APPENDIX 4

REAL-ONLY OPTIMIZATION IN AC-DC SYSTEM - SENSITIVITY RELATIONS BETWEEN DEPENDENT AND CONTROL VARIABLES

From equations (3.57) and (3.56),

\[ \Delta I_{dm} = - e^t \Delta I_d' \]  \hspace{1cm} (A4.1)

\[ \Delta V_d' = [R] \Delta I_d' + e \Delta V_{dm} \]  \hspace{1cm} (A4.2)

From equation (3.55), increments in \( P_d \) and \( Q_d \) can be expressed as

\[ \Delta P_d = [S_{82}'] \Delta I_d' + I_d \Delta V_{dm} \]  \hspace{1cm} (A4.3)

\[ \Delta Q_d = [S_{92}'] \Delta I_d' + [S_{93}'] \Delta V_{dm} \]  \hspace{1cm} (A4.4)

where \([S_{ij}']\) denote sensitivity matrices related to \( P \)-optimization.

A4.1 INCREMENTS IN LOAD BUS VOLTAGES

Since real and reactive power demands are assumed to be constants, \( \Delta P_D = 0, \Delta Q_D = 0 \).

Equation (3.54) is written as

\[
\begin{bmatrix}
\Delta P_S \\
\Delta P_N \\
\Delta Q_G \\
\Delta Q_L
\end{bmatrix} + \begin{bmatrix}
\Delta P_{ds} \\
\Delta P_{dn} \\
\Delta Q_{dg} \\
\Delta Q_{dl}
\end{bmatrix} - \begin{bmatrix}
\Delta P_{Gs} \\
\Delta P_{Gn} \\
\Delta Q_{Gg} \\
0
\end{bmatrix} = 0
\]  \hspace{1cm} (A4.5)
where subscripts s, N, G and L represent respectively the slack bus, all buses other than the slack bus, generator buses and load buses. The vector $\Delta P_{dN}$ contains components of $\Delta P_d$ and zero elements corresponding to buses without converters. The vectors $\Delta Q_{dG}$ and $\Delta Q_{dL}$ contain components of $\Delta Q_d$ and zero elements corresponding to buses without converters. The vector $\Delta P_{GN}$ contains all components of $\Delta PG$ and zero elements corresponding to buses without generators. Equation (A4.5) is written as

$$
\begin{bmatrix}
\Delta P_S^{t} & \Delta P_N \\
\Delta Q_S & \Delta Q_N \\
\Delta Q_G & \Delta Q_L \\
\Delta V_L & \Delta V_L \\
\end{bmatrix}
\begin{bmatrix}
\Delta P_{ds} \\
\Delta P_{dN} \\
\Delta Q_{dG} \\
\Delta Q_{dL} \\
\Delta V_L \\
\end{bmatrix}
+ 
\begin{bmatrix}
\Delta P_{dS} \\
\Delta P_{dN} \\
\Delta Q_{dG} \\
\Delta Q_{dL} \\
\Delta V_L \\
\end{bmatrix}
= 0 \quad (A4.6)
$$

Let the above partial derivative submatrices be denoted by $J_{RC}$ where the row subscript R can be s, N, G or L representing respectively slack bus real power equation, all other real power equations, generator reactive power equations or load reactive power equations and the column subscript C can be $\delta$ or L, representing respectively $\delta$ or $V_L$.

Assuming $J_{L\delta} = 0$, in the fourth equation in (A4.6),

$$[J_{LL}] \Delta V_L = - \Delta Q_{dL}$$
Dividing by the corresponding load bus voltages,

\[
\frac{1}{V_L} J_{LL} \Delta V_L = -\left(\frac{1}{V_L}\right) \Delta Q_{dl}
\]  (A4.7)

Making use of the decoupling assumptions of the Fast Decoupled Power Flow (FDPF) method [14],

\[
\frac{1}{V_L} \frac{\partial Q_L}{\partial V_L} = B'' \quad \text{and}
\]

\[
[B''] \Delta V_L = -\left(\frac{1}{V_L}\right) \Delta Q_{dl}
\]

Defining \([K_1]\) such that

\[
[K_1] \Delta Q_d = \frac{1}{V_L} \Delta Q_{dl}
\]

\[
[B''] \Delta V_L = -[K_1] \Delta Q_d
\]

\[
\Delta V_L = -[B'']^{-1} [K_1] \Delta Q_d
\]  (A4.8)

Substituting \(\Delta Q_d\) from (A4.4),

\[
\Delta V_L = [S_{12}' S_{13}'] \begin{bmatrix} \Delta I_d' \\ \Delta V_{dm} \end{bmatrix}
\]  (A4.9)

where

\[
[S_{12}'] = -[B'']^{-1} [K_1] [S_{92}']
\]  (A4.10)

\[
[S_{13}'] = -[B'']^{-1} [K_1] [S_{93}']
\]  (A4.11)
A4.2 INCREMENTS IN CONVERTER TRANSFORMER TAPS

From (3.55), (A4.1) and (A4.2), it can be shown that

\[
\Delta a = [s_a] \Delta I_d' + [s_b] \Delta V_{dm} - [D_1] \Delta V_t \tag{A4.12}
\]

where \([D_1]\) is a diagonal matrix.

