APPENDIX 7

REACTIVE-ONLY OPTIMIZATION IN AC-DC SYSTEM - SENSITIVITY RELATIONS BETWEEN DEPENDENT AND CONTROL VARIABLES AND BETWEEN TRANSMISSION LOSS AND CONTROL VARIABLES

Increments in $I_{d,m}$, $V_d'$ and $P_d$ are given by the equations (A4.1)-(A4.3) of Appendix 4.

Let the matrices $[S^j]_m$ represent sensitivity matrices related to Q-optimization. Equation (A4.3) is written as

$$A*d * tS64"] Ald' + *d
AVdm (A7.1)$$

where $S64" = S82'$

From equations (4.41), (A4.1) and (A4.2), $AQ_d$ can be shown to be

$$AQ_d = [S92']_d I_d' + [S93']_d AVdm - [D2] Δcos θ (A7.2)$$

where the matrices $S92'$ and $S93'$ are the same as those used in Appendix 4, equation (A4.4).

A7.1 INCREMENTS IN POWER FACTORS

From (4.41), (A4.1) and (A4.3)

$$Δcos θ = [S74"]_d I_d' + [S75"]_d AVdm + [S76"]_d Δcos θ (A7.3)$$
A7.2 INCREMENTS IN LOAD BUS VOLTAGES

Since by assumption, $\Delta x_p = 0$ and real and reactive demands are constants, equation (4.40) is written as

$$
\begin{bmatrix}
\Delta F_s \\
\Delta F_N \\
\Delta F_G \\
\Delta F_L \\
\end{bmatrix} =
\begin{bmatrix}
\Delta P_s \\
\Delta P_N \\
\Delta Q_G \\
\Delta Q_L \\
\end{bmatrix} +
\begin{bmatrix}
\Delta P_{ds} \\
\Delta P_{dN} \\
\Delta Q_{dg} \\
\Delta Q_{dL} \\
\end{bmatrix} -
\begin{bmatrix}
\Delta P_G \\
\Delta Q_G \\
\Delta Q_G \\
\Delta Q_G \\
\end{bmatrix} = 0
$$

(A7.4)

where subscripts $s, N, G, L$ and increments $\Delta F_{dN}, \Delta Q_{dg}$ and $\Delta Q_{dL}$ are defined in the same manner as in Section A4.1.

Referring to equations (2.3) and (2.4), $P_k$ and $Q_k$ are functions of only ac system variables. Hence equation (A7.4) is written as

$$
\begin{bmatrix}
\frac{\partial P_s}{\partial \delta} & \frac{\partial P_s}{\partial V_G} & \frac{\partial P_s}{\partial V_L} & \frac{\partial P_s}{\partial T} \\
\frac{\partial P_N}{\partial \delta} & \frac{\partial P_N}{\partial V_G} & \frac{\partial P_N}{\partial V_L} & \frac{\partial P_N}{\partial T} \\
\frac{\partial Q_G}{\partial \delta} & \frac{\partial Q_G}{\partial V_G} & \frac{\partial Q_G}{\partial V_L} & \frac{\partial Q_G}{\partial T} \\
\frac{\partial Q_L}{\partial \delta} & \frac{\partial Q_L}{\partial V_G} & \frac{\partial Q_L}{\partial V_L} & \frac{\partial Q_L}{\partial T} \\
\end{bmatrix} \Delta \delta \\
\begin{bmatrix}
\Delta P_{ds} \\
\Delta P_{dN} \\
\Delta Q_{dg} \\
\Delta Q_{dL} \\
\end{bmatrix} \\
\begin{bmatrix}
\Delta P_G \\
\Delta Q_G \\
\Delta Q_G \\
\Delta Q_G \\
\end{bmatrix} = 0
$$

(A7.5)

As in Section (A4.1), $[J_{RC}]$ represent the partial derivative submatrices with the row subscript $R$ having the same meaning as in equation (A4.6). The column subscript $C$ can be $\delta, G, L$ or $T$ representing $\delta, V_G, V_L$ or $T$ respectively.
Assuming $[J_{LL}'] = 0$ in the last equation in (A7.5)

$$[J_{LL}'] \Delta V_L = -[J_{LG}'] \Delta V_G - [J_{LT}'] \Delta T + \frac{1}{V_L} \Delta Q_{GL} - \frac{1}{V_L} \Delta Q_{dL}$$

where $\frac{1}{V_R}$

By using the decoupling assumptions of FDPF method [14],

$$[J_{LL}'] = B''.$$

Defining [KA] $\Delta Q_s = \frac{1}{V_L} \Delta Q_{GL}$ and

[KB] $\Delta Q_d = \frac{1}{V_L} \Delta Q_{dL}$

using equation (A7.2),

$$\Delta V_L = [s_{11}'' s_{12}'' s_{13}'' s_{14}'' s_{15}'' s_{16}''] \Delta X_Q \quad (A7.6)$$

