2.1 INTRODUCTION

A decision problem can be either deterministic or non-deterministic. It is deterministic if for each action there is a single event, and hence, a single evaluated outcome. Then the action that is selected has the most beneficial outcome. In this circumstance, there is no uncertainty as to the consequence of choosing a specific action. A decision problem is non-deterministic if a single event cannot accurately describe the variation in the outcomes for an action. Non-deterministic decisions are of two kinds - decisions under risk and decisions under absolute uncertainty.

Risk is associated with those cases in which the probability distribution of the future event is known or can be assumed. In situations of absolute uncertainty on the other hand, nothing is known about the relative probabilities of future events.

Most of the decisions in long-range planning for any major project take place in an environment in which the objectives, the constraints and the consequences of possible actions are not known precisely. Effective decision analysis is important in long-range planning because of the requirements of large financial commitments and resource allocation.
This chapter reviews the concept of fuzzy sets and explains how fuzzy concepts can be applied to the statistical decision making methods such as Maximin, Maximax decision rules, Hurwicz-α method, minimax loss and worth expectation criterion etc. Fuzzy decision analysis is carried out for long-range planning where the system states and objectives are not known precisely. Fuzzy decision making approach is systematically explained through an example.

2.2 FUZZY SETS - BACKGROUND

The concept of fuzzy set theory was first introduced by Prof. L.A. Zadeh [12]. The concept of fuzzy sets theory has been applied recently in various fields such as switching theory, automata theory, decision theory etc.

The theory of fuzzy sets may be viewed as an attempt in developing a body of concepts and techniques for dealing in a systematic way with a type of imprecision which arises when the boundaries of a class of objects are not sharply defined. Among the very common examples of such classes are the classes of "young women", "small cars" and "funny jokes". Membership in such classes, as they are suggestively called, fuzzy sets, is a matter of degree rather than an all or nothing proposition. Thus, informally a fuzzy set may be regarded as a class in which there is a graduality of progression from membership to nonmembership or, more precisely, in which an object may have a grade of membership intermediate between unity (full membership) and zero (nonmembership). Therefore it is clear that fuzzy approach is based on the premise that the key elements in human thinking are not just numbers but can be approximated
to tables of fuzzy sets, or, in other words, classes of objects in which the transition from membership to non-membership is gradual rather than abrupt. The main distinguishing features of fuzzy approaches are (i) use of so-called linguistic variables 'in place of' or 'in addition to' numerical variables, (ii) characterisation of simple relations between variables by fuzzy conditional statements, and (iii) characterisation of complex relations by fuzzy algorithms. Fuzzy approach provides an approximate and yet effective and more flexible means of describing the behaviour of systems which are too complex or too ill-defined at the time of modelling and planning.

The following sections review the definition of fuzzy sets, basic operations on it and extensively used statistical decision making methods.

2.2.1 Definition

Let \( X \) be a space of points, with a generic element of \( X \) denoted by \( x \). Thus \( X = \{x\} \). A fuzzy set \( A \) in \( X \) is characterised by a membership function \( \mu_A(x) \) which associates with each point \( x \) a real number in the interval \((0,1)\), with a value of \( \mu_A(x) \) at \( x \) representing the grade of membership of \( x \) in \( A \). Thus the nearer the value of \( \mu_A(x) \) to unity, the higher the grade of membership of \( x \) in \( A \).

A fuzzy set is generally assumed to be embedded in a nonfuzzy universe of discourse, which may be any collection of objects, concepts, or mathematical constructs. For example, a universe of discourse, \( U \), may be the set of all real numbers; the set of integers \( 0,1,2,\ldots,100 \); the set of
all students in a course; the set of objects in a room; etc. Universe of discourse is denoted by the symbol $U$, with or without subscripts and/or superscripts. A fuzzy set in $U$ or, equivalently, a fuzzy subset of $U$, is usually denoted by $A$, with or without subscripts and/or superscripts.

