CHAPTER 3

COST MINIMIZATION IN AC AND AC-DC SYSTEMS THROUGH REAL POWER OPTIMIZATION

3.1 INTRODUCTION

A major goal in the operation of today's power systems is to have maximum economy of operation and to provide a reliable and quality supply to the consumers. The system quantities that are adjusted in an ac power system for establishing such optimal operating conditions are real power outputs of generators, voltage magnitudes of generators and synchronous condensers, tap ratios of OLTC transformers, reactive powers of switchable VAR sources. Among these control variables, only the real power generation has a major influence on the real power flow and cost of operation and the rest of the variables influence mainly the reactive power flow and voltage levels. Hence for fast computation of optimal power flow solution for cost minimization one can consider only the real power generations and the corresponding constraints on real power generation at slack bus and on real power flows over transmission lines. This sub-optimal power flow problem is called Real-only OPF and is especially useful for security-constrained optimization involving numerous line flow security constraints corresponding to the postulated contingency states of the system.

A few methods using SLP approach for Real-only OPF solutions in ac systems are available in the literature. One of these works [12] uses a decoupled Jacobian to solve a
Real-only security constrained OPF problem. This method requires the computation and triangulation of the base case Jacobian matrix at each LP formulation. To overcome the oscillatory convergence of the method, 'interpolation' was used, but the method was not successful in reaching the exact optimal solution.

The only work that uses constant matrix of FDPF method for obtaining LP model in the SLP based OPF method is that of reference [13] which is also a security constrained Real-only OPF method. To arrest oscillatory convergence, this method adopts a 'variable bounding and interpolation' strategy which increases the number of LP moves and reduces much of the computational efficiency that are obtained by the use of constant matrices. For a 30-bus system the method has taken seven LP moves and six interpolation power flow moves to obtain the optimal solution.

For obtaining the Real-only OPF solution in ac-dc systems, it is necessary to include the additional real power controls and constraints of the dc subsystem. So far, no LP based OPF methods have been developed for these ac-dc systems.

This Chapter deals with the development of SLP based Real-only OPF model using constant matrices for both ac and ac-dc systems and proposes an efficient method of solution for achieving reliable convergence. The OPF method developed is tested for its effectiveness on ac test systems and on an integrated ac-dc system and the results obtained are compared with those reported in earlier works.
3.2 STATEMENT OF REAL-ONLY OPTIMAL POWER FLOW IN AC SYSTEM

The Real-only optimization (P-optimization) of ac power system without dc lines is first considered. This Real-only OPF problem is a nonlinear optimization problem where the total generation cost is minimized by adjusting the real power control variables subject to various equality and inequality constraints. The vector of real power control variables (P-control variables) $X_p$, is the vector $PG'$ of controllable real power generations. Assuming that the line flow constraints are expressed in terms of line phase angle limits, the vector of dependent variables $Y_p$ to be constrained within suitable operating limits during optimization comprises $PG_s$, the slack bus real power generation and $\psi$, the vector of line phase angles.

i.e. $Y_p = (PG_s, \psi)^t$

The limits on $X_p$ constitute the control constraints and those on $Y_p$ constitute the operating constraints of the problem. The equality constraints are the power flow equations of the ac system. Since reactive power flow is not significantly modified while adjusting real power generations, bus voltage magnitudes and reactive generations are assumed to be within the permissible limits. It is further assumed that the cost of generation of a generating unit is a quadratic function of its real power generation.

With these assumptions, the Real-only OPF problem is stated as the following Nonlinear Programming (NLP) problem:
Determine : $X_p$

to minimize: the total generation cost

$$f = \sum_{i=1}^{NC} (c_{1i} P_{G_i}^2 + c_{2i} P_{G_i} + c_{3i})$$
$$+ (c_{1s} P_{G_S}^2 + c_{2s} P_{G_S} + c_{3s})$$

subject to: the control constraints

$$x_{P_{\text{min}}} \leq x_p \leq x_{P_{\text{max}}}
$$

the operating constraints

$$y_{P_{\text{min}}} \leq y_p \leq y_{P_{\text{max}}}
$$

and the power flow constraints

$$F(P_G, \delta) = 0
$$

The power flow constraints (3.4) are the real and reactive power equations given below.

$$F_k = P_k - (P_{G_k} - P_{D_k}) = 0
$$

and

$$F_{k+n} = Q_k - (Q_{G_k} - Q_{D_k}) = 0
$$

$k=1,2,...,n$

where $P_k$ and $Q_k$ are given by the equations (2.3) and (2.4).

3.3 DEVELOPMENT OF LP MODEL

For solving the NLP problem stated in equations (3.1) to (3.4), the SLP approach is adopted. The LP model is obtained by linearizing the objective function and the control, operating and power flow constraints around an
operating state. The solution of this LP problem is used to update the control vector and obtain the new system state which is an improvement over the previous state with regard to minimization of generation cost and enforcement of constraints. The LP formulation, its solution and updating of the control variables are repeated at the second and subsequent states till the optimal solution is reached.

Linearizing the objective function (3.1) the increment in generation cost is written as

\[ \Delta f = \sum_{i=1}^{NC} (2c_{1i} \Delta P_{G_i} + c_{2i}) \Delta P_{G_i} \]

\[ + (2c_{1s} \Delta P_{G_s} + c_{2s}) \Delta P_{G_s} \]  

(3.7)

The control constraints to be enforced during optimization are written as

\[ x_{P_{\min}} \leq (x_P^0 + \Delta x_P) \leq x_{P_{\max}} \]

\[ (x_{P_{\min}} - x_P^0) \leq \Delta x_P \leq (x_{P_{\max}} - x_P^0) \]  

(3.8)

where superscript '0' denotes the operating state and 'min' and 'max' denote minimum and maximum values respectively.

