CHAPTER 3

Analysis of MHD Non-Darcian boundary layer flow and heat transfer over an exponentially vertically stretching surface with Thermal Radiation
3.1. INTRODUCTION:

In industrial manufacturing process the heat and mass transfer problems are well used. This phenomena applicable in wire and fibre coatings and transpiration cooling etc. In astrophysics and geophysics the MHD flow basically used. Basically the MHD flow has wide applications. Usually used in Engineering and industrial. The fluid subjected to a magnetic field become a good agreement results. There is a wide application in Mechanical Engineering field. After the pioneering work of Sakiadis [1961] many researchers gave attention to study flow and heat transfer of Newtonian and non-Newtonian fluids over a linear stretching sheet. By considering quadratic stretching sheet, Kumaran and Ramanaiah [1996] analyzed the problem of heat transfer. Ali [1995] investigated the thermal boundary layer flow on a power law stretching surface with suction or injection.

Elbashbeshy [2001] analyzed the problem of heat transfer over an exponentially stretching sheet with suction. Magyari and Keller [1999] discussed the heat and mass transfer in boundary layers on an exponentially stretching continuous surface. Sanjayanand [2006], and Khan [2005] extended the work of Elbashbeshy [2001] to viscoelastic fluid flow, heat and mass transfer over an exponentially stretching sheet. Raptis et al. [1981] constructed similarity solutions for boundary layer near a vertical surface in a porous medium with constant temperature and concentration. Bejan and Khair [1985] used Darcy’s law to study the features of natural convection boundary layer flow driven by temperature and concentration gradients. Forchheimer [1901] proposed quadratic term in Darcian velocity to describe the inertia effect in porous medium. Plumb and Huenefeld [1984] studied the problem of non-Darcian free convection over a vertical isothermal flat plate. Rees and Pop [1994] also studied the free convection flow along a vertical wavy surface with constant wall temperature. Rees and Pop [1995] studied the case where the heated surface displays waves while the Darcy’s law is supplemented by the Forchheimer terms. They stated that the boundary flow remains self similar in the presence of surface waves but where inertia is absent and when inertia is present but surface waves are absent. However, the combination of the two effects yields non
Tsou et al. [1967] studied flow and heat transfer in the boundary layer on a continuous moving surface while Gupta and Gupta [1977] solved boundary layer flow with suction and injection. Andresson and Bech [1992] have studied the MHD flow of the power law fluid over stretching sheet. Pavlov [1974] gave an exact similarity solution to the MHD boundary layer equation for the steady and two dimensional flow caused solely by the stretching if an elastic surface in the presence of uniform magnetic field. M S Abel and Mahesha [2008] heat transfer in MHD viscoelastic fluid flow over a stretching sheet with variable thermal conductivity non uniform heat, source and radiation. In this paper we analyzed thermal radiation effect in a exponentially vertical stretching surface on a MHD flow. And effect of various physical parameters are also discussed in detail.

3.2. MATHEMATICAL FORMULATION AND SOLUTION

Fig. 1: Schematic of the stretching sheet problem

FLOW ANALYSIS

The sheet lies in the plane $y = 0$ with the flow being confined to $y > 0$. Two equal and opposite forces are applied along the x-axis, so that the sheet is stretched, keeping the origin fixed. Under the boundary layer approximation and the assumption that the contribution due to the normal stress is of the same order of magnitude as the shear stress, the basic boundary layer equations governing equations are given by
\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \]  
\[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\nu}{k} u - \frac{C_b}{\sqrt{k}} u^2 + \Phi(T-T_x) - \sigma \frac{B_0^2}{\rho} u \]  
\[ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{\sigma}{\rho C_p} B_0^2 u^2 + \frac{\mu}{\rho C_p} \left( \frac{\partial T}{\partial y} \right)^2 + \frac{Q}{\rho C_p} (T-T_x) - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y}, \]  

