An equation is linear if each term contains not more than one variable and each variable appears to the first power. If not, the equation is said to be nonlinear.

When the given equation or a set of equations is nonlinear, it is not possible to obtain the solution in one step. It can be obtained by iterative procedures one of which is known as generalised Newton-Raphson method. These iterative techniques require approximate solution to start the iterations.

Let

\[ \begin{align*}
F(x,y) &= 0 \\
G(x,y) &= 0
\end{align*} \quad \ldots (A1.1) \]

be a system of two nonlinear equations in \( x \) and \( y \) and let \( x_0 \) and \( y_0 \) be the approximate solutions.

Expanding \( F(x,y) \) in Taylor's series and neglecting terms except those linear in \( x \) and \( y \), the following system of equations are obtained:

\[ \begin{align*}
F(x,y) &= F(x_0,y_0) + (x-x_0) \frac{\partial F}{\partial x} + (y-y_0) \frac{\partial F}{\partial y} \\
G(x,y) &= G(x_0,y_0) + (x-x_0) \frac{\partial G}{\partial x} + (y-y_0) \frac{\partial G}{\partial y} \quad \ldots (A1.2)
\]
are equal to zero. But \( F(x, y) \) and \( G(x, y) \) are not equal to zero, since \( x_0, y_0 \) are only approximate solutions. Taking this fact into account and rearranging the terms in the system of Equations (A1.2) leads to

\[
\begin{align*}
\frac{\partial F}{\partial x} x + \frac{\partial F}{\partial y} y &= -F(x_0, y_0) + \frac{\partial F}{\partial x} x_0 + \frac{\partial F}{\partial y} y_0 \\
\frac{\partial G}{\partial x} x + \frac{\partial G}{\partial y} y &= -G(x_0, y_0) + \frac{\partial G}{\partial x} x_0 + \frac{\partial G}{\partial y} y_0 
\end{align*}
\] ...

(A1.3)

In this system, all the partial derivatives are known since they are evaluated for \( x = x_0 \) and \( y = y_0 \), also the right hand sides of both the equations are known.

The two equations in the system of Equations (A1.3) are linear in \( x \) and \( y \), which can be solved directly. The values of \( x \) and \( y \) thus obtained, are the new, hopefully better approximations to the solution of the system of Equations (A1.1). If the new approximations are called \( x_1 \) and \( y_1 \), then the process is repeated with these values of \( x_1 \) and \( y_1 \) replacing \( x_0 \) and \( y_0 \). In this way another new, but better approximation is obtained. The process of iterations will be stopped when the variables at any level satisfy or very nearly satisfy the given system of Equations (A1.1).

The extension of this technique to systems of three or more equations in an equal number of variables is straightforward. Derivation of the algorithm for \( m \) variables requires the solution of a system of \( m \) linear equations. But if the number of variables are not equal to the number of equations, this procedure cannot be applied, since the resulting system of linear equations will be overspecified or underspecified depending on number of variables and number of equations.

In implicit technique of flood routing, one has to solve
The function $F_i$ is given by continuity, whereas $G_i$ is the result of momentum equation. The first and second equations of the system of Equations (A1.4), will contain only three unknowns, namely $y_i$ or $Q_i$, $y_{i+1}$ and $Q_{i+1}$, since $y_1$ or $Q_1$ is known from inflow hydrograph. The last equation of the system will contain only two unknowns $y_N$ and $Q_N$ as a result of downstream boundary condition.

As can be seen from the above equations, each equation of the system of Equations (A1.4) contains a maximum of only four unknowns.

The application of the generalized Newton iteration method to Equations (A1.4) is made by assigning trial values to the unknowns. When these trial values are substituted in Equations (A1.4), the right side of the equations may not vanish but acquire values known as the residuals. Solutions are obtained by adjusting the values until each residual vanishes or...
is reduced to a tolerable quantity. The computations may be organized in a series of iteration steps.

