CHAPTER 2
METHODS OF FLOOD ROUTING

2.1 UNSTEADY FLOW

The flow of water in a situation where the velocity or depth changes with time is defined as unsteady flow. In natural channels the flow is basically an unsteady one. Unsteady flow can be further divided into (i) gradually varied unsteady flow, and (ii) rapidly varied unsteady flow. Flood waves are gradually varied unsteady flows. There are two equations which govern the unsteady flow in open channels, which were first published by Saint-Venant [1]. They are the equations of continuity and momentum which are respectively based on the conservation of mass and energy.

The continuity equation is expressed as

\[ D \frac{\partial V}{\partial x} + V \frac{\partial D}{\partial x} + \frac{\partial y}{\partial t} = 0 \] ... (2.1)

The momentum equation is given as:

\[ \frac{\partial V}{\partial x} + \frac{V}{g} \frac{\partial y}{\partial x} + \frac{1}{g} \frac{\partial V}{\partial t} = \frac{\partial z}{\partial x} - S_f \] ... (2.2)

where, D is hydraulic mean depth
V is mean velocity of flow
y is the depth of flow
g is acceleration due to gravity
z is level of the bed of the channel from an arbitrary datum
S_f is the friction slope
x is the distance along the channel, and
t is time.

If the channel is prismatic, \( \frac{\partial z}{\partial x} \) can be replaced by \( S_o \), the bed slope of the channel.
The derivation of unsteady flow equations is given in standard reference works, for instance Thomas [4], Gilcrest [5], Stoker [6], Chow [7], Henderson [8], Fang [9], Yevjevich and Barnes [10] and Ligget [11], and is based on the following assumptions:

a. The wave surface gradually varies which is equivalent to stating that the vertical pressure distribution is hydrostatic or that the vertical acceleration is small.

b. Velocity distribution across wetted area does not substantially affect the wave propagation.

c. The wave movement can be considered as two dimensional.

d. The average bed slope of the channel is so small that \( \sin \alpha \) may be replaced by \( \tan \alpha \) and \( \cos \alpha \) by unity, where \( \alpha \) is the angle made by the channel bottom with the horizontal.

These two equations contain two independent variables \( x \) and \( t \) and two basic dependent variables \( V \) and \( y \). The rest of the parameters are constant, such as \( g \) and \( S_0 \) or functions of dependent variables \( V \) and \( y \) such as \( D \) and \( S_1 \). Fundamentally, dependent variables are one representing the flow \( (V) \) and the other representing the cross-section \( (y) \). Consequently, they may be considered in different forms such as the rate of flow \( (Q) \) and the area of cross-section \( (A) \). Hence, the basic equations can be expressed mathematically in different forms, though the basic terms constituting the equations will remain the same.

The first two terms \( (D \frac{\partial V}{\partial x} \) and \( V \frac{\partial y}{\partial x}) \) in the continuity equation represent the change in storage due to change in velocity of flow with the distance. The third term \( (\frac{\partial y}{\partial t}) \) represents the change in storage due to change in depth of
flow or cross-section with time. The terms in the momentum equation are grouped into two categories, viz., the acceleration terms and the force terms. The acceleration terms are $\frac{\partial V}{\partial t}$ and $V \frac{\partial V}{\partial x}$. They are respectively local (due to time variation) and convective (due to spatial variation) accelerations. The force terms are $\frac{\partial y}{\partial x}$, $\frac{\partial z}{\partial x}$, or $S_o$ and $S_f$. They are respectively change in pressure force, gravity body force due to bed slope of the channel and the frictional force effects.

2.1.1 Uniformity progressive flow

One particular form of unsteady flow that could be simulated in prismatic channels is the uniformly progressive flow. It has a stable wave profile that will not change in shape as it moves downstream. In this flow successive positions of wave fronts at different times are parallel. The celerity is greater than the mean water velocity of the wave. The wave travels with constant velocity but the mean water velocity in the cross-section may vary from section to section as the hydraulic radius and surface slope change.

