3.1 INTRODUCTION

Planning and operation of complex systems involve many uncertainties. Many probabilistic concepts are available for complex system modelling problems. All uncertainties are treated in a probabilistic way. There are some statements in complex systems modelling problem which are not treated or viewed in a probabilistic way. A power system example is considered because of its high complexity due to the uncertain operation of the system. For example, a statement such as 'the available reserve capacity is very large and the present demand is average' used in the operation of power systems, has much imprecision. Subjective informations are used to characterize the above statement. The terms 'large' and 'average' are called linguistic values. These subjective informations are well modelled by fuzzy sets. As an illustration, the fuzzy concepts are applied here to measure the generation outages of a power system. The main objective of this chapter is to explain how power engineers can apply the fuzzy set concepts in power generation system problems.

Fuzzy set theory is a useful tool for the expression of professional judgements. These judgements appear as verbal statements viz., the system is in the moderate outage state, the quality control is adequate. Our ability to make precise and significant statements concerning a given system diminishes with increasing complexity of the system. The
closer one looks at the real world problem which is usually complex, the fuzzier the manner of solution becomes. The complex real-world problem can be understood by asking a sequence of simple questions. These questions can best be answered by experienced engineers with descriptive words or phrases which are said to be the values of a given linguistic variable. The theory of fuzzy sets is a useful tool with which such words and phrases can be interpreted with the use of the membership functions. These express numerically the meaning of the linguistic variables. The construction of the membership function can be accomplished with the co-operation and assistance of a panel of experienced engineers in specific cases. The resulting membership functions for answers to various simpler questions can then be manipulated in a logical manner following the theory of fuzzy sets to obtain a meaningful answer to the originally complex problem.

3.2 FURTHER OPERATIONS ON FUZZY SETS [18,19,20]

Some basic operations on fuzzy sets were discussed in Chapter 2. Fuzzy relation and composition are discussed here. Examples are shown in the forthcoming sections.

1. A fuzzy relation \( R \) indicates the support for the ordered pairs obtained from fuzzy sets

\[
A = \sum_i \mu_A(x_i) \mid x_i
\]

and

\[
B = \sum_j \mu_B(y_j) \mid y_j
\]
This relation is defined by,

\[ R = A \times B = \sum_i \sum_j Z_{Ej} \mu_R(x_i, y_j) \quad \text{(3.1)} \]

in which \( \mu_R(x_i, y_j) = \min \{ \mu_A(x_i), \mu_B(y_j) \} \)

The ordered pairs are the members \((x, y)\) and their support is the minimum of either the support of \(x\), namely \(\mu_A(x)\) or the support of \(y\), \(\mu_B(y)\).

2. The conditional expression \(R = A \times B\) is a fuzzy relation between \(A\) and \(B\) where \(A\) is a fuzzy subset of the universe \(U = \sum_i 1|x_i\) and \(B\) of \(V = \sum_j 1|y_j\). Fuzzy composition defines what fuzzy subset \(y\) of \(V\) is induced by \(x\), a fuzzy subset of \(U\). A fuzzy composition is defined by,

\[ y = x \cdot R \quad \text{(3.2)} \]

where each grade of membership is obtained from

\[ \mu_y = \bigvee [\mu_A(y) \land \mu_R(x, y)] \]

The operation in the above equation (3.2) is very similar to matrix multiplication.

3.3 APPLICATION OF FUZZY SETS IN POWER GENERATION SYSTEMS

The application of basic operations of fuzzy sets in power generation system problems is explained through an example.
If the generation capacity available is very large, it is expressed as the fuzzy set $A$,

$$A = 1|1 + 0.7|0.9 + 0.4|0.8 + 0.1|0.7$$

and if the load demand is average, it is expressed as the fuzzy set $B$,

$$B = 0.2|0.7 + 0.8|0.6 + 1|0.5 + 0.8|0.4 + 0.2|0.3$$

The fuzzy set operations are shown in the Table 3.1.

