CHAPTER 1
INTRODUCTION

1.1 INTRODUCTION

The Linear Programming Problem (LPP) of the Mathematical Programming consists of a linear objective function and a set of linear constraints to be fulfilled. It is simple in structure but powerful in applicability to a wide range of problems in engineering, medicine, agriculture, industry, business, social science, management, defence, administration, communication etc. It has become an important model in modern theoretical and applied mathematics and has proved to be one of the most effective tools in operations research for decision making. The success of it stems from its flexibility in depicting real world situations varying from military operations to behavioural and social sciences. It is a model to get the best out of a given situation. Linear Programming Problems are concerned with the optimal allocation of limited resources to meet certain desired objective.

The linear function, which is to be optimized, is called the objective function and the conditions to be satisfied are expressed as simultaneous linear equalities or inequalities referred to as constraints. A solution vector, which satisfies the set of constraints and the given objective, is termed as an optimal solution. The logical analysis and conclusions of all decision making problems are based on the concept of models and model building, which is an abstraction of the reality and
may appear to be less complex than the reality itself. The collection of data is the most difficult part of constructing a model. The formulation of a linear programming model involves a detailed study of the system, data collection, identification of decision variables, construction of the objective function and system constraints. In the course of formulating a model, model builders often tend to include inadvertently or innocently, constraints and variables that may not play a role at all in defining the feasible set. The presence of such constraints / variables is hardly disputed and can play havoc with Linear Programming solution procedures and greatly hamper the solution effort. It is well known that every additional constraint / variable in a Linear Programming Problem demands more computational effort and computer memory. The identification of such embedded non-role play constraints / variables in the model without affecting the character of the system is as difficult as solving the Linear Programming Problem itself.

1.2 MOTIVATION

The cost to prevent inclusion of constraints / variables that are irrelevant to the LP model will largely depend on the kind of problem being formulated, objective of the problem, the method used to solve it and most importantly, the specifics of the practical context of the problems. There is always a greater advantage in solving a row-wise / column-wise reduced model than the original before applying any algorithm to find the optimal solution. The earlier researchers (Dantzig (1955), Zionts (1965), Thompson (1966), Tischer (1968), Boneh (1979), Telgen (1979), Bradley (1982), Bixby (1987), Danny (1988) and Imbert(1995)) have proposed several methods for minimizing the computational effort by identifying redundant constraints and variables.
But most of these methods proposed for the identification of redundancies are computationally expensive. However, the heuristic algorithms proposed by recent researchers (Paulraj (1998) and Rhymend (1999)) have shown significant improvement in computational efficiency. But still these algorithms did not identify all the redundancies leaving scope for further improvement. This has motivated to think and develop an integrated algorithm, which will further reduce the size of the original model and speed up the process of computation and saving memory too. The apparently identified redundant constraints are validated by substituting the tentatively obtained solution of the reduced model and necessary amendments are made to incorporate the faulting constraints into the reduced model and re-seek the optimal solution with minimal computational effort.

1.3 THE GENERAL LPP AND COMPUTATIONAL PROCEDURES

The aim of formulating a general Linear Programming Problem is to find values for a set of m out of n decision plus m surplus variables meeting the objective (minimize or maximize) of a linear function 

\[ z = c_1 x_1 + c_2 x_2 + \ldots + c_n x_n \]

and fulfilling the m linear constraints of inequalities or equalities, of the form

\[ a_{ij} x_1 + a_{i2} x_2 + \ldots + a_{ij} x_j + \ldots + a_{in} x_n \{ \leq, =, \geq \} b_i, i = 1, 2, \ldots, m \]

and \( x_j \geq 0, \ j = 1, 2, \ldots, n \)

where \( a_{ij}, b_i \) and \( c_i \) are numerically known values and the signs \( \leq, =, \geq \) can vary from constraint to constraint. Now adding surplus or artificial variables or both, as the case may be to the constraints, they may be written in matrix form \( Ax = b \), where A is the augmented constraint
coefficient matrix of m rows and (n+k) columns lying in the range 
(n+m) ≤ (n+k) ≤ (n+2m).

Several Simplex based algorithms are available to solve the Linear Programming Problems. Some of the following computational procedures are used to solve the Linear Programming Problems – Revised Simplex, Dual Simplex and recently developed Multiplex and Dual Multiplex [Sakthivel (1984), Ramunjan (1986) and Narayanasamy (1989)]. These are already proven, well founded and established procedures. Simplex algorithm in any form is essentially a univariate search technique and exhibits slow and sluggish convergence properties. This was overcome recently by the development of a Multiplex algorithm, which possesses the property of a rapid convergence. It brings into fore a cluster of variables into the basis and reduces the number of iterations considerably. The dual Simplex or the dual Multiplex algorithm removes infeasibility, if any, from the solution. All through this research work, the original and the reduced models are solved using the above methods.