The associated boundary conditions to the problem are

\[ U = U_w(x), v = 0, T = T_w(x), \text{at } y = 0, \]
\[ u = 0, T \rightarrow T_x, \text{as } y \rightarrow \infty \]  

where \( u \) and \( v \) are the \( x \) and \( y \) components of the velocity field of the steady plane boundary flow, respectively. \( \nu \) denotes the kinematic viscosity and \( \alpha \) is the thermal diffusivity of the ambient fluid. Both are assumed to be constant, \( \sigma \) is the electrical conductivity and \( B_0 \) is the magnetic field flux density. The fluid flow is independent of the temperature field, \( T_x \) is the temperature of the ambient fluid and \( Q \) is internal heat generation/absorption coefficient. \( C_b \) is the drag coefficient. The stretching velocity \( U_w \) and exponential temperature distribution \( T_w \) are defined as

\[ U_w(x) = U_0 e^{\frac{x}{L}}, \]
\[ T_w(x) = T_x + (T_0 - T_x) e^{\frac{ax}{L}}, \]  

where \( T_0 \) and \( a \) are parameters of temperature distribution over the stretching surface. \( T \) is the temperature; \( K \) is the thermal conductivity, \( C_p \) is the Specific heat and \( Q_r \) is the radioactive heat flux.
where $\kappa^*$ is the mean absorption coefficient and $\sigma^*$ is the Stefan-Boltzmann Constant. $T^4$ is expressed as a linear function of temperature, hence

$$T^4 = 4T_x^3T - 3T_x^4$$  \hspace{1cm} (3.2.7)

Introducing the following non-dimensional parameter:

$$\eta = \sqrt{\frac{\text{Re} y}{2L}} e^{\frac{s}{2L}}$$

$$\psi(x, \eta) = \sqrt{2 \text{Re} v} e^{\frac{s}{2L}} f(\eta),$$  \hspace{1cm} (3.2.8)

$$T(x, y) = T_e + (T_0 - T_e) e^{\frac{s}{2L}} \theta(\eta),$$  \hspace{1cm} (3.2.9)

Where $\psi$ is the stream function which is defined in the usual form as

$$u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x}$$  \hspace{1cm} (3.2.10)

Thus substituting (3.2.8) and (3.2.9) into Eq(3.2.10), we obtain $u$ and $v$ as follows:

$$u(x, y) = u_0 e^{\frac{s}{2L}} f' (\eta), \quad v(x, y) = -\frac{v}{L} \sqrt{\frac{\text{Re}}{2L}} e^{\frac{s}{2L}} \{ f(\eta) + \eta f' (\eta) \}.$$  \hspace{1cm} (3.2.11)

Eqs. (3.2.1) to (3.2.4) is transformed into the ordinary differential equation with the aid of equations (3.2.8)-(3.2.11). Thus, the governing equations takes the form,

$$f'' + ff' - (2 + N_2) f^2 - 2G e^{\frac{x}{2L}} e^{2s} \theta - 2e - f' \left( \frac{Ha^2}{\text{Re}} + N_1 \right) = 0$$  \hspace{1cm} (3.2.12)

$$\text{Pr}^{-1} \theta' (1 + \frac{4K}{3}) + f \theta - af' \theta + e^{\frac{x}{2L}} \left( \frac{Ha^2}{\text{Re}} f^2 + f^2 e^x \right) + 2e^{-x} \theta = 0$$  \hspace{1cm} (3.2.13)
The boundary conditions (3.2.5) reduces to,

\[ C_f \sqrt{Re} = \sqrt{2X} f'(0), f(0) = 0, f'(0) = 1, \theta(0) = 1, \]

\[ f'(\infty) = 0, \theta(\infty) = 0, \]

