of actions is available for making the decision. The term “best” merely defines a particular class of programming problems that meets the conditions. The criterion for selecting the best values for the decision variables can be described by a linear function of these variables. The criterion function is generally referred to as the objective function. The operating rules governing the process can be expressed as a set of linear equalities and inequalities. This set is referred to as the constraint set.

Linear programming models are widely used to solve real life problems in diverse fields for:

- representing or approximating a large variety of confronting problems in diverse fields as linear programming models
- using many efficient techniques available in linear programming
- using sensitivity analysis which re-engineer the model in case of environmental changes and
• making meaningful inferences available during the solution process

The formulation of a linear programming model from the given verbal statement of a problem involves a detailed study of the system, data collection, identification of the decision variables, construction of system constraints and the objective function. In the course of formulating the model, model builders often tend to include inadvertently, constraints and variables that may not play a role in defining the feasible set. Inclusion of such constraints and variables although does not alter the optimal solution of a given problem, nevertheless increases the computational complexity. It is a well-established fact that every additional constraint / variable in a linear programming problem demands more computational effort and computer memory.

1.2 MOTIVATION

The extent to which the simplification can be performed on a LP model is a matter of judgement. There is a significant advantage in reducing the model row-wise and column-wise prior to applying any algorithm to find the optimal solution. Primal-dual properties are used to detect such redundancies. The identified redundant constraints are validated with the optimal solution obtained with the reduced model and necessary amendments are made to incorporate the faulting constraints into the reduced model and re-seek the optimal solution with minimal computational effort. Algorithms for an apriori identification and disposal of redundancies (rows and columns) in a linear programming model have been developed to quicken the process of finding solution for a given LP problem and save computational effort.
Earlier researchers have proposed methods for minimizing the computational effort by identifying redundant constraints. Many of these methods are generally computationally expensive. A conjunctive algorithm making use of the “matrix of intercepts” enables one to identify \textit{apriori} redundancies. The motive for detecting redundancies is to reduce the original model size and thus the cost of solution.

1.3 METHODS USED TO SOLVE THE LP PROBLEMS

Many efficient algorithms are available to solve the linear programming problems. Simplex algorithm is a proven, well-founded and established procedure to solve linear programming problems. It is essentially a univariate search technique and exhibits slow convergence properties. Multiplex algorithm is a multivariate search technique. It possesses the property of rapid convergence. It brings a cluster of variables into the basis and reduces the number of iterations considerably. A matrix of intercepts of the promising variables is constructed to select more than one variable at a time to enter the basis. This matrix enables to select a set of linearly independent vectors to enter the basis. This algorithm also minimizes the tendency of variables popping in and out of the basis. The dual Simplex or the dual Multiplex algorithm removes infeasibility if any, from the solution. In this study, the original and the reduced linear programming models are solved using the Simplex and the Multiplex algorithms.

1.4 OUTLINE OF THE THESIS

Linear programming problems have many real life applications. Revised Simplex algorithm, is a time tested, well-founded and established procedure to solve linear programming problems. The Multiplex and the dual Multiplex algorithms applied to linear
programming problems, accept or reject more than one variable at a time and this approach has accelerated the rate of convergence. The effect of the generalized model reduction algorithm in conjunction with the Simplex and the Multiplex algorithms is analyzed first.

Most of the real life linear programming problems will have their decision variables bounded depending on the market conditions either on one or both ends. Dantzig developed a separate algorithm for bounded variable problem and it accounts for the bounds implicitly. The Multiplex and the dual Multiplex algorithms act in a similar way when it comes to handling bounded variable problems also. The effect of the generalized model reduction algorithm on bounded variable problems in conjunction with the above said methods is also analyzed.

As an improvement to solve large-scale linear programming problems having block angular structure, decomposition principle has been applied. This principle transforms the original problem into a master program and a sequence of sub-problems. The master program corresponding to the central constraints coordinates the sub-problems through the primal-dual properties. Dantzig and Wolf’s algorithm to solve such problems uses decomposition principle in conjunction with Simplex procedure. In order to make the convergence rapid, Multiplex and dual Multiplex algorithms are applied to solve decomposition problems. Usually for most of the large-scale real life linear programming problems, it is not uncommon to find quite a few resource constraints turn out to be redundant. Under such circumstances relatively a small percentage of the decision variables bind the solution at the optimal stage. Significant efforts have been directed to develop methods and models to identify those constraints and variables other than the ones, which are most likely to be tight at optimal solution.
It will be an interesting and meaningful exercise to apply the model reduction algorithm to decomposition problem on parallel and distributed environments and study the effect of the computational efficiency. The block angular structure of the decomposition problem and the applications of the model reduction algorithm to its correlated de-coupled LP models make this implementation cost effective.

1.5 CONCLUSION

In this chapter, the requirement of identification of redundancies and the various linear programming models in which they appear are discussed. This chapter also outlines the application of the model reduction algorithm along with the Simplex and the Multiplex algorithms to various linear programming models.