Let $A$ be the fuzzy subset of $U$, with generic element $x$. $\mu_A(x)$ represents the grade of membership of $x$ in $A$. The support of $A$ is the set of points in $U$ at which $\mu_A(x)$ is positive. The height of $A$ is the supremum of $\mu_A(x)$ over $A$. A crossover point of $A$ is the point in $U$ whose grade of membership in $A$ is 0.5.

Example 2.1

Let the universe of discourse be the interval $(0,100)$, with $x$ interpreted as age. A fuzzy subset of $U$ labelled old may be defined by a membership function such as

$$\mu_A(x) = \begin{cases} 0 & \text{for } 0 \leq x \leq 50 \\ \frac{1+[(x-50)/5]-2}{1} & \text{for } 50 \leq x \leq 100 \end{cases}$$

In this case, the support of old is the interval $(50,100)$, the height of old is effectively unity and the crossover point of old is 55.

Though the fuzzy membership function is in some respect similar to the probability density function, they are conceptually different. Probability is about how frequently a sample occurs in a population while fuzzy membership value means how closely or how accurately a sample resembles an ideal element of a population. It is important to note that the meaning attached to particular
numerical value of the membership function is purely subjective in nature.

To understand the distinction between fuzziness and randomness, it is helpful to interpret the grade of membership in a fuzzy set as a degree of possibility rather than probability. As an illustration, consider the proposition "they got out of John's car" (which is a Fiat). The question is: How many passengers got out of John's car? - assuming for simplicity that the individuals involved have the same dimensions. Let \( n \) be the number in question. Then, with each \( n \) we can associate two numbers \( \mu_n \) and \( P_n \) representing, respectively, the possibility and the probability that \( n \) passengers got out of the car. For example, we may have for \( \mu_n \) and \( P_n \):

\[
\begin{array}{cccccccc}
 n & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
 \mu_n & 0 & 1 & 1 & 1 & 0.7 & 0.2 & 0 \\
 P_n & 0 & 0.6 & 0.3 & 0.1 & 0 & 0 & 0 \\
\end{array}
\]

\( \mu_n \) is interpreted as the degree of ease with which \( n \) passengers can squeeze into a Fiat. Thus \( \mu_5 = 0.7 \) means that, by some specified or unspecified criterion, the degree of ease of squeezing 5 passengers into a Fiat is 0.7. On the other hand, the probability that Fiat may be carrying 5 passengers might be zero. Similarly, the possibility that a Fiat may carry 4 passengers is 1; by contrast, the corresponding probability in the case of Fiat might be 0.1.

Normally the construction of the membership function can be accomplished with the co-operation and assistance of
a panel of experienced engineers in specific cases. The resulting membership function can then be manipulated in a logical manner following the theory of fuzzy sets to obtain a meaningful result to the originally complex problem.

2.2.2 Basic operations on fuzzy sets [13]

To simplify the representation of fuzzy sets, it is convenient to employ the following notation.

A nonfuzzy finite set such as

\[ U = \{x_1, \ldots, x_n\} \]

is expressed as

\[ U = x_1 + x_2 + \ldots + x_n \]  \hspace{1cm} (2.1)

or \[ U = \sum_{i=1}^{n} x_i \]

with the understanding that equation (2.1) is a representation of \( U \) as the union of its constituent singletons, with + playing the role of the union rather than the arithmetic sum. Thus

\[ x_i + x_j = x_j + x_i \]

and \[ x_i + x_i = x_i \]

for \( i, j = 1, \ldots, n \).
As an extension of this notation, a finite fuzzy subset, $A$, of $U$ is expressed as the linear form

$$A = \mu_1 x_1 + \ldots + \mu_n x_n$$  \hspace{1cm} (2.2)