In a similar manner, the linearized constraints on the dependent variables are obtained as

\[ (y_{P_{\min}} - y_P^0) \leq \Delta y_P \leq (y_{P_{\max}} - y_P^0) \]  

(3.9)

From the first order approximation of the Taylor's series expansion of the power flow equation (3.4)
which is written as

$$\Delta f = 0$$

The approximate LP model is given by:

Determine : $$\Delta x_p = \Delta p g'$$

to minimize : $$\Delta f = \sum_{i=1}^{NC} (2c_{1i} p g_i + c_{2i}) \Delta p g_i$$

$$+ (2 c_{1s} p g_s + c_{2s}) \Delta p g_s$$ \hspace{1cm} (3.10)

subject to : $$\Delta x_{p_{min}} \leq \Delta x_p \leq \Delta x_{p_{max}}$$ \hspace{1cm} (3.11)

$$(y_{p_{min}} - y_p^o) = \Delta y_{p_{min}} \leq \Delta y_p \leq \Delta y_{p_{max}} = (y_{p_{max}} - y_p^o)$$ \hspace{1cm} (3.12)

and $$\Delta f = 0$$ \hspace{1cm} (3.13)

The dimension of the above LP model is reduced by expressing the dependent variables $$\Delta p g_s$$ and $$\Delta y_p$$ in equations (3.10) and (3.12) in terms of the control vector $$\Delta x_p$$ alone. The necessary sensitivity relations between $$\Delta y_p$$ and $$\Delta x_p$$ are derived making use of equation (3.13).

Referring to equations (3.5) and (3.6), equation (3.13) in expanded form is
\[
\begin{bmatrix}
\Delta P_s \\
\Delta P_N \\
\Delta P_G \\
\Delta P_L
\end{bmatrix}
= \begin{bmatrix}
\Delta P_s \\
\Delta P_N \\
\Delta Q_G \\
\Delta Q_L
\end{bmatrix}
= \begin{bmatrix}
\Delta P G_s - \Delta P D_s \\
\Delta P G_N - \Delta P D_N \\
\Delta Q G_G - \Delta Q D_G \\
\Delta Q G_L - \Delta Q D_L
\end{bmatrix} = 0 \quad (3.14)
\]

where subscripts s, N, G and L represent respectively the slack bus, all the buses other than the slack bus, the set of generator buses and the set of load buses.

Noting that \( \Delta P D = 0 \), \( \Delta Q D = 0 \), \( \Delta Q G_L = 0 \) and assuming \( \Delta Q G_G \neq 0 \), equation (3.14) is rewritten as

\[
\begin{bmatrix}
\frac{\partial \Delta P_s}{\partial \delta} \\
\frac{\partial \Delta P_N}{\partial \delta} \\
\frac{\partial \Delta Q_G}{\partial \delta} \\
\frac{\partial \Delta Q_L}{\partial \delta}
\end{bmatrix}
\begin{bmatrix}
\Delta P G_s \\
\Delta P G_N \\
\Delta Q G_G \\
\Delta Q G_L
\end{bmatrix}
= 0 \quad (3.15)
\]

In equation (3.15), \( \Delta P G_N \) contains all components of \( \Delta P G \) and zero elements corresponding to buses without generation.

3.3.1 **Sensitivity relation between bus phase angles and control variables**

Sensitivity relation between \( \delta \) and \( X_P \) is crucial in the development of LP model. Instead of this relation, the sensitivity of \( \delta \) with respect to \( \Delta X_P \) is obtained to
enable the use of constant matrix instead of the Jacobian matrix in the derivation of the LP model.

Dividing each of the component equation in the second matrix equation in (3.15) by the corresponding bus voltage, we obtain

\[
\frac{1}{V_N} \frac{\partial P_N}{\partial \delta} \Delta \delta = \frac{1}{V_N} \Delta P G_N
\]

(3.16)

Making use of the assumptions

\[
\cos \delta_{km} = 1, \ G_{km} \sin \delta_{km} << B_{km} \quad \text{and} \quad Q_k << B_{kk} V_k^2
\]

where \( \delta_{km} = (\delta_k - \delta_m) \) and \( G_{km} \) and \( B_{km} \) are the real and imaginary parts of \( km \)th element \( Y_{km} \) of the bus admittance matrix, the LHS of equation (3.16) is approximated to

\[
[B'''] V. \Delta \delta \quad \text{where} \quad [B'''] \quad \text{is the negated bus susceptance matrix excluding the slack bus.}
\]

The RHS of equation (3.16) is written as \([W] \Delta P G'\)

where \([W]\) is an \((n-1) \times NC\) matrix. Equation (3.16) becomes

\[
[B''''] V. \Delta \delta = [W] \Delta P G'
\]

\[
V. \Delta \delta = [S_d] \Delta P G' = [S_d] \Delta X_p
\]

(3.17)

where the sensitivity matrix \([S_d]\) is

\[
[S_d] = [B''']^{-1} [W]
\]

(3.18)
3.3.2 Sensitivity relation between slack bus real power generation and control variables

From the first equation in (3.15),

$$\Delta P_G = \left( \frac{\partial P_G}{\partial \delta} \right)^t \Delta \delta = \left( \frac{\partial P_G}{\partial \delta} \cdot \frac{1}{V} \right)^t V \Delta \delta$$

Substituting $V \Delta \delta$ from equation (3.17),

$$\Delta P_G = \left( \frac{\partial P_G}{\partial \delta} \cdot \frac{1}{V} \right)^t [S_d] \Delta X_p$$

$$\Delta P_G = s_1^t \Delta X_p$$  (3.19)

where the sensitivity vector $s_1^t$ is given by

$$s_1^t = \left( \frac{\partial P_G}{\partial \delta} \cdot \frac{1}{V} \right)^t [B''']^{-1} [W]$$  (3.20)

3.3.3 Sensitivity relation between line phase angles and control variables

The incremental relation connecting line phase angles to the bus phase angles is

$$\Delta \psi = [M_1] \Delta \delta$$  (3.21)

where $[M_1]$ is the bus incidence matrix which gives the incidence of 'limiting lines' in the network to the buses.