**TABLE 3.1 ILLUSTRATION OF FUZZY BASIC OPERATIONS**

<table>
<thead>
<tr>
<th>Level of reserve margin</th>
<th>Capacity available A</th>
<th>Demand B</th>
<th>$A \cup B$</th>
<th>$A \cap B$</th>
<th>$\bar{A}$</th>
<th>$\bar{B}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0.9</td>
<td>0.7</td>
<td>0</td>
<td>0.7</td>
<td>0</td>
<td>0.3</td>
<td>1</td>
</tr>
<tr>
<td>0.8</td>
<td>0.4</td>
<td>0</td>
<td>0.4</td>
<td>0</td>
<td>0.6</td>
<td>1</td>
</tr>
<tr>
<td>0.7</td>
<td>0.1</td>
<td>0.2</td>
<td>0.2</td>
<td>0.1</td>
<td>0.9</td>
<td>0.8</td>
</tr>
<tr>
<td>0.6</td>
<td>0</td>
<td>0.8</td>
<td>0.8</td>
<td>0</td>
<td>0</td>
<td>0.2</td>
</tr>
<tr>
<td>0.5</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.4</td>
<td>0</td>
<td>0.8</td>
<td>0.8</td>
<td>0</td>
<td>0</td>
<td>0.2</td>
</tr>
<tr>
<td>0.3</td>
<td>0</td>
<td>0.2</td>
<td>0.2</td>
<td>0</td>
<td>0</td>
<td>0.8</td>
</tr>
<tr>
<td>0.2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0.1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>
3.3.1 Fuzzy relation between available capacity and demand satisfaction \cite{6,21}

Consider the Fuzzy subsets A and B.

Let \[ A = \sum_i \mu_A(x_i) / x_i \] and \[ B = \sum_j \mu_B(y_j) / y_j \]

As an example of fuzzy relation, consider the generation capacity available to meet the required demand. Assume the capacity available is low and the demand is met to some extent and can be represented as the following fuzzy sets.

\[
A = 1|0 + 0.9|0.1 + 0.5|0.2 + 0.2|0.3 + 0.1|0.4
\]

\[
B = 1|0 + 0.8|0.1 + 0.6|0.2 + 0.2|0.3
\]

The fuzzy relation between these two fuzzy sets is given by equation (3.1)

\[ R = A \times B \]

The grade of membership \( \mu_R(x_i, y_j) \) can be evaluated as follows:

\[
\mu_R(0,0) = \mu_A(0) \land \mu_B(0) = \min(1,1) = 1
\]
\[
\mu_R(0,0.1) = \mu_A(0) \land \mu_B(0.1) = \min(1,0.8) = 0.8
\]
\[
\mu_R(0,0.2) = \mu_A(0) \land \mu_B(0.2) = \min(1,0.6) = 0.6
\]
\[
\mu_R(0,0.3) = \mu_A(0) \land \mu_B(0.3) = \min(1,0.2) = 0.2
\]

where \( \land \) is the minimum operator.
These grades of membership form the first row in the tabular representation of such fuzzy relation $R$ and is shown in Table 3.2.

TABLE 3.2 FUZZY RELATION BETWEEN AVAILABLE CAPACITY AND DEMAND SATISFACTION

<table>
<thead>
<tr>
<th>Low capacity available</th>
<th>Demand able to meet to some extent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.0</td>
</tr>
<tr>
<td>0.0</td>
<td>1.0</td>
</tr>
<tr>
<td>0.1</td>
<td>0.9</td>
</tr>
<tr>
<td>0.2</td>
<td>0.5</td>
</tr>
<tr>
<td>0.3</td>
<td>0.2</td>
</tr>
<tr>
<td>0.4</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Addition in the available capacity can be expressed as a fuzzy set,

$$A' = 0.5|0 + 0.6|0.1 + 1|0.2 + 0|0.3 + 0|0.4$$

Then the expectation of the demand being satisfied is given by the fuzzy composition (equation 3.2),

$$B' = A' \cdot R$$

whose grades of membership are obtained as follows:

$$\mu_{B'} = \max [\min(\mu_{A'}, \mu_R)]$$
The first element of the above vector is obtained as follows:

\[
\max[\min(0.5,1),\min(0.6,0.9),\min(1,0.5),\min(0,0.2),\min(0,0.1)]
\]

\[
= \max [ 0.5, 0.6, 0.5, 0, 0 ]
\]

\[
= 0.6
\]

The fuzzy set of interest \( B' \) that is the expectation of demand being satisfied is given as follows:

\[
B' = 0.6|0 + 0.6|0.1 + 0.6|0.2 + 0.2|0.3
\]

In this way, a known fuzzy relation between two opinions serves as a basis for deciding on the unknown characteristic when the opinion on the other is available.