or

$$A = \sum_{i=1}^{n} \mu_i x_i \hspace{1cm} (2.3)$$

where $\mu_i$, $i = 1, \ldots, n$, is the grade of membership of $x_i$ in $A$. In cases where the $x_i$ are numbers, there might be some ambiguity regarding the identity of the $\mu_i$ and $x_i$ components of the string $\mu_i x_i$. In such cases, it is convenient to employ a separator symbol such as $|$ for disambiguation. It can be written as,

$$A = \mu_1 x_1 + \ldots + \mu_n x_n$$  \hspace{1cm} (2.3)

or

$$A = \sum_{i=1}^{n} \mu_i x_i \hspace{1cm} (2.3)$$

The basic operations which can be performed on fuzzy sets are the following:

Let $A$ and $B$ be two fuzzy sets and $\mu_A(x)$ and $\mu_B(x)$ are the respective grades of membership of $x$ in $A$ and $B$.

1. The compliment of $A$ is denoted by $A'$ and is defined by

$$A' = 1 - \mu_A(x) \hspace{1cm} \forall x$$  \hspace{1cm} (2.4)
2. The union of fuzzy sets $A$ and $B$ is denoted by $A \cup B$ and is defined by

$$A \cup B = \mu_A(x) \lor \mu_B(x) \quad \forall x$$

$$= \max \left[ \mu_A(x), \mu_B(x) \right] \quad \forall x$$

where $\lor$ is the symbol for maximum.

In the representation of $A$ if we have $x_i = x_j$, then we can make the substitution expressed by

$$\mu_i x_i + \mu_j x_i = (\mu_i \lor \mu_j) x_i$$

For example,

$$A = 0.3|a + 0.8|a + 0.5|b$$

may be rewritten as

$$A = (0.3 \lor 0.8)|a + 0.5|b$$

$$= 0.8|a + 0.5|b$$

3. The intersection of $A$ and $B$ is denoted by $A \cap B$ and is defined by

$$A \cap B = \mu_A(x) \land \mu_B(x) \quad \forall x$$

$$= \min \left[ \mu_A(x), \mu_B(x) \right] \quad \forall x$$

where $\land$ is the symbol for minimum.
4. The product of A and B is denoted by $AB$ and is defined by

$$AB = \mu_A(x)\mu_B(x) \quad \forall x$$  \hspace{1cm} (2.8)

Thus, $A^\alpha$, where $\alpha$ is any positive number, should be interpreted as

$$A^\alpha = [\mu_A(x)]^\alpha \quad \forall x$$  \hspace{1cm} (2.9)

Similarly, if $\alpha$ is any nonnegative real number such that

$$\alpha \sup_{x} \mu_A(x) \leq 1$$

where $\sup$ represents supremum over $x$, then,

$$\alpha A = \alpha \mu_A(x) \quad \forall x$$  \hspace{1cm} (2.10)

5. The algebraic sum of two fuzzy sets A and B is denoted by $A + B$ and is defined by

$$A + B = \mu_A(x) + \mu_B(x) - \mu_A(x)\mu_B(x) \quad \forall x$$  \hspace{1cm} (2.11)

Example 2.2

Let us consider two fuzzy subsets

$A = 0.2|x_1| + 0.8|x_2| + 0.6|x_3|$

and $B = 0.4|x_1| + 0.6|x_2| + 1.0|x_3|$
Then \( A \cup B = 0.4x_1 + 0.8x_2 + 1.0x_3 \)
\( A \cap B = 0.2x_1 + 0.6x_2 + 0.6x_3 \)
\( A' = 0.8x_1 + 0.2x_2 + 0.4x_3 \)
\( AB = 0.08x_1 + 0.48x_2 + 0.6x_3 \)
\( A + B = 0.52x_1 + 0.92x_2 + 1.0x_3 \)

### 2.3 Statistical Decision Principles - State of Art [14,15]

The following methods are extensively used by the statistical decision makers under various situations.