Dividing the elements of each column of $[M_1]$ by the corresponding bus voltage,

$$\Delta \psi = [M] (V \Delta \delta)$$  (3.22)
Substituting $V \Delta \delta$ in (3.22) from (3.17),

$$\Delta V = \begin{bmatrix} S_2 \end{bmatrix} \Delta X_p$$  \hspace{1cm} (3.23)

where

$$\begin{bmatrix} S_2 \end{bmatrix} = [M] [B^\prime]^{-1} [W]$$  \hspace{1cm} (3.24)

### 3.3.4 LP model

The incremental relation between the dependent variables and the control variables is obtained using equations (3.19) and (3.23) as

$$\Delta Y_p = [S'] \Delta X_p$$  \hspace{1cm} (3.25)

where the sensitivity matrix $[S']$ is given by

$$[S'] = \begin{bmatrix} s_1^t \\ s_2 \end{bmatrix}$$

The incremental generation cost $\Delta f$, is expressed in terms of $\Delta X_p$ by substituting $\Delta P_G$ in (3.10) from (3.19).

That is,

$$\Delta f = c_p^t \Delta X_p$$  \hspace{1cm} (3.26)

where

$$c_p = (2c_{1i} P_G + c_{21}) + (2c_{1s} P_G + c_{2s}) s_{1i}$$
The reduced order LP problem obtained from equations (3.10) to (3.13) is

Determine : $\Delta X_P$

to minimize : $\Delta f = c_p^T \Delta X_P$ \hspace{1cm} (3.27)

subject to : $\Delta X_{P_{min}} \leq \Delta X_P \leq \Delta X_{P_{max}}$ \hspace{1cm} (3.28)

$(Y_{P_{min}} - Y_P^o) = \Delta Y_{P_{min}} \leq [B^T] \Delta X_P \leq \Delta Y_{P_{max}} = (Y_{P_{max}} - Y_P^o)$ \hspace{1cm} (3.29)

Converting the vector of free variables $\Delta X_P$ into a vector of non-negative variables $\beta_P$ given by

$\beta_P \triangleq \Delta X_P - \Delta X_{P_{min}}$ \hspace{1cm} (3.30)

the LP problem in (3.27) to (3.29) is rewritten in terms of $\beta_P$ as

Determine : $\beta_P$

to minimize : $v_P = c_p^T \beta_P$ \hspace{1cm} (3.31)

subject to : $[A_P] \beta_P \leq b_P$ \hspace{1cm} (3.32)

The number of constraints in the above LP problem is equal to the number of control variables plus twice the number of limiting lines plus two. Therefore instead of solving this, its dual given below is solved for which the number of constraints is equal only to the number of control variables.
Determine $\mu_P$ to maximize: $W_P = b_P^T \mu_P$ \hspace{1cm} (3.33)
subject to: $[A_P]^{-1} p_c \geq c_P$ \hspace{1cm} (3.34)

From the optimal solution of the dual problem, the optimal values of the primal variables $\beta_P$ are obtained.

3.4 METHOD OF SOLUTION

The solution to the NLP problem (3.1) to (3.4) is obtained through the SLP approach. The details of the method adopted are given in the flow chart in Figure 3.1.

At the beginning (block 2 of the flow chart) the constant matrices $B'$, $B''$, and $B'''$ are assembled, factorized and their sparse upper triangular factors are compactly stored. The initial state and initial cost are obtained by conducting a power flow using FDPF method (block 3). The dual LP problem (DLPP) is set up by computing $[A_P]^T$, the RHS vector $c_P$ (block 4) and the objective vector $b_P$ (block 5). The dual LP problem is solved by two-phase Revised Simplex method (block 6) and optimal increments in the control variables $X_P$ are obtained. The control vector $X_P$ is then updated and a power flow is conducted to obtain the new state and new value of cost (block 7). Blocks 4 to 7 in the flow chart represent an 'LPstep' in the SLP method of solution. These LPsteps are repeated till convergence.

3.4.1 Controlling oscillatory convergence

SLP method is inherently susceptible to oscillations in the value of the objective function and in the values of
READ INITIAL STEP SIZE, XPSTEP AND MAXIMUM
NUMBER OF STEP SIZE REDUCTIONS, NRMX

CONSTRUCT AND FACTORIZE $B', B''$ AND $B'''$
MATRICES

PERFORM POWER FLOW AND COMPUTE COST

$\text{LPS} = 0, \text{LPSR} = 0, \text{NRDN} = 0$

$\text{LPS} = \text{LPS} + 1$

STORE PRESENT STATE AND COST; COSTO = COST

CONSTRUCT $[A_p]^t$ AND RHS VECTOR $c_p$ OF
DUAL LP PROBLEM (DLPP)

COMPUTE OBJECTIVE VECTOR $b_p$ OF DLPP

$\text{LPSR} = \text{LPSR} + 1$

SOLVE DLPP

UPDATE XP, PERFORM
POWER FLOW & COMPUTE COST

XPSTEP = 0.5 * XPSTEP

NRDN = NRDN + 1

COST < COST0

NO

REVERT TO OLD STATE

YES

STOP

FIG. 3-1 REAL POWER OPTIMIZATION IN AC AND AC-DC
SYSTEMS.
control variables especially when the solution approaches the optimal point. To overcome this, reference [13] uses a 'variable bounding and interpolation' technique. This method does not use restricted bounds for the control variables at the beginning and whenever cost increases in an LP step, cost reduction is achieved by taking one or more 50 percent interpolation on control variables. In addition, at the end of each LP step the control variables are monitored to identify oscillations and bounds of the oscillatory variables are reduced for the next LP step to prevent cost oscillation in the subsequent LP steps. However, the above technique slows down the convergence and increases the number of LP moves and the number of power flow computations.