### 3.3.2 Fuzzy measure of generation system outages

Two types of data are available from the inspection and testing of the generating system. One type of observation made from within the system is the outage of subsystems. Such information can be incorporated in a
logical manner to obtain an estimate of the outage state of the whole generating system. The other type of data are obtained from environmental conditions.

Let $X$ denote the event that causes the whole generation system in a severe outage state and $x_i$ denote the severe outage state of the system using $i$th group of data. For example, let $i = 1$ correspond to the information on component outages, and $i = 2$ correspond to the outages due to the environmental conditions. Therefore for $n$ groups of data,

$$X = \bigcup_{i=1}^{n} x_i$$

where $U$ is the union operator.

or in terms of grade of membership function,

$$\mu_X = \bigvee_i (\mu_{x_i})$$

where $\bigvee$ is the maximum operator.

There are $m$ components in the generating system. Let $Y_{ij}$ denote the severe outage state of the $j$th component due to $i$th group of data. Then $x_i$ can be considered as the algebraic sum of the outage of each component. that is,

$$x_i = \sum_{j=1}^{m} Y_{ij}$$

whose grade of membership function is given by,

$$\mu_{x_i} = 1 - \prod_{j=1}^{m} (1 - \mu_{Y_{ij}})$$

Let $x_1$ denote the severe outage state of the system caused by component outages and $x_2$ denote the severe outage
state of the system due to environmental conditions. There are two major components which are detected in the failure state and their respective grades of membership will be $\mu_1 = 0.8$ and $\mu_2 = 0.6$, then

$$\mu_1 = 0.92$$

By experience, it has been stated that 20 percent of the time, the whole system operation is affected by environmental changes. The truth (grade of membership) value for the above statement is 0.75 as obtained from the established membership function which has been developed by the advice from various experts in this field.

Therefore $\mu_2 = 0.75$

Then, the grade of membership for the total generation system in the severe outage state is given as,

$$\mu_X = \max(\mu_1, \mu_2) = 0.92$$

In this way, the subjective information that we have obtained from complex power systems can be numerically quantified by fuzzy set theory.

3.4 FUZZY PROBABILISTIC MEASURES - BACKGROUND [22]

In probability theory, an event $A$, is a member of a sample space $S$. The notions of an event and its probability constitute the most basic concepts of probability theory.
An event is a precisely specified collection of points in the sample space. By contrast, in everyday experience one frequently encounters situations in which an 'event' is a fuzzy, rather than a sharply defined collection of points. For example, the ill-defined events: 'the power generation system is in the high reserve capacity state during early morning hours', 'most of the time the hydel unit is in the good state' are fuzzy because of the imprecision of the meaning of the words like high reserve capacity and good state.

The main objective is to show how the notion of a fuzzy event can be given a precise meaning in the context of fuzzy sets.

By using the concept of a fuzzy set, the notions of an event and its probability can be extended in a natural fashion to fuzzy events. It is possible that such an extension may eventually significantly enlarge the domain of applicability of probability theory, especially in those fields in which fuzziness is a pervasive phenomenon.