1.4 REDUNDANCIES AND CAUSES OF OCCURRENCE

A constraint / variable in a LP model is redundant if it can be omitted without affecting the feasible set. In other words, the constraints / variables that do not play a role in defining the feasible set may be termed as redundant constraints / variables. Redundancies in Mathematical programming models are nothing uncommon and Linear Programming Problems are no exception. Redundancy may occur in the LP formulation phase because of bad source data or reluctance to run
the risk of omitting some relevant constraints while modeling the problem.

1.5 OVERVIEW OF STATISTICAL ANALYSIS

A detailed study of the behaviour of the existing model reduction algorithms and those proposed in this research work for the identification of redundant constraints in a LP model have been made by solving many problems that were either randomly generated or down loaded from the net-library apart from those chosen from textbooks, journals and some Ph.D. theses. For a better study of both computational efficiency and identification of redundancies by the model reduction algorithms, LP problems containing an appreciable number of constraints and variables were considered. In the course of a statistical analysis of the results, some of the commonly used statistical techniques and Statistical Package for Social Sciences (SPSS) were used. The factors involved in the performance of the developed algorithm are compared with the factors in the original model under different categories and are tested using the Randomised Block Design (RBD) in Multivariate Analysis of Variance (MANOVA). The ranking of the model reduction algorithms on the basis of their performance is done using the test based on the proportion of the identified redundant constraints. The above test is justified provided that the more the identification of redundant constraints, the less is the time required to solve the LP model.
1.6 OUTLINE OF THE THESIS

Linear Programming Problem is widely used for decision making and has many real life applications. Decision making is the most important aspect of any business and industry. The revised Simplex and Multiplex are some of the existing algorithms available to solve LP models. Presence of redundancies in an LP model is hardly disputed. It is necessary to understand the modeling implications of these structural redundancies, which place a need for efficient as well as effective identification techniques.

The earlier researcher's heuristic algorithms based on the matrix of intercepts - Theta Max. and Theta Min. identify as many redundant constraints as possible for minimization and maximization objective with lower and upper bound constraints respectively. The extra computational effort required to identify the a priori redundant constraints using the matrix of intercepts algorithms depends on the size of Theta matrix because mxn number of divisions have to be performed to construct this matrix at start in the worst case. The construction of the gradient matrix algorithm comparatively needs virtually no computational effort. A refined hybrid heuristic algorithm for model reduction has been proposed in this research work to identify redundant constraints for each of the objective function independently by mixing inequalities in different proportions. The hybrid algorithms make use of the union of the redundancies identified by the gradient matrix and Theta Max. for minimization, and the union of the gradient matrix and Theta Min. for maximization objectives respectively. This union approach has shown substantial improvements in the identification of redundant constraints for problems with minimization and maximization objectives.
Also the study reveals that the more the identification of redundancies, the less is the time required to reach optimal solution, which is obvious and expected.

The quest for identification of more redundancies and a general algorithm for identifying redundancies for a LP model have led to the development of the conjunctive algorithm. The conjunctive algorithm, which is the union of the individual algorithms, identified more number of redundancies which otherwise escaped detection in the earlier algorithms. The computational efficiency is much higher than the earlier approaches.

A generalized LP model consists of upper and lower bound constraints besides few equalities. Equalities are the most stringent constraints and ordinarily cannot be redundant if there is an optimal solution for a given problem. Whenever the model reduction algorithm is applied to a LP Problem containing mixed set of constraints, the algorithm initially identifies equality and inequality constraints as redundant. This study has shown that if an optimal solution exists, then equality will have a role to play in the feasible set. Further a detailed analysis by retaining all the equality constraints within the reduced model first and then keeping all of them outside next was made using the conjunctive algorithm. This study has shown that keeping all the equality constraints a priori in the reduced model obtained by applying the conjunctive algorithm yields better results.

The application of the Multiplex algorithm along with the model reduction algorithm has given encouraging results for large scale LPP. Because of the property of rapid convergence of the Multiplex algorithm
due to minimizing the tendency of popping variables, the number of iterations required to solve the LP model has been considerably reduced when compared to the revised Simplex algorithm. Further studies show that the application of the model reduction algorithm along with the Multiplex has yielded better computational results than the model reduction algorithm along with the revised Simplex for large scale problems.

The improved gradient algorithm identifies 30% more redundancies than the earlier constraint gradient algorithm for a minimization objective. The study on redundant variables has shown improvement compared to the earlier researchers' algorithm. The application of the gradient algorithm to Karmarkar's model has shown encouraging identification of redundancies.

1.7 CONCLUSION

In this chapter, the general computational procedures used to solve LPP, the causes for redundancy in LP models and some of the commonly used statistical tools have been discussed. An outline of the application of the model reduction algorithms has also been highlighted.