The monoclinal rising wave, presented in Figure 2.1 is a case of uniformly progressive flow. This type of wave can be approximated to flood waves in natural channels. The wave can also be subjected to simple mathematical treatment. Hence it is of interest in flood routing. Here, it is possible to express both the acceleration terms $\frac{\partial V}{\partial t}$ and $V \frac{\partial V}{\partial x}$ as functions of change in pressure force.

![Figure 2.1 Monoclinal rising wave](image-url)
\[ \frac{\partial y}{\partial x} \]. When these relationships are substituted in Equation (2.2), the momentum equation is simplified to

\[ \frac{\partial y}{\partial x} = \frac{S_0 - (V_y A - Q_0^2)/K^2}{1 - Q_0^2/gA^2} \] (2.3)

where \( Q_0 \) is the overrun, which is given by

\[ Q_0 = (V_w - V)A \] (2.4)

where \( V_w \) is the celerity of the monoclinal wave and \( K \) is the conveyance of the section given by

\[ K = Q \sqrt{S_0} \] (2.5)

in which \( Q \) is the flow through the channel.

If the channel is further simplified to a wide rectangle and Chezy's formula is used to evaluate \( S_0 \), Equation (2.3) with partial derivative replaced by full derivative, becomes

\[ \frac{dy}{dx} = \frac{S_0}{3} \left( \frac{y^3 - (V_y y - Q_0^2)/C^2 S_0}{y^3 - Q_0^2/g} \right) \] (2.6)

in which \( C \) is the Chezy's constant. Equation (2.6) can be integrated analytically leading to the shape of monoclinal wave profile in a wide rectangular channel.

### 2.2 APPROXIMATIONS TO BASIC EQUATIONS

The Saint-Venant equations have no exact analytical solution relevant to a flood wave propagation in natural channels. It is true that a monoclinal wave can be analytically solved, but it would amount to assuming a uniform wide channel; see Chow [7], Henderson [8], Price [12] and Karmegam [14]. In the absence of a more general exact solution, it may be necessary to approximate the basic equations and obtain simplified solutions. Such approximations can also be justified because some of the terms in Equation (2.2) may be less important and less influential than others for a particular situation.
The relative importance of the terms in the momentum equation were studied in the Flood Studies Report [13] and by Karmegam [14] who based their studies on the order of magnitude analysis as propounded by Henderson [8] [15]. They concluded that

i. The momentum of flow in the river is governed primarily by the bed and friction slopes and is modified by the water surface slope, \( \frac{\partial y}{\partial x} \), which is defined relative to the bed slope of the channel.

ii. The acceleration and the convection of momentum terms can be ignored except possibly in flows in steep rivers and in local variations such as flow through bridges, weirs and other obstructions in the river.

iii. The contribution to the momentum in the main channel from the tributaries and lateral inflow can be ignored.

iv. The lateral inflow from small tributaries and direct runoff can be significant under snow melt conditions but in general its effect is small.

It should be possible to deduce the relative importance of the terms of the basic equations for any river flow situation which will prove useful in obtaining satisfactory solution with approximate equations.

2.3 FLOOD WAVE PROPAGATION

The movement of flood wave along the reach is subject to the balance of various forces included in the momentum equation. Due to the combined effect of several physical and hydraulic factors, the shape of the flood wave will be undergoing constant change as the flood proceeds downstream. These factors include rate and height of rise, length and crest of the hydrograph, rate of fall, slope of the channel, stages and channel section downstream, length and storage.
of the reach and the lateral inflow due to tributaries etc. For an enumeration and explanation of these factors with respect to flood wave propagation reference can be made to 'Flood Routing' [16], Hickman [17], Linsley et al. [18], Henderson [8] and Wormleaton [19].