Let $S$ be a real linear space $\mathbb{R}^n$. A fuzzy event $A$ in $\mathbb{R}^n$ is a fuzzy set in $\mathbb{R}^n$. The probability of a fuzzy event $A$ in $\mathbb{R}^n$ with membership function $\mu_A(x)$ is defined by

$$P(A) = \int_{\mathbb{R}^n} \mu_A(x) dP (3.3)$$

In other words, the probability of a fuzzy event is the expectation $E(\mu_A)$ of its membership function.
Let $A$ and $B$ be two fuzzy events in $\mathbb{R}^n$, then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad (3.4)$$

$$P(A + B) = P(A) + P(B) - P(AB) \quad (3.5)$$

The two fuzzy events $A$ and $B$ are said to be statistically independent if

$$P(A|B) = P(A)P(B) \quad (3.6)$$

The conditional probability of two fuzzy events is defined by,

$$P(A|B) = P(AB)/P(B) \quad (3.7)$$

If $A$ and $B$ are statistically independent,

then $P(A|B) = P(A)$

and $P(B|A) = P(B) \quad (3.8)$

### 3.5 Fuzzy Measure of Capacity Outages

Capacity outages in power generation systems can be given subjective values like large capacity outage, medium and low capacity outages etc. These classifications are meaningful but are not clearly defined. The theory of fuzzy sets can be used to interpret such adjectives with membership functions. The assignment of the membership function of a fuzzy set is subjective in nature and in
general reflects the context in which the problem is viewed. Although the assignment of the membership function of a fuzzy set is subjective, it cannot be assigned arbitrarily. The construction of a membership function can be accomplished with the co-operation and assistance of a panel of experienced engineers in specific cases.

Let \( y \) denote the capacity outage proportion (in MWs) of a power system. If the total capacity of the whole generating system is 1000 MW, the large capacity outage state of the system can be written as a fuzzy set \( X \),

\[
X = \{ (0, y < 100 \text{ MW} ), \\
(2(y-100)/1000, 100 \leq y \leq 600 \text{ MW} ), \\
(1, y > 600 \text{ MW} ) \}
\]

whose grade of membership function is given as,

\[
\mu(y) = \begin{cases} 
0 & y < 100 \text{ MW} \\
2(y-100)/1000 & 100 \leq y \leq 600 \text{ MW} \\
1 & y > 600 \text{ MW}
\end{cases} \quad (3.9)
\]

The large capacity outage state is said to have occurred when 600 MW unit has failed or when two or more low capacity units have failed whose total capacity exceeds 600 MW. When large capacity outage state is considered, the grade of membership is given as 1 for the capacity outages of 600 MW or more. When the capacity outage is around 300 MW, its grade of membership is given as,

\[
2(300-100)/1000 = 0.4
\]
Let $D_i$ denote the large capacity outage state of the system with membership function $\mu_{D_i}$. The grade of membership function is given as in equation (3.9). Let $Q$ represent an overall outage state with membership function $\mu_Q$. Consider the following 4 units with capacities,

\[ y_1 = 50 \text{ MW}, \quad y_2 = 500 \text{ MW}, \quad y_3 = 400 \text{ MW} \quad \text{and} \quad y_4 = 80 \text{ MW} \]

If these four units have failed, their respective grades of membership obtained from (3.9) are as follows:

\[ \mu_D^1 = 0; \quad \mu_D^2 = 0.8; \quad \mu_D^3 = 0.6; \quad \mu_D^4 = 0. \]

If these outages of units have occurred with large time duration between each unit's outage, then we may say that

\[ Q_1 = \bigcup_{i=1}^{4} D_i \quad \text{(3.10)} \]

where $\bigcup$ represents the union operation.

The grade of membership is given as

\[ \mu_Q = \max (\mu_D^1, \mu_D^2, \mu_D^3, \mu_D^4 ) \]

\[ = \max (0.0, 0.8, 0.6, 0.0) \]

\[ = 0.8 \]

If these outages have occurred with less time duration between each unit's outage, the outage state is represented as follows:
whose membership function is given as

\[ \mu_{Q_2} = 1 - \prod_{i=1}^{4} [1 - \mu_{D_i}] \]

\[ = 1 - (1-0)(1-0.8)(1-0.6)(1-0) \]

\[ = 0.92 \]

\( \mu_Q \) and \( \mu_{Q_2} \) represent the minimum and maximum bounds for the outage state considered (\( \mu_Q \)). Hence,

\[ \mu_{Q_1} \leq \mu_Q \leq \mu_{Q_2} \]

While considering a overall system performance during a time period, the outages of these four units make a strong membership to the large capacity outage category.