The sensitivity submatrices in (A7.6) involve inversion of $B''$ as shown below for one submatrix $s_{11}''$.

$$[s_{11}''] = [B'']^{-1} [-J_{LG}']$$

A7.3 INCREMENTS IN CONVERTER TRANSFORMER TAPS

From equations (4.41), (A4.1) and (A4.2),

$$\Delta a = [s_a] \Delta I_d' + [s_b] \Delta V_{dm} - [d_1] \Delta V_t - [d_3] \Delta \cos \theta$$
Writing $\Delta V_t = [D_{G1}] \Delta V_G + [D_{L1}] \Delta V_L$ and substituting $\Delta V_L$ using equation (A7.6).

$$\Delta a = [S_{51}'' S_{52}'' S_{53}'' S_{54}'' S_{55}'' S_{56}'''] \Delta x_Q \quad (A7.7)$$

### A7.4 INCREMENTS IN REACTIVE POWERS OF GENERATORS

From the third equation in (A7.5) and assuming $[J_{G\delta}'] = 0$,

$$\Delta Q_G = [J_{GG}] \Delta V_G + [J_{GL}] \Delta V_L + [J_{GT}] \Delta T + \Delta Q_{dg}$$

Writing $\Delta Q_{dg}$ as $[Kc] \Delta Q_d$ and substituting $\Delta Q_d$ and $\Delta V_L$ using equations (A7.2) and (A7.6),

$$\Delta Q_G = [S_{21}'' S_{22}'' S_{23}'' S_{24}'' S_{25}'' S_{26}'''] \Delta x_Q \quad (A7.8)$$

### A7.5 SENSITIVITY RELATION BETWEEN $V$, $\Delta \delta$ AND $\Delta x_Q$

The second equation in (A7.5) is written as

$$[J_{NS'}'] \Delta \delta = -[J_{NG'}] \Delta V_G - [J_{NL'}] \Delta V_L - [J_{NT'}] \Delta T - \frac{1}{V_N} \Delta P_{dN}$$

where $[J_{NC'}] = \frac{1}{V_N} [J_{NC}]$

As explained in Section A4.4,

$$[J_{NS'}'] \Delta \delta = [B'''] (V, \Delta \delta)$$

Writing $\frac{1}{V_N} \Delta P_{dN}$ as $[Kd] \Delta P_d$ and substituting $\Delta P_d$ and $\Delta V_L$
using equations (A4.3) and (A7.6).

\[ \mathbf{v} \Delta \delta = \begin{bmatrix} g_{e1} & g_{e2} & g_{e3} & g_{e4} & g_{e5} & g_{e6} \end{bmatrix}^t \Delta \mathbf{X}_Q \]

\[ = [g_e]^t \Delta \mathbf{X}_Q \quad \text{(A7.9)} \]

The sensitivity submatrices in (A7.9) involve inversion of \( B^{-1} \) as shown below for one submatrix \( S_{e2} \).

\[ [S_{e2}] = [B^{-1}]^{-1} \left[ [-J_W] \right] [S_{12}] \]

### A7.6 INCREMENT IN TRANSMISSION LOSS

Total transmission loss is given by

\[ P_L = P_{La} + P_{Ld} = \sum_{k=1}^{n} P_k + \sum_{j=1}^{m} P_{dj} \]

where \( P_{La} \) and \( P_{Ld} \) are losses in ac and dc networks respectively.

\[ \Delta P_L = \left( \frac{\partial P_{La}}{\partial V_g} \right)^t \Delta V_g + \left( \frac{\partial P_{La}}{\partial V_L} \right)^t \Delta V_L + \left( \frac{\partial P_{La}}{\partial T} \right)^t \Delta T \]

\[ + \left( \frac{\partial P_{La}}{\partial \delta} \right)^t \frac{1}{V} \mathbf{v} \cdot \Delta \delta + [e^t 1] \Delta P_d \]

Substituting \( \Delta V_L \), \( \mathbf{v} \cdot \Delta \delta \) and \( \Delta P_d \) from (A7.6), (A7.9) and (A4.3),

\[ \Delta P_L = c_Q^t \Delta \mathbf{X}_Q \quad \text{(A7.10)} \]