The oscillatory convergence of the SLP method is due to the fact that the linearized constraints and objective function in an LP move which are valid over incremental $\Delta x_p$ are used with wide limits of $\Delta x_p$. In the proposed method, properly chosen restricted bounds ($x_{PSTEP}$) on the decision variables $\Delta x_p$ (control variable increments) are applied right from the first LP step (block 1 in the flow chart). Too small a value for the initial step size $x_{PSTEP}$ will avoid oscillations but will require a large number of LP steps before convergence is reached and too large a value will give rise to cost oscillations. Hence suitable value of initial $x_{PSTEP}$ is selected for obtaining good convergence and accordingly the bounds for $\Delta x_p$ in equation (3.28) are chosen as given below:

$$\Delta x_{Pmax} = \min \{ (x_{Pmax} - x_P^o), x_{PSTEP} \}$$  \hspace{1cm} (3.35)

$$\Delta x_{Pmin} = -\min \{ (x_P^o - x_{Pmin}), x_{PSTEP} \}$$  \hspace{1cm} (3.36)
Further, after the first LPstep, the initial step sizes are successively reduced by 50% (block 8) in each of the subsequent LPsteps. This technique of applying diminishing step sizes makes the control space around the successive solution points to shrink gradually till it becomes very small and arrests the oscillating tendency of the objective function and control variables giving rise to good convergence behaviour and improved accuracy of the converged solution. The LPsteps are stopped when the number of reductions made in the initial step size, XPSTEP exceeds a prespecified number, NRDMX (block 9) as no further improvement in the objective function is possible due to the shrunk decision space.

If the cost increases at the end of any LPstep, the new state obtained is discarded, the previous state is restored (block 10) and another LPstep is taken from the old state after having reduced the step sizes by 50 percent in block 8 in Figure 3.1. This LPstep does not require computation of $[A_p]^\top$ and $c_p$ of the DLPP again and so it is termed as a 'reduced LPstep' (LPSR in the flow chart).

### 3.5 COMPUTATIONAL DETAILS

In the proposed algorithm, the constraint matrix $[A_p]^\top$ and the objective vector $c_p$ of the dual LP problem are computed through the sensitivity vector $s_1$ and sensitivity matrix $[S_2]$ which are obtained using the factors of constant matrix $B''$ in all the LPsteps. The factors of $B'$ and $B''$ are used for power flow solution at each LPstep. The power flow Jacobian matrix, either full or decoupled are not at all required in this algorithm. The sparse constant matrices are constructed in compact arrays and converted into their factors using sparsity-oriented bifactorization.
method [39] only once at the beginning of the solution. The bus admittance matrix construction is also done in compact arrays.

The dimension of the problem and computing time are reduced at each LP step by selecting only those line flow constraints that are found to be 'effective' where an effective constraint is considered as one for which the line flow expressed in terms of line phase angle is more than 75% of the maximum rating of the line.

3.6 NUMERICAL EXAMPLES AND DISCUSSION

The Real-only OPF algorithm presented in Section 3.4 was tested on the following two test systems.

(i) The IEEE 14-bus test system [40] and
(ii) The modified IEEE 30-bus test system [8].

3.6.1 IEEE 14-bus system

The line data, load data, transformer data and static capacitor data for the IEEE 14-bus system are given in reference [40]. This system is assumed to have 3 generators and 2 synchronous condensers. The generator cost data was chosen as in reference [31] and this data is furnished in Appendix 2.

Real power generations of the two generators viz., PG2 and PG6 were taken as control variables for minimizing the fuel cost. Five lines were taken as limiting lines for which line flow constraints were applied. In order to compare the results with those of reference [31] which does not consider the line flow constraints, the ratings of the
chosen limiting lines were assumed to be large so as not to restrict the optimal solution. The minimum and maximum limits of the control variables and slack bus real power generation and the assumed initial state are given in Table 3.1. Based on a number of trial studies on this as well as on the 30-bus test system, the initial step size for the control variable was chosen as 40 percent of the rated output (i.e. 40 MW for both the generators). The number of reductions (NRDMX) in the step size was chosen as three. With this value of step size and NRDMX, the optimal state obtained after application of the proposed algorithm is presented in Table 3.1. The cost convergence pattern is shown in Figure 3.2. It is seen that the generation cost is reduced from 1151.40 $/hr to 1136.13 $/hr in 4 full LPsteps (4 LPS). There was no violation in the slack generation in any of the intermediate states. The algorithm took 1.03 seconds on a VAX 11/780 computer to give the optimal solution.

The optimal solution for this IEEE test system has been obtained in reference [31] for the same limits of control variables and slack generation by applying the Quadratic Programming (QP) technique. The results of the proposed method are compared with those of [31] in Table 3.2. It is seen that the proposed method gives almost the same optimal cost in 4 LPsteps compared to the QP method of reference [31].

In order to test the effectiveness of the proposed algorithm in enforcing the line flow constraints, the rating of one of the lines viz., the line connecting buses 2 and 4 was reduced from 5° to 4° resulting in 4.56% limit violation in the initial state.
### TABLE 3.1 RESULTS OF REAL-ONLY OPTIMIZATION ON 14-BUS AC SYSTEM

<table>
<thead>
<tr>
<th>Variable</th>
<th>Limits</th>
<th>Initial state</th>
<th>Optimal state</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Minimum</td>
<td>Maximum</td>
<td></td>
</tr>
<tr>
<td>I Control variables</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( P_{G2} (\text{MW}) )</td>
<td>20.0</td>
<td>100.0</td>
<td>90.0</td>
</tr>
<tr>
<td>( P_{G6} (\text{MW}) )</td>
<td>20.0</td>
<td>100.0</td>
<td>60.0</td>
</tr>
<tr>
<td>II Dependent variable</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( P_{G1} (\text{MW}) )</td>
<td>50.0</td>
<td>200.0</td>
<td>116.59</td>
</tr>
<tr>
<td>III Generation cost ($/hr)</td>
<td></td>
<td></td>
<td>1151.40</td>
</tr>
</tbody>
</table>