### 3.6 State Transition Matrix for Reserve Capacity Variations

Generators experience frequent load changes. The load may vary from a minimum load to a peak load in a day itself. A typical daily load curve is shown in Figure 3.1. The Table 3.3 shows the various reserve capacity states that may occur during a day and their representations. The state 1 (i.e. very low reserve capacity state) represents the very high demand and state 5 represents the very low demand. The sample space \( S \) is represented by the 5 states shown in Table 3.3.

\[ S = \{ VL, L, M, H, VH \} \]
<table>
<thead>
<tr>
<th>State</th>
<th>Subjective description of reserve capacity states</th>
<th>Representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Very Low</td>
<td>VL</td>
</tr>
<tr>
<td>2</td>
<td>Low</td>
<td>L</td>
</tr>
<tr>
<td>3</td>
<td>Medium</td>
<td>M</td>
</tr>
<tr>
<td>4</td>
<td>High</td>
<td>H</td>
</tr>
<tr>
<td>5</td>
<td>Very High</td>
<td>VH</td>
</tr>
</tbody>
</table>
The state transition matrix for the various reserve capacity states is given as follows:

<table>
<thead>
<tr>
<th></th>
<th>VL</th>
<th>L</th>
<th>M</th>
<th>H</th>
<th>VH</th>
</tr>
</thead>
<tbody>
<tr>
<td>VL</td>
<td>1/2</td>
<td>1/4</td>
<td>1/8</td>
<td>1/16</td>
<td>1/16</td>
</tr>
<tr>
<td>L</td>
<td>1/8</td>
<td>5/8</td>
<td>1/8</td>
<td>1/16</td>
<td>1/16</td>
</tr>
<tr>
<td>M</td>
<td>3/16</td>
<td>3/16</td>
<td>7/16</td>
<td>2/16</td>
<td>1/16</td>
</tr>
<tr>
<td>H</td>
<td>1/8</td>
<td>1/8</td>
<td>1/4</td>
<td>1/4</td>
<td>1/4</td>
</tr>
<tr>
<td>VH</td>
<td>1/8</td>
<td>3/16</td>
<td>6/16</td>
<td>4/16</td>
<td>1/16</td>
</tr>
</tbody>
</table>

From the daily load curve shown in Figure 3.1, it can be seen that the peaking units are switched on after 3 P.M. Eventhough the customers are satisfied with the required demand at very low reserve capacity state (VL), this state is considered to be a very cautious one because of the possible random outages of the units.

When around 50 percent of the capacity is in reserve and the operating cost is low (peaking units are not switched on during 7 A.M. to 11 A.M.), it is said to be in the medium reserve capacity state and is considered to be a good state.

Suppose a fuzzy event A is defined as a medium reserve capacity state in S, then the membership function of this state is subjectively defined by

\[ \mu_A(VL) = 0.02; \quad \mu_A(L) = 0.2; \quad \mu_A(M) = 1; \quad \mu_A(H) = 0.5; \quad \mu_A(VH) = 0.1 \]

The elements of the first row of the state transition matrix are the probabilities for various kinds of
reserve capacity states following a very low reserve capacity state. At two units of time after the very low reserve capacity state, the probabilities for the various kinds of reserve capacity states are calculated as

\[
\begin{bmatrix}
0.5 & 0.25 & 0.125 & 0.0625 & 0.0625 \\
0.25 & 0.5 & 0.25 & 0.125 & 0.0625 \\
0.125 & 0.25 & 0.5 & 0.25 & 0.125 \\
0.0625 & 0.125 & 0.25 & 0.5 & 0.25 \\
0.0625 & 0.0625 & 0.125 & 0.25 & 0.5 \\
\end{bmatrix}
\]

Therefore, at two units of time after the very low reserve capacity state, the probability of having a good state (medium reserve capacity state) is

\[
P(A) = \sum_{i=1}^{5} \mu_A(x_i) p(x_i) = 0.02 \times 41/128 + 0.2 \times 83/256 + 1.0 \times 24/128 + 0.5 \times 12/128 + 0.1 \times 19/256
\]

\[
= 0.02 \times 41/128 + 0.2 \times 83/256 + 1.0 \times 24/128 + 0.5 \times 12/128 + 0.1 \times 19/256
\]

\[
= 0.313
\]