where, \( X = \frac{x}{L} \) \( Ha = \left( \frac{\sigma B_0^2 L^3}{\rho \nu} \right)^{\frac{1}{2}} \) is Hartman number, \( Ec = U_0^2 / c_p (T_0 - T_\infty) \) is Eckert number, \( \lambda = \frac{QL^2}{\mu c_p \text{Re}} \) is the dimensionless heat generation/absorption parameter, \( Gr = g \beta (T_0 - T_\infty) \frac{L^3}{\nu^2} \) is the Grashof number, \( \text{Re} = U_0 L / \nu \) is Reynolds number, \( Gr = Gr_i / \text{Re}^2 \) is the thermal buoyancy parameter and \( Pr = \frac{\nu}{\alpha} \) is the Prandtl number, \( \lambda = L, Z = Ha^2 / \text{Re}, a = W, K = \frac{4\sigma^* T_\infty^3}{K^* K} \) Radiation number in the above system of local similarity equations, the effect of the magnetic field is included as a ratio of the Hartman number to the Reynolds number. The physical quantities of interest in the problem are the local skin friction acting on the surface in contact with the ambient fluid of constant density which is defined as

\[ \tau_{wx} = \rho \nu \left( \frac{\partial u}{\partial y} \right)_{y=0} = \left( \frac{\rho \nu U_0}{L} \right) \left( \frac{Re}{2} \right)^{\frac{1}{2}} e^{\frac{x}{2}} f'(0) \]

And the non-dimensional skin friction coefficient, \( C_f \), which can be written as,

\[ C_f = \frac{2\tau_{wx}}{(\rho U_w^2)} \text{ or } C_f \sqrt{Re} = \sqrt{2X} f'(0). \]

The local surface heat flux through the wall with \( k \) as thermal conductivity of the fluid is given by

\[ q_{wx} = -k \left( \frac{\partial T}{\partial y} \right)_{y=0} = \frac{k(T_0 - T_\infty)}{L} \left( \frac{Re}{2} \right)^{\frac{1}{2}} e^{\frac{x}{2}} \theta(0). \]
The local Nusselt number, $Nu_x$, which is defined as

$$Nu_x = \frac{xq_w(x)}{k(T_w - T_s)},$$

(3.2.19)

$$Nu_x / \sqrt{Re_x} = -(X / 2)^2 \theta(0),$$

(3.2.20)

where $Re_x$ is the local Reynolds number based on the surface velocity and is given by

$$Re_x = \frac{xU_w(x)}{v}$$

(3.2.21)

### 3.3. NUMERICAL SOLUTION

The above Non linear equations that is (3.2.14) and (3.2.15) are subjected with similarity transformations and the obtained governing equations solved by finite difference scheme kellor box method by gauss elimination method.

The governing equations of the problem are given by

$$f^{'''} + (2 + N_2) f^{''} - 2\left(\frac{Ha^2}{Re}\right) f^{'} + 2\left(\frac{Ha^2}{Re}\right) N_1 e^{-x} f^{'} + 2Gre^{\alpha x^2} e^{-x} \theta = 0$$

$$Pr^{-1} \theta^{'} (1 + \frac{4K}{3}) + f \theta^{'} - a f^{'} \theta + e^{-\frac{x(1-a)}{2}} Ec(2\frac{Ha^2}{Re} f^{''} + f^{'''} e^{-x}) + 2\lambda e^{-x} \theta = 0$$

$$f(0) = 0, f^{'}(0) = 1, \theta(0) = 1 as \eta \to 0$$

$$f^{'}(\infty) = 0, \theta(\infty) = 0 as \eta \to \infty$$

(3.3.1)

In this method the third and second order non-linear differential equations:
FINITE DIFFERENCE SCHEME

THIS SCHEME INVOLVES 5 STEPS

Step1: This is scheme or method involves 5 steps

Step1: Decomposing of given differential equations into a set of first order ordinary differential equations.

Step2: a) Approximate the first order derivatives with standard forward difference

\[
\frac{dy}{dx} \approx \frac{y_i - y_{i-1}}{\Delta x}
\]

b) Approximate the dependent variables with two point averages

\[
y \approx \frac{y_i + y_{i+1}}{2}
\]

Using these approximation the ordinary differential equations is transformed to finite difference equations. Solution, say \(y_i = y_i + \delta y_i\) and substituting this in the finite difference equation and drop terms non-linear in \(\delta y_i\) to arrive at linear F.D.E’s.