1. **Maximin principle**
2. **Maximax principle**
3. **Hurwicz Method**
4. **Minimax loss**
5. **Worth expectation criterion**

The Maximin principle says to "pick the best of the worst" or "maximize the minimum". That is the minimum possible payoff of each action is determined and then the action with the maximum minimum worth is selected.

The Maximax principle is used to select an action whose best consequences is as good as every other consequence.

Maximin method is pessimistic because it finds the best from the worst. Maximax method is optimistic because it finds the best of the best. The Hurwicz method combines the maximin and maximax methods with an index of optimism-pessimism \( \alpha \). The Hurwicz values of actions are computed by using the relation

\[
H_i = \alpha V_i + (1-\alpha)V_i
\]  
\[ (2.12) \]
where \( V_i \) and \( v_i \) are the largest and smallest payoff values for action \( i \). The action with the largest \( H_i \) value is selected as the best action.

In minimax regret principle, the concern is the amount of worth lost by not choosing a better action for the same event. For each action-event combination, a "regret" is computed as the difference between the profit or worth that will result and the maximum profit that could be obtained with the another action. The maximum regret for each action is found and the alternative with the minimum value is chosen.

Worth Expectation is the most used decision rule. Let \( W_i \) represent the expected worth of action \( i \), then

\[
W_i = \sum_{j=1}^{m} g_j U_{ij}
\]

(2.13)

where 
\( j = \) event or outcome 
\( U_{ij} = \) worth of action \( i \) given outcome \( j \) 
\( g_j = \) belief that outcome \( j \) occurs

When worths are known and beliefs are quantified numerically then the expected worth of action \( i \) can be computed directly. The vector \( W \) is the expected worth for all actions in the set. The rule of maximum expected worth selects the action \( i \) associated with maximum \( W_i \).

2.4 FUZZY DECISION MAKING APPROACH [1,16,17]

In long-range planning, since the knowledge of the system events is not precisely known and the utilities associated with different alternatives are also not known
precisely. Therefore, no alternative can be considered as the best or the optimal. The decision analysis therefore requires a fuzzy set of alternatives 'A' be found representing the relative merits of all alternatives by the grades of membership. A simple fuzzy decision is illustrated by the payoff Table 2.1 in the following section.

**TABLE 2.1 PAYOFF TABLE FOR EXAMPLE FUZZY DECISION SITUATION**

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Events</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>e1</td>
</tr>
<tr>
<td>a1</td>
<td>2</td>
</tr>
<tr>
<td>a2</td>
<td>6</td>
</tr>
<tr>
<td>a3</td>
<td>3</td>
</tr>
</tbody>
</table>

The evaluated outcomes shown here numerically, are the quantitative indices of performance (cost or profit).

**2.4.1 Fuzzy concepts applied to existing decision making methods**

Let S represent the set of all possible events over a time period and A represent the set of proposed projects' alternatives. Associated with each alternative and event there is a payoff value (Table 2.1) $P_{ij}$. Over the proposed planning time horizon, the events are not known precisely. The following Table 2.2 shows the events and their corresponding grades of membership. The grades of membership are subjectively assigned.
The fuzzy decision analysis is outlined in this section. The possible events over a planning period is represented by the following fuzzy set $S_1$:

$$S_1=\{(1,0.25),(2,0.50),(3,0.80),(4,0.5)\} \quad (2.14)$$

The elements of the payoff matrix are awarded with grades of membership based on the subjective judgement of the events. The fuzzy payoff values associated with each alternative are

$$P_1 = \{(0.25,2),(0.5,5),(0.8,1),(0.5,3)\} \quad (2.15)$$
$$P_2 = \{(0.25,6),(0.5,4),(0.8,2),(0.5,0)\} \quad (2.15)$$
$$P_3 = \{(0.25,3),(0.5,1),(0.8,5),(0.5,2)\}$$

