8 CONCLUSIONS

Ever since Dantzig proposed his simplex method for solving linear programming problems many researchers had tried many variations to this method. One of the variations is to bring into the basis two variables at a time. Revised simplex algorithm is a univariate search technique which replaces a single vector at each iteration. Multiple column entry was discouraged by saying that it will increase computations instead of decreasing the computations[47]. Ragner Frisch of the University of Oslo succeeded in bringing more variables into the basis at start of each iteration and at the end of each iteration one may be moving from a point inside the feasible region to another point which may either be inside the feasible region or on the boundary of the constrained set and not necessarily the vertices of the feasible region as in the case of simplex method. But it has the drawback of guessing the promising variables and selecting the starting basis. It also has to guess the preference direction[42,43]. If a wrong selection is made, the number of computations and time involved to reach the optimal solution will be more than the revised simplex method.

In all the above methods, one will be moving from a basic feasible solution to another basic feasible solution. In other words, the feasibility is always maintained. Since the crash algorithm, multiple column selection and suboptimization procedure[7,40] were all used to find a feasible solution from an infeasible starting solution which may be created because of the presence of lower bound constraints and equality constraints, the author likes to find a basis which is feasible or infeasible but the optimality conditions should be satisfied at infeasible points. This led to the present multiplex algorithm in which one can bring one or more variables into the basis at the beginning of each pass and one will be moving from one basic solution to another basic solution till there is improvement in the objective function. When there is no improvement, the feasibility of the solution can be checked. In the event of the optimality condition being satisfied but the solution is infeasible either the dual simplex procedure or the dual multiplex algorithm can be used to remove the infeasibility and reach the optimal solution.

The 'θ' matrix enables to select a set of linearly independent vectors to construct a basis at the beginning of each pass. The author feels that if the selection criterion using 'θ' matrix is suitably modified, the solution to any linear programming problem can be obtained exactly in m number of iterations where m is the number of resource constraints.

Redundant constraints, if any, in the problem will increase the computational affort. The 'θ' matrix may be used advantageously to detect such redundancies. Hence if redundant constraints are removed at the beginning itself, it will increase the computational efficiency. Cycling is a phenomenon which may occur while solving a linear programming problem using simplex algorithm. These variants of simplex procedure are avoided if the multiplex algorithm is used.

The multiplex algorithm can accept a starting basis and proceed until there is no improvement in the objective function value irrespective of the solution being feasible or infeasible. Feasibility may be obtained using the dual simplex procedure.

The following problems are suggested for further investigation.

i. It has been observed while solving medium to large scale linear programming problems that if the multiplex algorithm finds a solution beyond the optimal point, at the end of first pass, then all the subsequent solutions lie to the left of this first basic solution. This is an indication of the convergence property of the multiplex algorithm. It will be an interesting exercise if this could be formalized and the convergence is established mathematically.

ii. A few alternate methods of selecting the variables using the 'θ' matrix are suggested in Chapter 6. Though the author has tried those algorithms for text book problems, this can be extended to large scale problems also. Any other selection procedure that would solve the linear programming problem in m number of iterations may also be attempted.

iii. Sparsity of constraint matrix is one other factor which has a direct bearing on the computational efficiency of linear programming problems. It may be observed from the discussions in Chapter 5 that multiplex algorithm works more efficiently for problems with a sparse constraint matrix than one with a dense matrix. The same problem with differing degree of sparsity can be taken up to study the relationship between sparsity and solution path of the linear programming problem.

iv. The multiplex algorithm could be easily extended to solve problems amenable to solution by the decomposition principle.

v. This can also be extended to solve quadratic programming problems.

vi. It will be interesting to develop a graphical plot on a VDU showing the entering and leaving variables. Such a package may be used as a tool for computer aided instruction. This will be an excellent educational aid and may be attractive to novice learners of simplex procedure to solve linear programming problems.

vii. It would be an interesting exercise to modify the multiplex algorithm code to store only the nonzero elements of the constraint matrix using the concepts of data structure as it is done in the commercial linear programming package.