Step3: Laniaries F.D.E using Newtons method this involves to start with a guess sealed, or Jacobi method and obtain \(\delta y_i\) and add the correction to initial solution.

Step4: Solve the linearised F.D.E’s using the standard method Gauss elimination.

Step 5. Repeat step 3 & Step 4 until we obtain the required result

3.4. RESULTS AND DISCUSSIONS

Present results, are displayed in table 1 and are noticed to be well in agreement with the present work.
Fig. 2. represents the effect of magnetic field parameter $\frac{Ha^2}{Re}$, on velocity profile $f'$. Here magnetic field produces a drag in the form of Lorentz force. Due to this effect, the magnitude of velocity decreases and the thermal boundary layer thickness increases.

Fig. 3. represents the various values of parameter $a$ with velocity profile. From this figure, it is observed that the value of $a$ increases with increase in the velocity flow and maximum velocity occurs at $a=7$.

Fig. 4. represents the dimensionless parameter $X$ with horizontal velocity profile. From this figure, it is noticed that the value of $X$ increases with decreases in the velocity profile here the flow is adjacent to a stretching sheet.

Fig. 5. It is observed from this figure that temperature decreases with increase in the values of $a$. Further, it is noticed that the thermal boundary layer thickness increases with increase in the value of $a$. For positive value of $a$, heat transfer decreases. Which indicates that, the flow of heat transfer is directed from the wall to the ambient fluid whereas the rate of heat transfer in the boundary layer increases near the wall.

Fig. 6. depicts the temperature profile in the fluid for various values of $\frac{Ha^3}{Re}$, for $a = -2$ and $Gr = 0, 0.5$. It is noticed that an increase in the strength of magnetic field i.e Lorentz force leads to an increase in the temperature far away from the wall, within the thermal boundary layer but the effect of magnetic field near the wall is to decrease the
temperature in the absence of Grashof Number. When the magnetic field increases, the thermal boundary layer thickness increases.

Fig 7. it is noticed that increase in Grashof number, increase in temperature up to certain value of n and suddenly decreases and decays asymptotically to zero. Further it is observed that this increase in temperature is due to the temperature difference between stretched wall and the surrounding fluid. When Grashof number leads to increases, the thermal boundary layer thickness decreases.

Fig.8 represents the temperature profile $\theta(\eta)$ for various values of X along $\eta$ for different values of $a = -1, -2$ and also Grashof number $Gr = 1.0$. It is noticed that the effect of increasing X on $\theta(\eta)$ is more effective for $a = -2$ than compared to the results obtained in the case when $a = -1$. It is interesting to note the behavior of X on $\theta(\eta)$, is that the temperature overshoots near the wall for small value of X, for $a = -2$, whereas the overshoot diminishes when a is enhanced to -1 for all other values of X. It is also observed that the boundary layer thickness decreases with an increase in X.

Fig.9 represents the variation of temperature profiles $\theta(\eta)$ for various values of magnetic field parameter ($Ha^2/Re = 0, 6, 8$) for two values of X. When X increases increases temperature decreases all other fixed values of other involved parameters except when the value of parameter $a = 5$. It is also to be noticed that thermal boundary layer thickness increases as X decreases and the effect of magnetic field is to increase the temperature for both values of X. This is due to the Lorentz force the temperature increases.
Fig.10. represents the effect of Prandtl number Pr on dimensionless heat transfer parameter \( \theta \). It is noticed from this figure that as Prandtl number Pr increases, temperature profile decreases. When Prandtl number Pr is small, heat diffuses quickly compared to the velocity (momentum), especially for liquid metals, (low Prandtl number) the thickness of the thermal boundary layer is much bigger than the momentum boundary layer. Fluids with lower Prandtl number have higher thermal conductivities where. Hence the rate of cooling in conducting flows increases due to the Prandtl number.