### TABLE 3.2 COMPARISON OF THE RESULTS OF REAL-ONLY OPTIMIZATION ON 14-BUS AC SYSTEM WITH THOSE OF REFERENCE [31]

<table>
<thead>
<tr>
<th></th>
<th>Proposed method</th>
<th>Method in reference [31]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Optimal generations (MW)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( P_{G1} )</td>
<td>165.02</td>
<td>160.60</td>
</tr>
<tr>
<td>( P_{G2} )</td>
<td>65.00</td>
<td>69.00</td>
</tr>
<tr>
<td>( P_{G6} )</td>
<td>38.32</td>
<td>38.60</td>
</tr>
<tr>
<td>2. Optimal cost ($/hr)</td>
<td>1136.13</td>
<td>1135.92</td>
</tr>
<tr>
<td>3. No. of LPsteps/iterations</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>4. Solution time(secs)</td>
<td>1.03*</td>
<td>5.58**</td>
</tr>
</tbody>
</table>

* VAX 11/780 system
** IBM 370/155 system
FIG. 3-2 COST CONVERGENCE IN THE REAL-ONLY OPTIMIZATION ON 14-BUS A.C. SYSTEM.
After application of the present algorithm, the flow violation in the line 2-4 was practically removed in the first LPstep and in the optimal state all the constraints were satisfied. The optimal cost obtained in this case was 1139.63 $/hr which is 3.5 $/hr higher than that of the first case with larger rating for the flow in line 2-4.

3.6.2 Modified IEEE 30 - bus system

The single line diagram of the 30-bus system is shown in Figure 3.3. The line data, load data, transformer data and generation cost data of this system are furnished in Appendix 3 and these are the same as in reference [8]. Real power generations of 5 generators at buses 2, 5, 8, 11 and 13 were adjusted to minimize the total generation cost. Seven lines viz., lines 1 to 4, 6, 7 and 15 were chosen as the limiting lines; but their ratings were chosen initially to be large so as not to restrict the optimal solution. This was done in order to compare the results with those of reference [8] in which the line flow constraints were not considered.

The initial step sizes of the control variables were selected as in the case of 14-bus test system. The number of reductions of the step sizes was chosen as three. The minimum and maximum limits of all the controllable generations, the slack bus real power generation, the initial state and the optimal state obtained after application of the proposed algorithm are presented in Table 3.3. The pattern of cost convergence is portrayed in Figure 3.4. When the cost increases at the end of an LPstep the previous state is restored and this resetting to the previous state is indicated by dotted horizontal line in Figure 3.4. It is seen that the total generation cost is decreased from an
FIG. 3.3  MODIFIED IEEE 30-BUS A.C. SYSTEM.
### TABLE 3.3 RESULTS OF REAL-ONLY OPTIMIZATION ON 30-BUS AC SYSTEM

<table>
<thead>
<tr>
<th>Limits</th>
<th>Variable</th>
<th>Initial</th>
<th>Optimal</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Minimum</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I Control variables</td>
<td>PG(_2)</td>
<td>20.0</td>
<td>80.0</td>
<td>80.0</td>
<td>44.00</td>
</tr>
<tr>
<td>Real power</td>
<td>PG(_5)</td>
<td>15.0</td>
<td>50.0</td>
<td>50.0</td>
<td>22.50</td>
</tr>
<tr>
<td>generations</td>
<td>PG(_8)</td>
<td>10.0</td>
<td>35.0</td>
<td>20.0</td>
<td>15.25</td>
</tr>
<tr>
<td>(MW)</td>
<td>PG(_{11})</td>
<td>10.0</td>
<td>30.0</td>
<td>20.0</td>
<td>11.50</td>
</tr>
<tr>
<td></td>
<td>PG(_{13})</td>
<td>12.0</td>
<td>40.0</td>
<td>20.0</td>
<td>12.00</td>
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<tr>
<td>II Dependent variables</td>
<td>Slack bus real power generation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(MW)</td>
<td>PG(_1)</td>
<td>50.0</td>
<td>200.0</td>
<td>99.22</td>
<td>189.11</td>
</tr>
<tr>
<td>III Generation cost</td>
<td>($/hr)</td>
<td>-</td>
<td>-</td>
<td>901.95</td>
<td>806.27</td>
</tr>
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</table>
FIG. 3-4 COST CONVERGENCE IN THE REAL-ONLY OPTIMIZATION ON 30-BUS A.C. SYSTEM.
initial value of $901.95 \$/hr to $806.27 \$/hr in 3 full LP steps (3 LPS) and 1 reduced LP step (1 LPSR) corresponding to three reductions in step size. The solution time on VAX 11/780 system was 1.78 seconds.

Starting from the same initial state and for the same limits of real power generations, an optimal cost of $804.853 \$/hr is reported in reference [8] which is a Real-Reactive OPF method for cost minimization (involving both P - and Q - control variables) employing 'Gradient Projection' technique. The final value obtained by the proposed linear method without considering the reactive power controls and the corresponding constraints is very close to that obtained in the above reference. This indicates that the proposed Real-only OPF method gives OPF solution with acceptable accuracy.

To verify the ability of the algorithm in enforcing the line flow constraints, the maximum rating of one of the limiting lines viz., line 15 was reduced from $9.5^\circ$ to $3.75^\circ$ causing 9.07% overload in the initial state. The application of the SLP algorithm removed the flow violation in the line 15 in the first LP step itself and thereafter practically no violation occurred in any of the dependent variables. The optimal cost obtained with NRDMX = 3, was $812.84 \$/hr which is higher than the cost of $806.27 \$/hr obtained in the first case with larger limit for the rating of line 15.