The set of possible payoff values is given as,

$$X = [0,1,2,3,4,5,6]$$

$$P_{\text{max}} = \text{SUP}_x(X) = 6 \quad (2.16)$$
The maximizing sets for various alternatives are determined as follows:

Each payoff value is divided by $P_{\text{max}}$ and the result is the grade of membership of the actual payoff value;

that is,
\[
X_{M_1} = [(2/6, 2), (5/6, 5), (1/6, 1), (3/6, 3)] \\
X_{M_2} = [(0.33, 2), (0.83, 5), (0.17, 1), (0.5, 3)] \\
X_{M_3} = [(1.0, 6), (0.67, 4), (0.33, 2), (0, 0)] \quad (2.17) \\
X_{M_4} = [(0.5, 3), (0.17, 1), (0.83, 5), (0.33, 2)]
\]

Next the fuzzy payoff (decision making) sets are obtained as follows:

\[
P_{S_1} = f_{P_1} \land f_{X_{M_1}} \quad (2.18)
\]

where $\land$ is minimum operator.

$f_{P_1}$ and $f_{X_{M_1}}$ are grade of membership functions of $P_1$ and $X_{M_1}$ respectively;

that is,
\[
P_{S_1} = [(0.25, 2), (0.5, 5), (0.17, 1), (0.5, 3)] \\
P_{S_2} = [(0.25, 6), (0.5, 4), (0.33, 2), (0, 0)] \quad (2.19) \\
P_{S_3} = [(0.25, 3), (0.17, 1), (0.8, 5), (0.33, 2)]
\]

The highest grade of membership for each alternative is determined as follows:

\[
H_{f_{a_1}} = \vee(0.25, 0.5, 0.17, 0.5) = 0.5 \\
H_{f_{a_2}} = \vee(0.25, 0.5, 0.33, 0) = 0.5 \quad (2.20) \\
H_{f_{a_3}} = \vee(0.25, 0.17, 0.8, 0.33) = 0.8
\]
Alternative 3 has the highest grade of membership, therefore it is the optimal alternative for the given sample decision making situation (Payoff Table 2.2).

Therefore the fuzzy set of alternatives is given by,

\[ A = [0.5|a1,0.5|a2,0.8|a3] \] (2.21)

This fuzzy set of alternatives indicates the relative merits of all the alternatives.

From equation (2.19), the lowest grade of membership for each alternative is determined as given below:

\[ \text{Lf}_a = \bigvee [0.25,0.5,0.17,0.5] = 0.17 \]
\[ \text{Lf}_a = \bigvee [0.25,0.25,0.33,0] = 0 \] (2.22)
\[ \text{Lf}_a = \bigvee [0.25,0.25,0.8,0.33] = 0.17 \]

The Hurwicz values for \( \alpha = 0.5 \) can be determined as,

\[ \text{HV}_i = \alpha \text{Hf}_a + (1-\alpha) \text{Lf}_a \] (2.23)

\[ = 0.5 *[0.5,0.5,0.8] - 0.5 *[0.17,0,0.17] \]
\[ = [0.25,0.25,0.4] + [0.85,0,0.85] \]

According to algebraic sum of two fuzzy sets,

\[ \text{HV}_i = [0.3134,0.25,0.451] \]
The largest Hurwicz value is 0.451 and hence $a_3$ is the best alternative which has been already chosen in equation (2.21). Therefore, the optimal alternative for the given sample payoff matrix is $a_3$.

From equation (2.19), the fuzzy regret matrix is computed as follows:

Regret is found by subtracting each grade of membership from the largest value in its column. Fuzzy regret matrix is shown in the following Table 2.3.