Fig.11. represents the effect of porous parameter N1 over velocity profile. Porous parameter increases, velocity decreases. Due to this, the velocity decreases in the boundary layer.

Fig.12. represents the effect of inertia coefficient N2 in the velocity profile. From this we conclude that due to the N2, the thickness of momentum of boundary layer decreases.

Fig.13. depicts the effect of heat source/sink parameter \( \lambda \). It is observed that the boundary layer generates, the energy, which causes the temperature profiles to increase with increasing the values of \( \lambda > 0 \) (heat source) whereas in the case of \( \lambda < 0 \) (absorption) boundary layer absorbs energy resulting in the temperature to fall considerably with decreasing in the value of \( \lambda < 0 \).
Fig. 14. depicts dimensionless temperature field for various values of K, with fixed values of other involved parameters. It is observed from the figure that, K increases, the temperature profiles and the thermal boundary layer thickness also increase.

Fig. 15. effect of porous parameter N1 on a temperature profiles and it is noticed that, temperature increases with the increase of porous parameter, which offers resistance to the flow resulting in the increase of temperature in the boundary layer.

Fig. 16. effect of drag coefficient of porous medium N2. From the figure it is noticed that the effect of drag coefficient is to increase the temperature profile in the boundary layer. Which implies boundary layer thickness also increases.

3.5. CONCLUSIONS

A numerical method has been employed to study MHD boundary-layer flow and heat transfer due to a stretching sheet in the presence of a thermal radiation. The effects of the various governing parameters on the heat transfer characteristic have been examined.

The key observations can be summarized as follows.

1. The thickness of velocity boundary layer will decrease with an increase in the temperature distribution and magnetic parameters.
2. The temperature decreases with an increase in the value of the temperature distribution parameter, magnetic parameter, heat source or sink parameter, and the Prandtl number.
3. The thickness of thermal boundary layer diminishes, with increase in both the temperature distribution and Prandtl number parameters and opposite result is observed for the magnetic parameter.
4. An increase in the temperature distribution parameter will increase both the skin friction coefficients and local Nusslet number.

5. An increase in the magnetic parameter will increase the skin friction.

6. Due to the presence of porous parameter, the thermal boundary layer thickness increases. Effect of drag coefficient also enhances the thermal boundary layer thickness.

Table 1: Values of heat transfer coefficient, \( \theta'(0) \) for various values of \( K \) and \( Ec \) with \( Pr=1.0 \) and all parameters taken as 0.0

<table>
<thead>
<tr>
<th>( K )</th>
<th>( Ec=0.0 )</th>
<th>( Ec=0.5 )</th>
<th>( Ec=1.0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>-1.641723</td>
<td>-0.6609</td>
<td>0.3198</td>
</tr>
<tr>
<td>2.0</td>
<td>-0.57579</td>
<td>-0.29001</td>
<td>-0.00423</td>
</tr>
<tr>
<td>3.0</td>
<td>-0.4714</td>
<td>-0.26390</td>
<td>-0.05638</td>
</tr>
</tbody>
</table>
Fig 2 Effect of magnetic field on velocity profiles with $\eta$.

$\frac{Ha^2}{Re} = Z, a = W, \lambda = L, Gr = 2.0, Pr = 1.0, Ec = 0.1, a = -1.5, X = 1.5, \lambda = 0.1$

$\frac{Ha^2}{Re} = 0, 1, 3, 5, 8$

Fig. 3 Variation of velocity profiles with $\eta$ for various values of $a$.

$a = 1, 3, 4, 5, 6, 7, \lambda = 0.1, X = 1.5, Ha^2/Re = 0.5, Gr = 2.0, Pr = 1.0, Ec = 0.1$
Fig. 4. Variations of velocity profiles with $\eta$ for different values of $X$.