Writing \([D_1] \Delta V_t\) as \([D_L] \Delta V_L\) and since \(\Delta V_G = 0\),

\[
\Delta a = [s_a] \Delta I_d' + [s_b] \Delta V_{dm} - [D_L] \Delta V_L
\]

Using equation (A4.9),

\[
\Delta a = [s_{72}' s_{73}'] \begin{bmatrix} \Delta I_d' \\ \Delta V_{dm} \end{bmatrix} \tag{A4.13}
\]

A4.3 INCREMENTS IN GENERATOR REACTIVE POWERS

Assuming \([J_G] = \frac{3Q_G}{3s} = 0\) in the third equation in (A4.6),

\[
\Delta Q_G = [J_G] \Delta V_L + \Delta Q_d
\]

Writing \(\Delta Q_d\) as \([K_2] \Delta Q_d\) and substituting \(\Delta V_L\) and \(\Delta Q_d\) using equations (A4.9) and (A4.4),

\[
\Delta Q_G = [s_{22}' s_{23}'] \begin{bmatrix} \Delta I_d' \\ \Delta V_{dm} \end{bmatrix} \tag{A4.14}
\]
A4.4 \( V \Delta \delta \) IN TERMS OF \( \Delta X_P \)

Dividing the second equation in (A4.6) by the corresponding bus voltages, we get

\[
\begin{align*}
[J_{N\delta}] \Delta \delta + [J_{NL}] \Delta V_L &= \frac{1}{V_N} \Delta P_{GN} - \frac{1}{V_N} \Delta P_{dN} \\
 where \quad [J_{N\delta}] &= \frac{1}{V_N} \cdot \frac{2P_N}{\delta} \\
[J_{NL}] &= \frac{1}{V_N} \cdot \frac{2P_N}{\delta V_L} \\
\end{align*}
\]

Writing \( \frac{1}{V_N} \Delta P_{GN} \) as \([W_1] \Delta P_G\)

and \( \frac{1}{V_N} \Delta P_{dN} \) as \([W_2] \Delta P_d\)

\[
[J_{N\delta}] \Delta \delta + [J_{NL}] \Delta V_L = [W_1] \Delta P_G - [W_2] \Delta P_d \quad (A4.15)
\]

Using the decoupling assumptions of FDPF method [14],

\[
[J_{N\delta}] \Delta \delta = [B^{'''}] (V \cdot \Delta \delta)
\]

where \([B^{'''}]\) is the negated bus susceptance matrix excluding the slack bus. Substituting \( \Delta P_d \) and \( \Delta V_L \) in (A4.15) using equations (A4.3) and (A4.9),

\[
V \cdot \Delta \delta = [s_{d1} \quad s_{d2} \quad s_{d3}] \Delta X_P = [s_d] \Delta X_P \quad (A4.16)
\]
where \( [S_{d1}] = [B^{n+1}]^{-1} [W_1] \). (A4.17)

\[ [S_{d2}] = [B^{n+1}]^{-1} [-[J_{NL}'] [S_{12}'] - [W_2] [S_{82}']] \] (A4.18)

\[ [S_{d3}] = [B^{n+1}]^{-1} [-[J_{NL}'] [S_{13}'] - [W_2] I_d] \] (A4.19)

**A4.5 INCREASES IN SLACK BUS REAL POWER GENERATION AND LINE PHASE ANGLES**

From the first equation in (A4.6),

\[
\Delta P_G = \left[ \frac{\partial P_S}{\partial \delta} \cdot \frac{1}{V} \right]^t (V. \Delta \delta) + \left[ \frac{\partial P_S}{\partial V_L} \right]^t \Delta V_L + h^t \Delta P_d
\]

where \( h^t \Delta P_d = \Delta P_{ds} \)

Substituting \( V, \Delta \delta, \Delta V_L \) and \( \Delta P_d \) using (A4.16), (A4.9) and (A4.3),

\[
\Delta P_G = [S_{31}' S_{32}' S_{33}'] \Delta X_P
\] (A4.20)

As in equation (3.22), increments in phase differences across lines is written using a matrix \([M]\) as,

\[
\Delta \Psi = [M] (V. \Delta \delta)
\]

Substituting \( V, \Delta \delta \) using equation (A4.16),

\[
\Delta \Psi = (S_{41}' S_{42}' S_{43}') \Delta X_P
\] (A4.21)