The foregoing power system example is intended merely to demonstrate possible ways of defining some of the concepts of probability theory in a more general setting in which the fuzzy events are allowed [23].
Most of the analytical and simulation modelling of complex systems are based on the assumption that all data inputs required can be determined or assigned uniquely. This cannot be implied with regard to real complex system modelling problems since some of the inputs are not precisely known. If the input data is fuzzy in nature, a range of values around the input can be assumed for modelling the complex system. It is possible to obtain the solution for each small interval in the range and if the considered interval is too small, there are thousands of solutions to the same problem. We can select some particular values in the range like minimum value, maximum value, arithmetic mean of the range, Hurwicz values with optimism-pessimism index $\alpha = 0.5, 0.6, 0.8$, etc., and we can obtain some solutions according to the above inputs. These solutions are called compromise solutions. The solution obtained with minimum value in the range is too pessimistic and the solution obtained with maximum value in the range is too optimistic. The solution obtained with the arithmetic mean is neither optimistic nor pessimistic. The solution obtained with Hurwicz value with $\alpha = 0.6$ or $0.8$ is nearly approaching optimism. We can take the solutions which are nearly approaching optimism as the best solutions.

For example, we can consider the measures of intersystem bonding power [24]. Ten measures of intersystem bonding power $P$ at different times can be assumed. The results of these measurements are given in the Table 3.4.
TABLE 3.4 RANGE OF VALUES FOR INTERSYSTEM BONDING POWER

<table>
<thead>
<tr>
<th>Interval</th>
<th>Range of power P MW</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>335 - 355</td>
</tr>
<tr>
<td>2</td>
<td>290 - 310</td>
</tr>
<tr>
<td>3</td>
<td>305 - 325</td>
</tr>
<tr>
<td>4</td>
<td>365 - 390</td>
</tr>
<tr>
<td>5</td>
<td>385 - 410</td>
</tr>
<tr>
<td>6</td>
<td>315 - 335</td>
</tr>
<tr>
<td>7</td>
<td>360 - 380</td>
</tr>
<tr>
<td>8</td>
<td>300 - 320</td>
</tr>
<tr>
<td>9</td>
<td>375 - 395</td>
</tr>
<tr>
<td>10</td>
<td>330 - 360</td>
</tr>
</tbody>
</table>

The aim is to determine the compromise confidence intervals to estimate the mathematical expectation with a confidence probability $\beta = 0.9$ or more. The particular values chosen in the range in each interval to obtain compromise solutions are shown in Table 3.5.

The sample calculations to find the confidence interval to estimate mathematical expectation with minimum value in the range are shown in the following section.
<table>
<thead>
<tr>
<th>Interval</th>
<th>Minimum MW</th>
<th>Maximum MW</th>
<th>Mean MW</th>
<th>Hurwicz value α=0.6</th>
<th>Hurwicz value α=0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>335</td>
<td>355</td>
<td>345</td>
<td>347</td>
<td>351</td>
</tr>
<tr>
<td>2</td>
<td>290</td>
<td>310</td>
<td>300</td>
<td>302</td>
<td>306</td>
</tr>
<tr>
<td>3</td>
<td>305</td>
<td>325</td>
<td>315</td>
<td>317</td>
<td>321</td>
</tr>
<tr>
<td>4</td>
<td>365</td>
<td>390</td>
<td>377.5</td>
<td>380</td>
<td>385</td>
</tr>
<tr>
<td>5</td>
<td>385</td>
<td>410</td>
<td>397.5</td>
<td>400</td>
<td>405</td>
</tr>
<tr>
<td>6</td>
<td>315</td>
<td>335</td>
<td>325</td>
<td>327</td>
<td>331</td>
</tr>
<tr>
<td>7</td>
<td>360</td>
<td>380</td>
<td>370</td>
<td>372</td>
<td>376</td>
</tr>
<tr>
<td>8</td>
<td>300</td>
<td>320</td>
<td>310</td>
<td>312</td>
<td>316</td>
</tr>
<tr>
<td>9</td>
<td>375</td>
<td>395</td>
<td>385</td>
<td>387</td>
<td>391</td>
</tr>
<tr>
<td>10</td>
<td>330</td>
<td>360</td>
<td>345</td>
<td>348</td>
<td>354</td>
</tr>
</tbody>
</table>
3.7.1 Sample Calculations to estimate Confidence Intervals