$\frac{\eta}{0} 2 4 6 8 10$

$0.0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1.0$

$X=0.1, 1.0, 2.0, 4.0, 6.0$

$\frac{\eta}{0}$

Fig. 5. Temperature profiles vs. $\eta$ for various values of $a$.

$\frac{\eta}{0} 2 4 6 8 10$

$0.0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1.0 \quad 1.2 \quad 1.4$

$a=-5,-4,-2,-1,0,1,2,5,0$

$\frac{\eta}{0}$
Fig. 6. Temperature profiles vs. $\eta$ for various values of $\text{Ha}^2/\text{Re}$ and $\text{Gr}$.

$\text{Ha}^2/\text{Re}=0, 3, 8$

$\text{Ec}=0.001, \lambda=0.01, X=0.5, \text{Pr}=1.0, a=-2$

--- $\text{Gr}=0$

___ $\text{Gr}=0.5$

$\theta(\eta)$

Fig. 6. Temperature profiles vs. $\eta$ for various values of $\text{Ha}^2/\text{Re}$ and $\text{Gr}$.

--- $\text{Gr}=0$

___ $\text{Gr}=0.5$

$\theta(\eta)$

Fig. 7. Temperature profile for various values of $\text{Gr}$.

--- $\text{X}=0.5, \text{Pr}=1.0, a=-2, \text{Ha}^2/\text{Re}=3, \text{Ec}=0.001, \lambda=0.01$

--- $\text{Gr}=-1.0, -0.5, 0.0, 0.5, 2.0, 5.0$

--- $\text{Gr}=-1.0, -0.5, 0.0, 0.5, 2.0, 5.0$

$\theta(\eta)$

Fig. 7. Temperature profile for various values of $\text{Gr}$.
Fig. 8. Temperature profiles vs. $\eta$ for various values of $a$ and $X$. $X=1.5, 0.5, 0.0$, $Pr=1.0, Gr=1.0, Ha^2/Re=3, Ec=0.001, \lambda=0.01$.

--- $a=1.0$
—— $a=2.0$

Fig. 9. Temperature profiles vs. $\eta$ for various values of $Ha^2/Re$ and $X$ when $a=5$. $Ec=0.001, \lambda=0.01, Pr=1.0, Gr=1.0, a=5$

----- $X=0.1$
—— $X=0.7$

$Ha^2/Re=0.6, 0.8$.
Fig. 10. Variation of temperature with $\eta$ for different values of $Pr$.

$Gr=0.0, Ec=0.001, X=0.5, \lambda=0.01, a=-1.5, Ha^2/Re=5.0$

$N_1=1.0, N_2=1.5$

$Pr=1.0, 2.0, 3.0, 4.0$

Fig. 11. Temperature profile vs. $\eta$ for various values of $N_1$.

$Gr=2.0, Pr=1.0, Ec=0.1, W=-1.5, X=1.5, l=0.1, Z=1.0, N_2=1.5$

$N_1=0.0, 1.0, 3.0, 5.0, 8.0$
Fig. 12. Temperature profile vs. $\eta$ various values of $N_2$.

$N_2=0.0, 1.0, 3.0, 5.0, 8.0$

Gr=2.0, Pr=1.0, Ec=0.1, W=-1.5, X=1.5, L=0.1, Z=1.0, N_1=1.0

Fig. 13. Variation of temperature with $\eta$ for different values of $\lambda$.

Gr=0.0, Ec=0.001, X=0.5, a=-1.5, Ha$^2$/Re=5.0, Pr=1.0, N_1=1.0, N_2=1.5

$\lambda=0.01, 0.02, 0.03, 0.04$
Fig 14: Effects of K on the temperature profiles $\theta(\eta)$, where $K = \frac{4\sigma^* T^3_c}{k' k}$ Radiation number

Fig 15: Effect of N1 on the temperature profiles $\theta(\eta)$
Fig 16: Effect of N2 on the temperature profiles $\theta(\eta)$