**Suppression of oscillations**

To verify the effectiveness of the method in arresting the oscillations in the control variables, their variations in successive LP steps are shown in Figure 3.5. In
FIG. 3-5 MOVEMENT OF CONTROL VARIABLES IN THE REAL-ONLY OPTIMIZATION ON 30-BUS A.C. SYSTEM.

- O---O Resetting the values of the control variables to previous state values.
FIG. 3.5 (continued)

MOVEMENT OF CONTROL VARIABLES IN THE REAL-ONLY OPTIMIZATION ON 30-BUS A.C. SYSTEM.
this Figure, the space between the thick lines in an LPstep is the range allowed for the movement of a control variable. It is seen that the oscillatory tendency of all the control variables are effectively suppressed.

3.7 REAL-ONLY OPTIMAL POWER FLOW IN AC-DC SYSTEM

The objective of the Real-only OPF problem in ac-dc system is to minimize the operating cost $f$, of the integrated ac-dc system by adjusting all the real power control variables simultaneously satisfying the system equations and the operating constraints. The vector of control variables $X_p$, and the vector of dependent variables $Y_p$ to be constrained within limits during optimization are chosen as

$$X_p = [P_G', I_d', V_{dm}]^t$$

$$Y_p = [V_L, Q_G, P_G, \phi, I_{dm}, V_d', a, P_d, Q_d]^t$$

Suitable minimum values of control angles are assumed for the converters so as to keep the reactive power consumption of converters a minimum. Constraints are imposed on load bus voltages $V_L$ and reactive powers of generators $Q_G$ since adjusting the dc current settings alter the reactive power flow in the ac system.

3.7.1 Problem formulation

In an integrated ac-dc system, the real and reactive power balance equations at the n ac buses are given by the equations (2.1) and (2.2). These equations are written compactly as

$$F(X_p, Y_p, \delta) = 0$$  (3.37)
The equations (2.9) to (2.12) of the m converters are written as

\[ E_{1k} = V_{dk} - a_k V_{tk} \cos \theta_k + I_{dk} R_{ck} = 0 \] (3.38)

\[ E_{2k} = V_{tk} - a_k V_{tk} \cos \phi_k = 0 \] (3.39)

\[ E_{3k} = P_{dk} - V_{dk} I_{dk} = 0 \] (3.40)

\[ E_{4k} = Q_{dk} - \beta_k P_{dk} \tan \phi_k = 0 \] (3.41)

where \( k = 1, 2, \ldots, m \)

These equations (3.38) to (3.41) are stated in compact form as

\[ E(X_p, Y_p) = 0 \] (3.42)

The dc network equations are given by equations (2.13) and (2.14). Equation (2.13) can be stated in compact form as

\[ V_{d'} = [R] I_{d'} + e V_{dm} \] (3.43)

The Real-only OPF problem for minimization of operating cost in an ac–dc system is a NLP problem given by

Determine : \( X_p \)

to minimize : \( f = \sum_{i=1}^{NC} (c_{i1} P_{G1}^2 + c_{21} P_{G1} + c_{31}) + (c_{1s} P_{G2}^2 + c_{2s} P_{G2} + c_{3s}) \) (3.44)

subject to : the control constraints

\[ X_{pmin} \leq X_p \leq X_{pmax} \] (3.45)

the operating constraints

\[ Y_{pmin} \leq Y_p \leq Y_{pmax} \] (3.46)
the power flow constraints
\[ F(X_p, Y_p, \delta) = 0 \] (3.47)

the converter equations
\[ E(X_p, Y_p) = 0 \] (3.48)

the dc network equations
\[ V_d' - [R] I_d' - e V_{dm} = 0 \] (3.49)

and \[ \sum_{i=1}^{m} I_{di} = 0 \] (3.50)

3.7.2 LP model

Following the same procedure adopted for the ac system, the NLP problem given in equations (3.44) to (3.50) is linearized around an operating state and converted into the following LP problem.

Determine \[ \Delta X_p \]

to minimize : \[ \Delta f = \sum_{i=1}^{NC} (2c_{1i} P_{Gi} + c_{2i}) \Delta P_{Gi} \]
\[ + (2c_{1s} P_{Gs} + c_{2s}) \Delta P_{Gs} \] (3.51)

subject to : \[ \Delta X_{P_{min}} \leq \Delta X_p \leq \Delta X_{P_{max}} \] (3.52)

\[ \Delta Y_{P_{min}} = (Y_{P_{min}} - Y_p^O) \leq \Delta Y_p \leq (Y_{P_{max}} - Y_p^O) = \Delta Y_{P_{max}} \] (3.53)
A reduced order LP model is obtained by expressing the objective function and the inequality constraints in terms of $\Delta \mathbf{x}_p$ only, using equations (3.54) to (3.57). The linearized sensitivity relations between the dependent and control variables are derived in Appendix 4. From equations (A4.1) to (A4.4), (A4.9), (A4.13), (A4.14), (A4.20) and (A4.21), we obtain

\[
\begin{bmatrix}
\Delta v_L \\
\Delta q_G \\
\Delta p_G \\
\Delta f \\
\Delta i_{dm} \\
\Delta v_d \\
\Delta a \\
\Delta f_d \\
\Delta q_d 
\end{bmatrix}
=
\begin{bmatrix}
0 & s_{12}' & s_{13}' \\
0 & s_{22}' & s_{23}' \\
0 & s_{31}' & s_{32}' & s_{33}' \\
0 & s_{41}' & s_{42}' & s_{43}' \\
0 & -e^t & 0 \\
0 & R & e \\
0 & s_{72}' & s_{73}' \\
0 & s_{82}' & I_d \\
0 & s_{92}' & s_{93}'
\end{bmatrix}
\begin{bmatrix}
\Delta p_G \\
\Delta i_d' \\
\Delta v_{dm}
\end{bmatrix}
\]  

(3.58)

i.e. $\Delta \mathbf{y}_p = [s'] \Delta \mathbf{x}_p$  

(3.59)

where $[s']$ is the sensitivity matrix relating dependent and control variables.
The submatrices $S_{12}'$ and $S_{13}'$ in equation (3.58) are larger in size and are computed from equations (A4.10) and (A4.11), using the factors of the constant matrix $B''$ of the FDPF method. The sensitivity submatrices relating dependent variables of dc system and $\Delta X_p$ are computed without much effort. The sensitivity submatrices relating $PQ_s$ and $f$ to control vector $X_p$ are computed from (A4.20) and (A4.21) using matrix $S_d$ which in turn is computed from (A4.17) to (A4.19) using the factors of a constant matrix $B'''$. The present method does not require repeated computation and factorization of large Jacobian matrix at any of the LP steps for computing the matrix $S'$. 