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Events</th>
<th>$	ext{e}_1$</th>
<th>$	ext{e}_2$</th>
<th>$	ext{e}_3$</th>
<th>$	ext{e}_4$</th>
<th>max$_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.63</td>
<td>0.0</td>
<td>0.63</td>
<td></td>
</tr>
<tr>
<td>a2</td>
<td>0.0</td>
<td>0.0</td>
<td>0.47</td>
<td>0.5</td>
<td>0.50</td>
<td></td>
</tr>
<tr>
<td>a3</td>
<td>0.0</td>
<td>0.33</td>
<td>0.00</td>
<td>0.17</td>
<td>0.33</td>
<td>min$_i$</td>
</tr>
</tbody>
</table>

From the above Table 2.3, the maximum fuzzy regret for each action is found and the alternative with the minimum value is chosen. The alternative $a_3$ is chosen as the best one.

In the Worth Expectation criterion, the fuzzy belief vector is nothing but the subjective judgement of the events. Therefore the fuzzy belief vector is given as:

$$g_i = [0.25, 0.5, 0.8, 0.5]$$
The expected worth for each alternative \( a_i \) is shown in the following Table 2.4.

**TABLE 2.4 FUZZY WORTH EXPECTATION**

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>( e_1 )</th>
<th>( e_2 )</th>
<th>( e_3 )</th>
<th>( e_4 )</th>
<th>Expected worth</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_1 )</td>
<td>2</td>
<td>5</td>
<td>1</td>
<td>3</td>
<td>5.3</td>
</tr>
<tr>
<td>( a_2 )</td>
<td>6</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>5.1</td>
</tr>
<tr>
<td>( a_3 )</td>
<td>3</td>
<td>1</td>
<td>5</td>
<td>2</td>
<td>6.25 ( \text{max} )</td>
</tr>
</tbody>
</table>

For the given fuzzy belief set, alternative \( a_3 \) is preferred as shown in the above Table 2.4.

**EXAMPLE 2.3**

S.B. Dhar [1] explained the fuzzy algorithm for decision analysis through a sample interconnection project between two power systems. The same example is considered here to illustrate the application of fuzzy concepts to statistical decision making methods. Cost criteria is alone considered for illustration. Figure 2.1 shows the hypothetical interconnection plans between two power systems.

S.B Dhar considered the following criteria while developing the different feasible alternatives for interconnection.
FIGURE 2.1 HYPOTHETICAL INTERCONNECTION PLANS
a) Technology must hold promise for submarine cable implementation of large capacity.

b) It must offer an economically acceptable solution.

c) Each generating station must be tied to the transmission system via at least two lines.

d) During any single outage, the total power must be transmitted without overloading the remaining equipment.

The following feasible alternatives are considered for the interconnection between two power systems.

Plan A:

1) HVDC submarine cables only

2) HVAC submarine cables only

Plan B:

3) Partly HVDC submarine cables + Partly HVDC overhead cables

4) Partly HVAC submarine cables + Partly HVAC overhead cables

The power system in the horizon year may be in any one of the L states where L is the number of possible states. The system states represent all possible system
conditions in the horizon year which may have either positive or negative impacts on the proposed project. For simplicity, eleven possible system states are shown in Table 2.5. The objective is to find the optimal alternative of the project for the power system under consideration.

The utility matrix based on capital investment costs for the interconnection project is given in Table 2.6. The elements of the utility matrix are awarded based on subjective judgement of the system state, alternative considered and criteria used. A weighting value of 1 to 10 is used.

The fuzzy utilities associated with each alternative are,

\[
P_1 = [ (0.25,5), (0.50,4), (0.80,2), (0.50,1), \\
       (0.25,1), (0.30,2), (0.40,3), (0.70,6), \\
       (0.30,1), (0.70,1), (0.30,7) ]
\]

\[
P_2 = [ (0.25,5), (0.50,4), (0.80,2), (0.50,1), \\
       (0.25,1), (0.30,2), (0.40,3), (0.70,6), \\
       (0.30,1), (0.70,1), (0.30,7) ]
\]

\[
P_3 = [ (0.25,9), (0.50,8), (0.80,7), (0.50,2), \\
       (0.25,1), (0.30,9), (0.40,8), (0.70,10), \\
       (0.30,2), (0.70,3), (0.30,10) ]
\]

\[
P_4 = [ (0.25,10), (0.50,9), (0.80,8), (0.50,3), \\
       (0.25,2), (0.30,10), (0.40,9), (0.70,10), \\
       (0.30,2), (0.70,4), (0.30,10) ]
\]
### TABLE 2.5 HORIZON YEAR SYSTEM STATE TABLE