Natural channels exhibit variations in their physical properties and so propagation of flood wave in natural channels is more complex than in a prismatic channel. A river may vary erratically in cross-section, roughness and slope with distance downstream. The direct influence of the physical properties is to increase the storage of the channel. Also, the lateral inflow or abstraction may change the wave profile. Meanders present in natural channels also will influence wave propagation.

2.3.1 Storage and reservoir action

Natural channels tend to widen out at top and may have wide flood plains. Hence, for the same increase in depth, the increase in storage in a natural channel will be much more than in a prismatic channel, especially when in high flows, see Figure 2.2. When the channel storage is large, the channel tends to act like a reservoir, modifying the flood wave by reservoir action.

In a reservoir, the velocity of flow is almost zero due to its depth and width but an increase in flow is transmitted across the surface in a series of surges. This increases the depth of water and hence the head at the outfall. Thus the outflow will reflect the

![Figure 2.2 Comparison of increase in storages](image-url)
variations in inflow. It follows that while inflow exceeds outflow the level in the reservoir rises increasing the outflow. Hence the maximum outflow and storage will occur when the inflow and outflow are equal as shown in Figure 2.3. The effect of reservoir, as seen here, is to retard and attenuate the flood wave considerably. Though a deep channel with a large flood plain may transmit the flood wave essentially by reservoir action, many channels are such that, when a flood occurs, the wave is transmitted partly by reservoir action and partly by simple translation. However, in the case of a wave in regular prismatic channel, the storage has little or no effect in flood wave modification.

2.3.2 Lateral inflow into the channel

The flood wave, in natural channels, as it proceeds downstream, is constantly being added to and modified by lateral inflow from tributaries.
and other overland flow. The lateral inflow itself, except in the case of large tributaries, is mostly ungauged and so has to be estimated in a flood routing problem. A fairly accurate estimate may require the catchment characteristics, like its size, shape, slope, soil, vegetation etc. Also the rainfall distribution on the catchment of the tributary must be known for the flood period. Thus the estimation of ungauged local inflow may turn out to be a major problem in flood routing. In most cases, the lateral inflow is estimated by simplified means.

2.4 UNSTEADY FLOW EQUATIONS AND WAVE MODELS

Henderson [8] presents the following chart which clearly demonstrates how non-uniformity and unsteadiness introduce extra terms into the dynamic equation.

\[ S_f = S_o - \frac{dV}{dx} - \frac{dV}{dt} \]

Steady uniform flow

Steady non-uniform flow

Unsteady non-uniform flow

On the basis of relative importance of the terms in the dynamic equation and following the works of Gunaratnam and Perkins [20], Flood Studies Report [13] and Ponce and Simons [21], Karmegam [14] has formulated different wave models to solve the Saint-Venant equations analytically or otherwise for a flood wave.

It is necessary to note that under all conditions of flow the continuity equation has to be satisfied. If the storage effects completely control, as in the cases of reservoirs and under certain conditions of natural channels, the dynamic equation can be neglected, resulting in the classical storage routing model. If friction is considered important, the approximation leads to the kinematic wave model. If the
pressure term is used along with friction a diffusion model results. Incorporation the full dynamic equation gives the complete solution with a dynamic wave model. These mathematical models can be summarized in a chart as below.

Continuity equation

\[ \partial_x \frac{\partial V}{\partial x} + \partial_x \frac{\partial V}{\partial t} + \partial_x \frac{\partial V}{\partial t} = 0 \]

Storage routing model

Momentum equation

\[ \frac{1}{g} \frac{\partial V}{\partial t} + \frac{V}{g} \frac{\partial V}{\partial x} + \frac{\partial V}{\partial x} - \frac{S_o}{g} + \frac{S_f}{g} = 0 \]

+ Continuity

Kinematic wave model

+ Continuity

Diffusion wave model

+ Continuity

Dynamic wave model

The above chart read together with Henderson's chart clearly brings out the approximations and the presence of unsteadiness and non-uniformity, if any, associated with the model.