Let $P_{\text{min}}$ be a vector that contains all minimum values for each interval.

$$P_{\text{min}} = [335, 290, 305, 365, 385, 315, 360, 300, 375, 330]$$

The statistical estimate of mathematical expectation is given as:

$$M(P_{\text{min}}) = \frac{1}{10} \sum_{i=1}^{10} P_{\text{min}} \approx 336.00 \text{ MW}$$

where $n = \text{sample size} = 10$.

The variance,

$$D(P_{\text{min}}) = \frac{1}{n-1} \sum_{i=1}^{10} (P_{\text{min}} - M(P_{\text{min}}))^2 \approx 1132.22 \text{ MW}^2$$

From $\beta$ distribution table, for $n-1 = 9$ and $\beta = 0.9$, dependability factor $t_\beta = 1.833$

The error factor $\epsilon_\beta = t_\beta \sqrt{D(P_{\text{min}})/n} \approx 19.50$.

The confidence interval to estimate the mathematical expectation is given as:

$$M(P_{\text{min}}) \pm \epsilon_\beta$$

$$= [316.50, 355.50] \text{ MW}.$$
Table 3.6 shows the compromise confidence intervals to estimate the mathematical expectation of intersystem bonding power when the measurements are imprecisely defined or when the measurement values are given as a range.

**TABLE 3.6 COMPROMISE CONFIDENCE INTERVALS**

<table>
<thead>
<tr>
<th>Chosen values in the range</th>
<th>Confidence intervals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum</td>
<td>[316.50, 355.50]</td>
</tr>
<tr>
<td>Maximum</td>
<td>[337.77, 378.23]</td>
</tr>
<tr>
<td>Mean</td>
<td>[327.16, 366.84]</td>
</tr>
<tr>
<td>Hurwicz value:</td>
<td></td>
</tr>
<tr>
<td>$\alpha = 0.6$</td>
<td>[329.28, 369.12]</td>
</tr>
<tr>
<td>$\alpha = 0.8$</td>
<td>[333.53, 373.67]</td>
</tr>
</tbody>
</table>

We can choose the appropriate confidence interval according to the value of intersystem bonding power $P$. That is, if the measured power is 340 MW at a particular time which falls in the ranges 1 and 10 (Table 3.4) and, since this value is around minimum in both ranges, the confidence interval shown against the minimum value (Table 3.6) can be considered.

In general, the confidence intervals calculated from the Hurwicz values ($\alpha = 0.6$ and $\alpha = 0.8$) can be considered as the best solutions.

### 3.8 CONCLUSION

The process of accounting for subjective information and wisdom in power generation engineering decision making is a matter of debate. One effort to include
verbal ideas involves the use of the fuzzy set theory. This chapter mainly considers the application of fuzzy concepts to power generation problems. Two applications are explained.

The first considers the fuzzification of objective information (i.e., expressing the present capacity available, the load demand and improve the capacity available as fuzzy sets and in the second application, subjective information is used to represent the impact of environmental changes in power generation systems.

The application of fuzzy theory allows the inclusion of professional wisdom, knowledge and intuition into an analytical scheme. In this chapter some examples are shown for application of fuzzy concepts to complex power systems. It has been argued that the probability theory is a useful tool in power generation system modelling and planning but its use is limited in the sense that most complex situations that have arisen during power generation system modelling and planning are made with shortage of numerical evidence and depends on informed opinions.

A new procedure has been developed to determine the compromise confidence intervals for the mean of intersystem bonding power $P$. This has been achieved by considering the range of values of $P$ which are measured periodically. In the next chapter, the concept of compromise solutions has been applied to a sample power system planning optimization problem in which the cost coefficients are assumed imprecisely.