<table>
<thead>
<tr>
<th>Possible system states</th>
<th>Semantic description</th>
<th>Grades of membership</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Very low reserve</td>
<td>0.25</td>
</tr>
<tr>
<td>2</td>
<td>low reserve</td>
<td>0.50</td>
</tr>
<tr>
<td>3</td>
<td>usual reserve</td>
<td>0.80</td>
</tr>
<tr>
<td>4</td>
<td>high reserve</td>
<td>0.50</td>
</tr>
<tr>
<td>5</td>
<td>very high reserve</td>
<td>0.25</td>
</tr>
<tr>
<td>6</td>
<td>outage of base load units</td>
<td>0.30</td>
</tr>
<tr>
<td>7</td>
<td>outage of major transmission circuits</td>
<td>0.40</td>
</tr>
<tr>
<td>8</td>
<td>high production cost at system A</td>
<td>0.70</td>
</tr>
<tr>
<td>9</td>
<td>low production cost at system A</td>
<td>0.30</td>
</tr>
<tr>
<td>10</td>
<td>no restriction of new generation site in system A</td>
<td>0.70</td>
</tr>
<tr>
<td>11</td>
<td>restriction of new generation site in system A</td>
<td>0.30</td>
</tr>
</tbody>
</table>

### TABLE 2.6 UTILITY MATRIX FOR CAPITAL INVESTMENT COSTS

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>System states</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>5</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>6</td>
<td>1</td>
<td>1</td>
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</tbody>
</table>
The set of possible payoff values is given as,

\[ X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \]

\[ \text{P}_{\text{max}} = \text{SUP}_X(X) = 10 \]

The maximizing sets for various alternatives are,

\[ X_{M1} = [(0.50, 5), (0.40, 4), (0.20, 2), (0.10, 1),
            (0.10, 1), (0.20, 2), (0.30, 3), (0.60, 6),
            (0.10, 1), (0.10, 1), (0.70, 7)] \]

\[ X_{M2} = [(0.50, 5), (0.40, 4), (0.20, 2), (0.10, 1),
            (0.10, 1), (0.20, 2), (0.30, 3), (0.60, 6),
            (0.10, 1), (0.10, 1), (0.70, 7)] \]

\[ X_{M3} = [(0.90, 9), (0.80, 8), (0.70, 7), (0.20, 2),
            (0.10, 1), (0.90, 9), (0.80, 8), (1.00, 10),
            (0.20, 2), (0.30, 3), (1.00, 10)] \]

\[ X_{M4} = [(1.00, 10), (0.90, 9), (0.80, 8), (0.30, 3),
            (0.20, 2), (1.00, 10), (0.90, 9), (1.00, 10),
            (0.20, 2), (0.40, 4), (1.00, 10)] \]

Fuzzy utility sets are obtained using equation (2.18).

\[ \text{PS}_1 = [(0.25, 5), (0.40, 4), (0.20, 2), (0.10, 1),
                (0.10, 1), (0.20, 2), (0.30, 3), (0.60, 6),
                (0.10, 1), (0.10, 1), (0.30, 7)] \]

\[ \text{PS}_2 = [(0.25, 5), (0.40, 4), (0.20, 2), (0.10, 1),
                (0.10, 1), (0.20, 2), (0.30, 3), (0.60, 6),
                (0.10, 1), (0.10, 1), (0.30, 7)] \]
The highest grade of membership for each alternative is determined as follows:

\[ H_{f_{a1}} = \vee (0.25,0.40,0.20,0.10,0.10,0.20,0.30,0.60,0.10,0.10,0.30) \]

\[ = 0.60 \]

\[ H_{f_{a2}} = \vee (0.25,0.40,0.20,0.10,0.10,0.20,0.30,0.60,0.10,0.10,0.30) \]

\[ = 0.60 \]

\[ H_{f_{a3}} = \vee (0.25,0.50,0.70,0.20,0.10,0.30,0.40,0.70,0.20,0.30,0.30) \]

\[ = 0.70 \]

\[ H_{f_{a4}} = \vee (0.25,0.50,0.80,0.30,0.20,0.30,0.40,0.70,0.20,0.40,0.30) \]

\[ = 0.80 \]

Alternative 4 has the highest grade of membership, therefore it is the optimal alternative for the
interconnection project considered based on the capital investment costs.

From fuzzy utility sets, the lowest grade of membership for each alternative is determined as follows:

$L_{f_{a1}} = \cap (0.25, 0.40, 0.20, 0.10, 0.10, 0.20, 0.30, 0.60, 0.10, 0.10, 0.30) = 0.10$

$L_{f_{a2}} = \cap (0.25, 0.40, 0.20, 0.10, 0.10, 0.20, 0.30, 0.60, 0.10, 0.10, 0.30) = 0.10$

$L_{f_{a3}} = \cap (0.25, 0.50, 0.70, 0.20, 0.10, 0.30, 0.40, 0.70, 0.20, 0.30, 0.30) = 0.10$

$L_{f_{a4}} = \cap (0.25, 0.50, 0.80, 0.30, 0.20, 0.30, 0.40, 0.70, 0.20, 0.40, 0.30) = 0.20$

The Hurwicz values for $\alpha = 0.5$, are determined from equation (2.23).

$HV_1 = 0.5*[0.6, 0.6, 0.7, 0.8] - 0.5*[0.1, 0.1, 0.1, 0.2]$
The largest Hurwicz value is 0.460 and hence $a_4$ is the best alternative.

Table 2.7. shows the fuzzy regret matrix that can be constructed from the fuzzy utility sets derived for the interconnection project. The maximum fuzzy regret for each alternative is found and the alternative with the minimum value is chosen as the best one.

The fuzzy belief vector is given as,

$$g_i = [0.25, 0.50, 0.80, 0.50, 0.25, 0.30, 0.40, 0.70, 0.30, 0.70, 0.30]$$

The expected worth for each alternative can be estimated from equation (2.13) and is shown in Table 2.8. Alternative $a_4$ is preferred in all cases.

2.5 CONCLUSION

Fuzzy concepts are applied to various statistical decision making methods and unique results are obtained. This chapter also describes a technique for long-range planning decision analysis under a fuzzy environment. A single attribute (capital investment cost) is considered and a fuzzy algorithm is explained to choose a best alternative. This method can be extended for multiple attributes [1] like operating cost, maintenance cost and overall best alternative can be chosen. For each attribute, there is a
<table>
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<tr>
<th>alternatives</th>
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set of fuzzy optimal alternatives. A new fuzzy decision set has to be formed by combining all optimal alternatives of all attributes. This set contains the maximum grade of membership of each fuzzy alternative set. From this new fuzzy decision set, the alternative which corresponds to the maximum grade of membership has to be selected. All methods are tested with the grades of membership instead of with actual payoff values and the same result is obtained. A FORTRAN program has been developed for the above explained fuzzy algorithm. The program reads the Payoff matrix values and the subjective grades of membership of events which are expected to occur over a period of time. It returns the best alternative as its output.

A number of decision making problems have been tried with these methods and good results have been obtained. In chapter 4, the concepts of fuzzy decision making approach, fuzzy regret matrix, fuzzy Hurwicz rule and fuzzy worth expectation criteria have been applied to a practical power system planning problem.