The increment in cost $\Delta f$, is expressed in terms of $\Delta X_p$ by substituting $\Delta P_Q_s$ in equation (3.51) using equation (A4.20).

$$\Delta f = (c_{p1}^t \ c_{p2}^t \ c_{p3}^t) \ \Delta X_p = c_p^t \ \Delta X_p$$ (3.60)

The LP problem in (3.51) to (3.57) is rewritten as

Determine : $\Delta X_p$

to minimize : $\Delta f = c_p^t \ \Delta X_p$ (3.61)

subject to : $\Delta X_{pmin} \leq \Delta X_p \leq \Delta X_{pmax}$ (3.62)

$$\Delta Y_{pmin} \leq [S'] \ \Delta X_p \leq \Delta Y_{pmax}$$ (3.63)

Following a similar procedure adopted for ac system, the vector of free variables $\Delta X_p$ is transformed into a non-negative vector $\beta_p$ where $\beta_p = (\Delta X_p - \Delta X_{pmin})$ and the primal LP problem is converted into its dual. The final dual LP model is
Determine : $\mu_p$

to maximize : $w_p = b_p^T \mu_p$ \hspace{2cm} (3.64)

subject to : $[A_p]^t \mu_p \geq c_p$ \hspace{2cm} (3.65)

where $\mu_p$ is the vector of dual variables and $w_p$ is the dual objective function.

3.8 METHOD OF SOLUTION

The solution to the original Real-only NLP problem (3.44) to (3.50) is obtained by successively solving the approximately equivalent LP problem (3.64) and (3.65) using SLP approach as in the case of ac systems. The method adopted is the same as that outlined in the flow chart of Figure 3.1 used for the P-optimization in ac system. Blocks 4 to 7 represent one LPstep (LPS) and blocks 10, 5, 6 and 7 represent a reduced LPstep (LPSR). For obtaining the power flow solution of the ac-dc system in blocks 3 and 7 of Figure 3.1, the power flow method described in Chapter 2 is employed. For this Real-only OPF method of ac-dc system, 'I-control mode' is selected in block 3 of the ac-dc power flow algorithm (Figure 2.5) since the dc current schedules are modified after each LP step.

As in the case of ac system, oscillations in the solution are arrested by the proper choice of initial step sizes for the decision variables $\Delta x_p$ in the first LPstep followed by 50% reduction in subsequent LPsteps. The procedure is stopped when the number of reductions made in the initial step sizes exceeds NRDMX. If the cost increases at the end of any LPstep, the new state obtained is
discarded and a reduced LP step is taken from the previous state.

3.9 SYSTEM STUDY

A computer program was prepared based on the flow chart of Figure 3.1 and this program centres around the following three major routines.

1. AC-DC power flow analysis
2. Computation of objective vector, RHS vector and constraint matrix
3. LP solution by two-phase Revised Simplex method.

Studies were made on a sample 30-bus, 5-converter ac-dc system shown in Figure 3.6. This system is an adaptation of the IEEE 30-bus test system with a 5TDC system added to it [29]. Load data and generation schedules are different from those used in IEEE 30-bus test system. The system has 30 buses, 5 generators and 41 ac lines. The 5TDC subsystem has 5 converters and 4 dc lines. The two converters $C_1$ and $C_5$ operate as rectifiers and the rest as inverters.

The ac line data of the ac-dc system are the same as those of the IEEE 30-bus ac system and are given in Appendix 3. The load data, transformer data, generation cost data and dc system data are furnished in Appendix 5. The dc system data are given in Section A5.4 of Appendix 5 in per unit based on the per unit system described in Appendix 1. Reactive power compensation of value 30 MVAR each, is provided at buses 4, 6 and 28 which are converter terminal buses where no generators are connected. Two adjustable VAR sources are assumed to be connected at load buses 10 and 24.
FIG. 3-6 A 30-AC BUS 5-CONVERTER A.C.-D.C. SYSTEM.
For the Real-only OPF solution, the minimum delay angles of rectifiers were fixed at 7° and minimum extinction angles of inverters were fixed at 16°. Line flow constraints were imposed on 12 lines viz., lines 1,9 to 12,14,15,18,27,36,40 and 41. The assumed maximum ratings of the lines expressed in terms of line phase angles are given in Appendix 5.

The initial step sizes chosen after elaborate trial studies on two test systems including the system in Figure 3.6 are as follows:

i. Generator real power : 0.4 p.u. on machine rating
ii. Converter current : 0.4 p.u.
iii. Reference converter voltage : 0.1 p.u.