To show the procedure, let it be assumed that the computations have been carried through the kth iteration cycle so that the values of the unknowns have been approximated through the kth cycle. Let the values of the unknowns at the end of the kth cycle be denoted by the superscript \( k \). It is desired to approximate the values of the \((2N-1)\) unknowns through the \((k+1)\)th cycle. When the values of the unknowns approximated through the kth cycle, are substituted in Equations (A1.4), the right sides become the residuals at the end of the kth step iteration. Let the residuals be represented by \( R^k_{1,1} \) and \( R^k_{2,1} \), in which \( R_1 \) is associated with the function \( F \) and \( R_2 \) is associated with the function \( G \). The values of the residuals at the kth iteration cycle for functional forms between sections \( i \) and \( (i+1) \) will be

\[
\begin{align*}
F_i \left( y^k_i, Q^k_i, y^{k+1}_{i+1}, Q^{k+1}_{i+1} \right) &= R^k_{1,1} \\
G_i \left( y^k_i, Q^k_i, y^{k+1}_{i+1}, Q^{k+1}_{i+1} \right) &= R^k_{2,1}
\end{align*}
\]  

... (A1.5)

When these functions are expanded in Taylor's series as done before and terms are grouped, they lead to

\[
\begin{align*}
\frac{\partial F}{\partial y_i} y_i^k + \frac{\partial F}{\partial Q_i} Q_i^k + \frac{\partial F}{\partial y_{i+1}} y_{i+1}^k + \frac{\partial F}{\partial Q_{i+1}} Q_{i+1}^k + R^k_{1,1} = \\
\frac{\partial F}{\partial y_i} y_i^{k+1} + \frac{\partial F}{\partial Q_i} Q_i^{k+1} + \frac{\partial F}{\partial y_{i+1}} y_{i+1}^{k+1} + \frac{\partial F}{\partial Q_{i+1}} Q_{i+1}^{k+1}
\end{align*}
\]
\[
\frac{\partial G}{\partial y_1} y_1^k + \frac{\partial G}{\partial Q_1} Q_1^k + \frac{\partial G}{\partial y_{i+1}} y_{i+1}^k + \frac{\partial G}{\partial Q_{i+1}} Q_{i+1}^k + \delta^k_{2,1} = \\
\frac{\partial G}{\partial y_1} y^{k+1}_1 + \frac{\partial G}{\partial Q_1} Q^{k+1}_1 + \frac{\partial G}{\partial y_{i+1}} y^{k+1}_{i+1} + \frac{\partial G}{\partial Q_{i+1}} Q^{k+1}_{i+1} \quad \ldots \quad \text{(A1.6)}
\]

(or)

\[
\frac{\partial F}{\partial y_1} dy_1 + \frac{\partial F}{\partial Q_1} dQ_1 + \frac{\partial F}{\partial y_{i+1}} dy_{i+1} + \frac{\partial F}{\partial Q_{i+1}} dQ_{i+1} + \delta^k_{1,1} = R_{1,1}
\]

\[
\frac{\partial G}{\partial y_1} dy_1 + \frac{\partial G}{\partial Q_1} dQ_1 + \frac{\partial G}{\partial y_{i+1}} dy_{i+1} + \frac{\partial G}{\partial Q_{i+1}} dQ_{i+1} + \delta^k_{2,1} = R_{2,1} \quad \ldots \quad \text{(A1.7)}
\]

in which \( dy_1 = y^{k+1}_1 - y^k_1 \)

\( dQ_1 = Q^{k+1}_1 - Q^k_1 \)

and partial derivatives are evaluated at the kth iteration cycle. Any standard method such as Gaussian elimination method can be used for the solution of the above equations, which is a system of \((2N-1)\) linear equations in \((2N-1)\) unknowns. The solution of the system will provide values of \( y_1^{k+1} \) and \( Q_1^{k+1} \), which are the unknown values at the \((k+1)\)th iteration cycle. The values of the variables found in the terminal iteration cycle will be taken as the values of the variables for the time step \((j+1)\) and the computations will be advanced to time step \((j+2)\).