The limits on the control and dependent variables, the initial state and the optimal state obtained by applying the proposed algorithm are furnished in Table 3.4. The cost convergence pattern is depicted in Figure 3.7, which shows that the cost converges to optimal value in 5 LPsteps and the initial operating cost of 1653.65 $/hr is reduced to 1359.82 $/hr resulting in a cost reduction of 17.77%. The optimal solution is obtained in a solution time of 2.84 seconds on a VAX 11/780 computer. In the initial state real power flow in lines 15 and 36 exceeded the specified limits by 10.34% and 5.77%. These violations were removed after the first LPstep and no violation occurred thereafter in any of the dependent variables.
TABLE 3.4 RESULTS OF REAL-ONLY OPTIMIZATION ON 30-BUS, 5-CONVERTER AC-DC SYSTEM

<table>
<thead>
<tr>
<th>Variable</th>
<th>Limits</th>
<th>Initial state</th>
<th>Optimal state</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Minimum</td>
<td>Maximum</td>
<td>Minimum</td>
</tr>
<tr>
<td>I CONTROL VARIABLES</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real power generations (MW)</td>
<td>PG₂</td>
<td>50.0</td>
<td>200.0</td>
</tr>
<tr>
<td></td>
<td>PG₅</td>
<td>15.0</td>
<td>50.0</td>
</tr>
<tr>
<td></td>
<td>PG₁₁</td>
<td>10.0</td>
<td>30.0</td>
</tr>
<tr>
<td></td>
<td>PG₁₃</td>
<td>15.0</td>
<td>50.0</td>
</tr>
<tr>
<td>DC current settings (p.u.)</td>
<td>Id₁</td>
<td>0.25</td>
<td>1.00</td>
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<tr>
<td></td>
<td>Id₂</td>
<td>-0.75</td>
<td>-0.20</td>
</tr>
<tr>
<td></td>
<td>Id₃</td>
<td>-0.75</td>
<td>-0.20</td>
</tr>
<tr>
<td></td>
<td>Id₄</td>
<td>-0.75</td>
<td>-0.20</td>
</tr>
<tr>
<td>Reference converter voltage (p.u.)</td>
<td>V₄₅₁</td>
<td>0.90</td>
<td>1.10</td>
</tr>
<tr>
<td>II DEPENDENT VARIABLES</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slack bus real power generation (MW)</td>
<td>PG₁</td>
<td>75.0</td>
<td>300.0</td>
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</table>
Table 3.4 (continued)

<table>
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<th>Variable</th>
<th>Limits</th>
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<tr>
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<td>Minimum</td>
<td>Maximum</td>
<td>state</td>
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<td>( \psi_1 )</td>
<td>-4.0</td>
<td>4.0</td>
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<tr>
<td>( \psi_9 )</td>
<td>-3.0</td>
<td>3.0</td>
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<tr>
<td>( \psi_{10} )</td>
<td>-1.0</td>
<td>1.0</td>
<td>0.830</td>
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<tr>
<td>( \psi_{11} )</td>
<td>-6.0</td>
<td>6.0</td>
<td>3.632</td>
</tr>
<tr>
<td>( \psi_{12} )</td>
<td>-9.0</td>
<td>9.0</td>
<td>6.893</td>
</tr>
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<td>Line phase</td>
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<td>( \psi_{14} )</td>
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<td>angles (degrees)</td>
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<td>( \psi_{15} )</td>
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<td>6.5</td>
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<tr>
<td>( \psi_{18} )</td>
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<td>3.0</td>
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<td>( \psi_{27} )</td>
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<td>2.0</td>
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<td>( \psi_{36} )</td>
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<td>6.0</td>
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<td>( \psi_{40} )</td>
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<td>4.0</td>
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<td>( \psi_{41} )</td>
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<tr>
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<td>V_{10}</td>
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<td>1.05</td>
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<th>Optimal state</th>
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<td>v14</td>
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<td>v15</td>
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<td>0.992</td>
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Reference converter current (p.u.)

<table>
<thead>
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<th>Limits</th>
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<th>Maximum</th>
<th>Initial state</th>
<th>Optimal state</th>
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<td>1.000</td>
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<td></td>
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<td>1.009</td>
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<td>Vd3</td>
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<td>1.10</td>
<td>0.988</td>
<td>1.044</td>
</tr>
<tr>
<td></td>
<td>Vd4</td>
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<td>1.10</td>
<td>0.993</td>
<td>1.051</td>
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</table>
Table 3.4 (continued)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Limits</th>
<th>Initial state</th>
<th>Optimal state</th>
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<tbody>
<tr>
<td></td>
<td>Minimum</td>
<td>Maximum</td>
<td></td>
</tr>
<tr>
<td>Converter taps (p.u.)</td>
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<td>1.020</td>
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</tr>
<tr>
<td>Pd2 (MW)</td>
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<td>-20.0</td>
<td>-58.92</td>
</tr>
<tr>
<td>Pd3 (MW)</td>
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<td>-20.0</td>
<td>-59.28</td>
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<tr>
<td>Pd4 (MW)</td>
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<td>-20.0</td>
<td>-69.51</td>
</tr>
<tr>
<td>Pd5 (MW)</td>
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<td>125.0</td>
<td>100.00</td>
</tr>
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<td>Qd1 (MVAR)</td>
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<td>22.82</td>
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<tr>
<td>cost ($/hr)</td>
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<td>-</td>
<td>1653.65</td>
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</tbody>
</table>

Note: The values of the variables underlined are those that exceed the specified limits in the initial state.
FIG. 3-7 COST CONVERGENCE IN THE P-OPTIMIZATION ON 30-BUS, 5-CONVERTER A.C.-D.C. SYSTEM
3.10 SUMMARY

Real-only optimal power flow methods have been developed for ac and ac-dc systems using the SLP approach. The factors of constant matrices of FDPF method have been used to obtain the sensitivity relations and construct the LP model in all the LP steps of the SLP method. Hence, evaluation and factorization of large Jacobian matrices at every LP step are not necessary in these methods. The proposed methods have been successfully tested on a few ac and ac-dc systems. Suitable choice of initial step sizes for the control variables and their gradual reduction in successive LP steps arrest cost oscillation and result in fast convergence behaviour. The results obtained for ac systems have been compared with those obtained for the same systems using QP and NLP methods. These comparisons demonstrate the satisfactory solution accuracy of the proposed methods.